Computer Algebra
Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.4-Improper/50-1.2.4.2-d-x-^m-a-x^q+b-x^n+c-x^-2-n-q-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [140]. This is test number [50].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (140)	0.00 (0)
Mathematica	99.29 (139)	0.71 (1)
Maple	97.14 (136)	2.86 (4)
Fricas	96.43 (135)	3.57 (5)
Giac	77.86 (109)	22.14 (31)
Mupad	51.43 (72)	48.57 (68)
Sympy	33.57 (47)	66.43 (93)
Maxima	17.14 (24)	82.86 (116)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

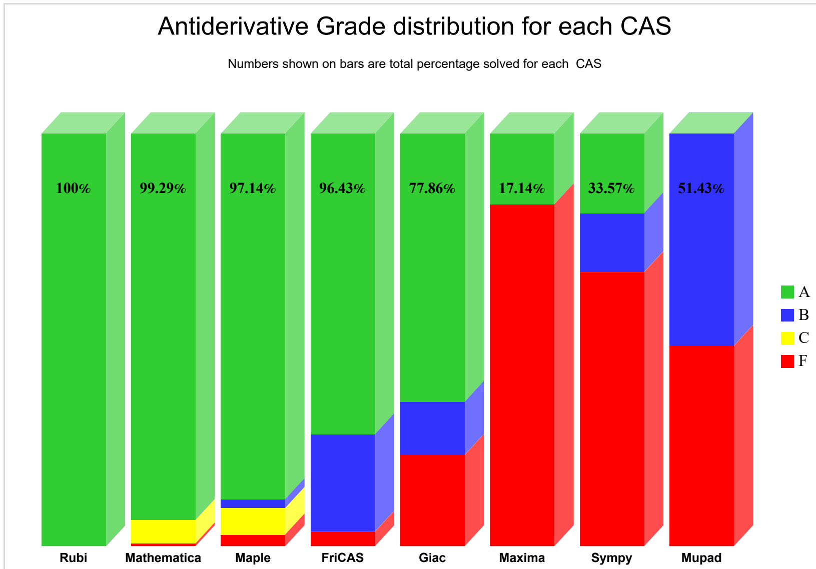
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

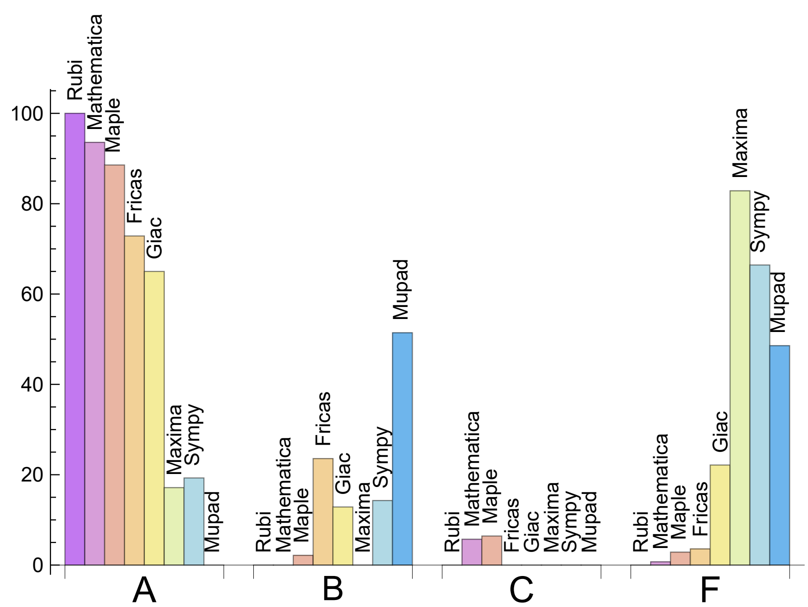
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	93.571	0.000	5.714	0.714
Maple	88.571	2.143	6.429	2.857
Fricas	72.857	23.571	0.000	3.571
Giac	65.000	12.857	0.000	22.143
Sympy	19.286	14.286	0.000	66.429
Maxima	17.143	0.000	0.000	82.857
Mupad	0.000	51.429	0.000	48.571

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	1	100.00	0.00	0.00
Maple	4	100.00	0.00	0.00
Fricas	5	80.00	0.00	20.00
Giac	31	45.16	16.13	38.71
Mupad	68	0.00	100.00	0.00
Sympy	93	72.04	27.96	0.00
Maxima	116	81.90	0.86	17.24

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.13
Maxima	0.19
Maple	0.26
Fricas	0.29
Giac	0.39
Mathematica	0.73
Sympy	1.00
Mupad	4.78

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	32.71	0.81	28.50	0.81
Maple	133.00	0.95	96.00	0.85
Mathematica	136.34	1.02	108.00	0.98
Rubi	139.59	1.00	103.50	1.00
Sympy	199.02	2.69	124.00	0.94
Giac	358.72	2.07	79.00	1.09
Fricas	466.01	2.98	297.00	2.69
Mupad	1266.07	6.91	172.00	2.35

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

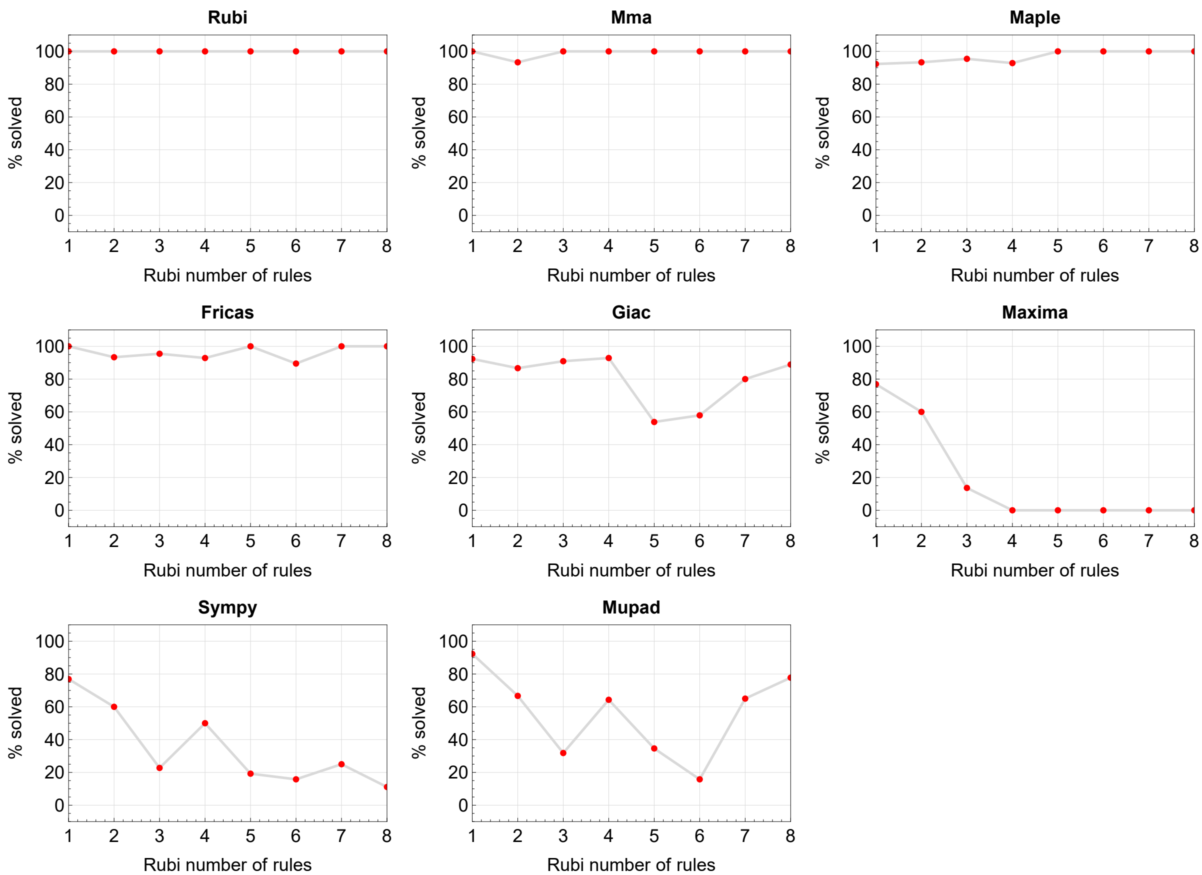


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

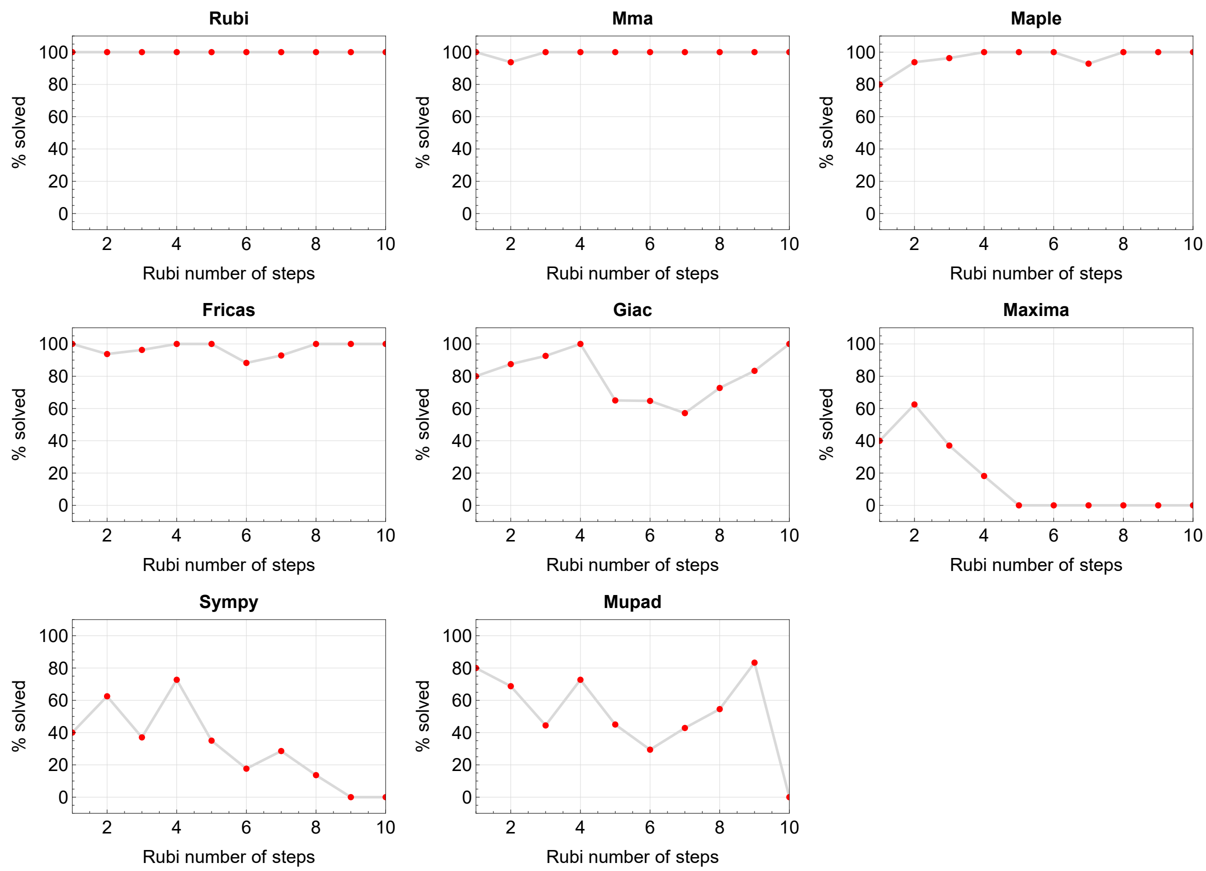


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

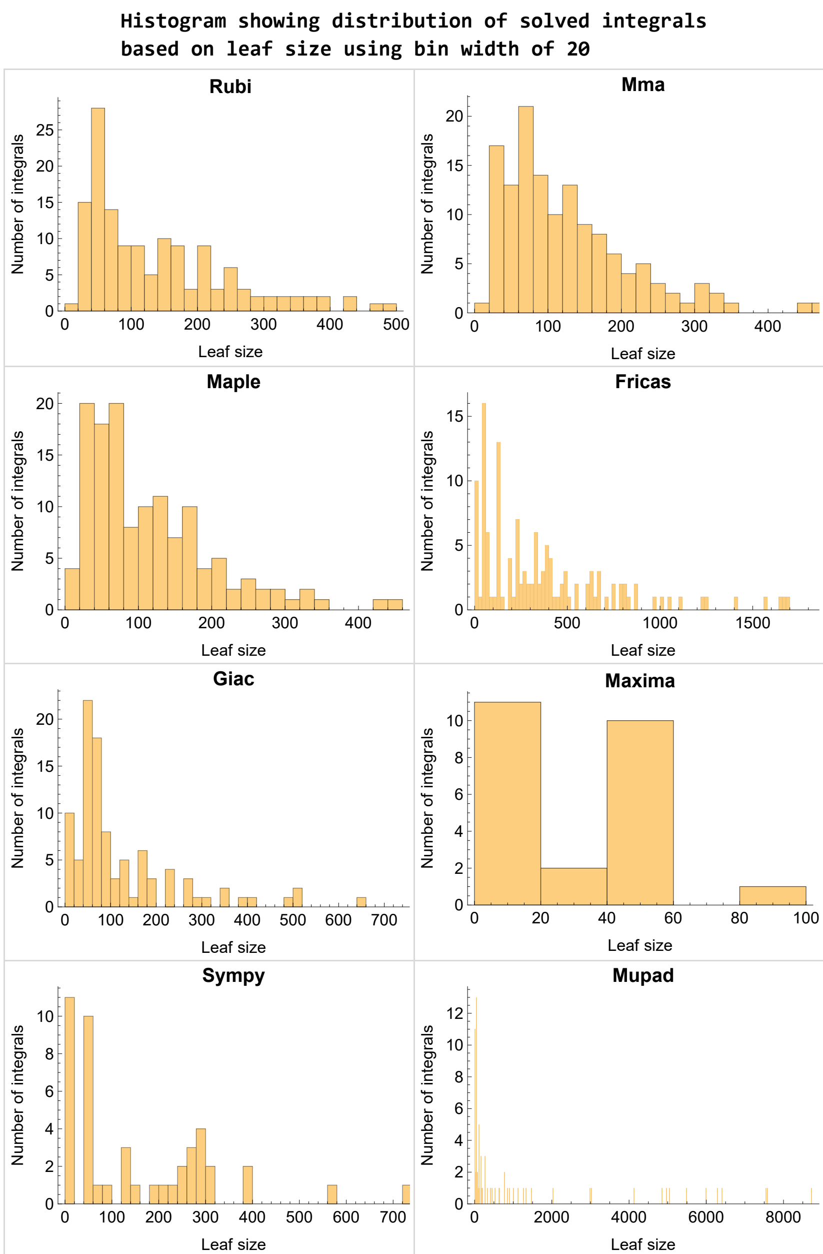


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

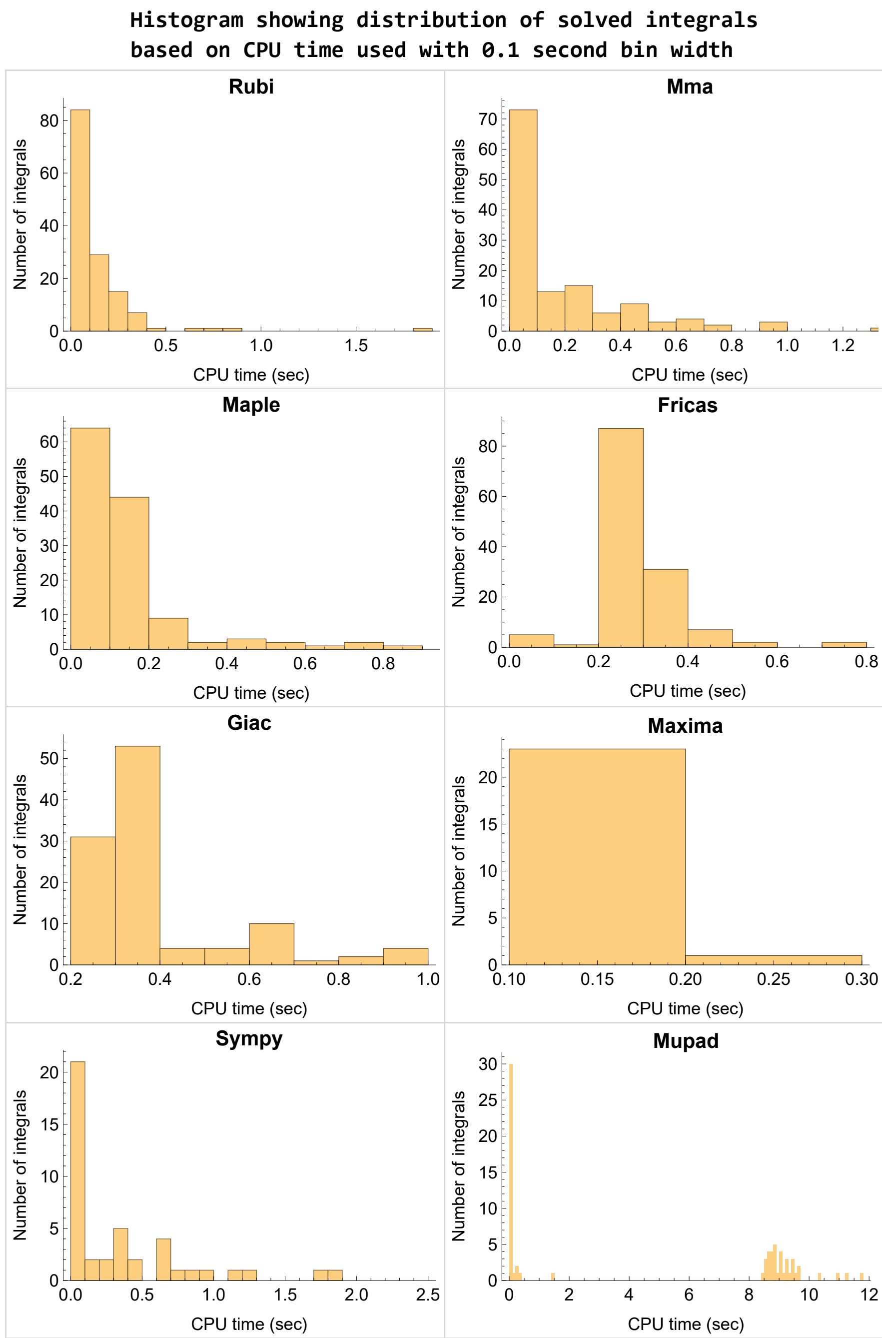


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

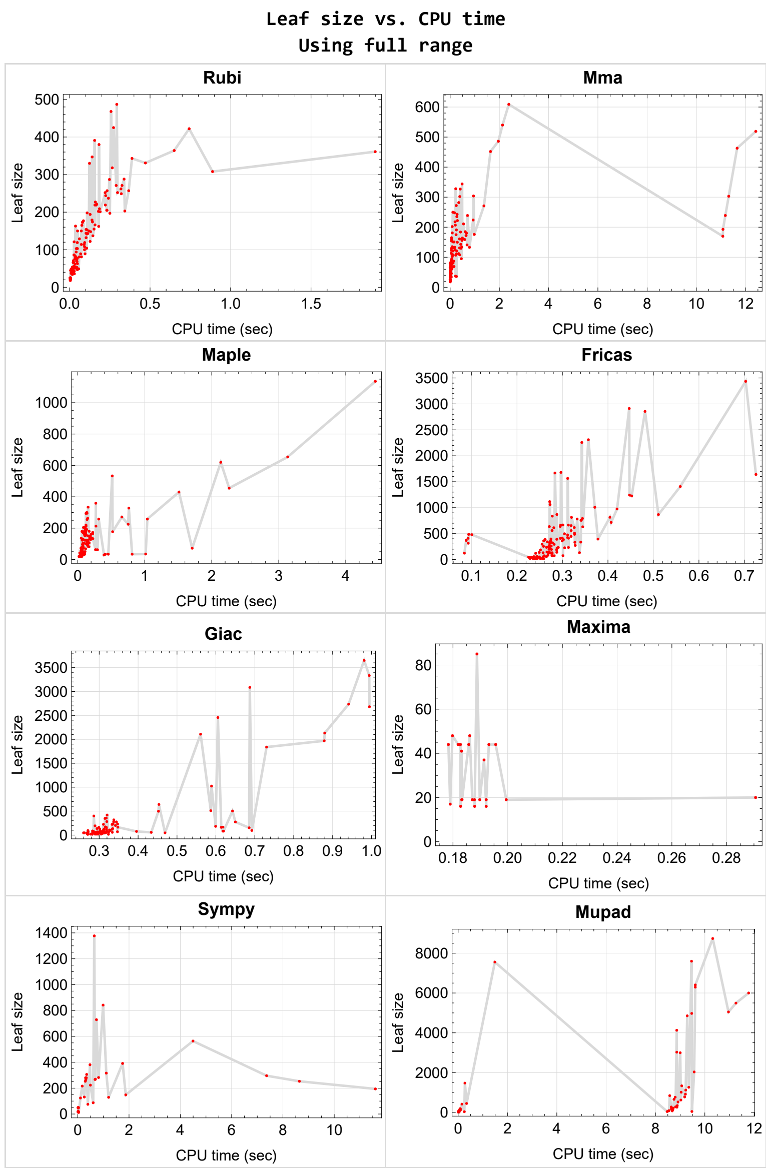


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

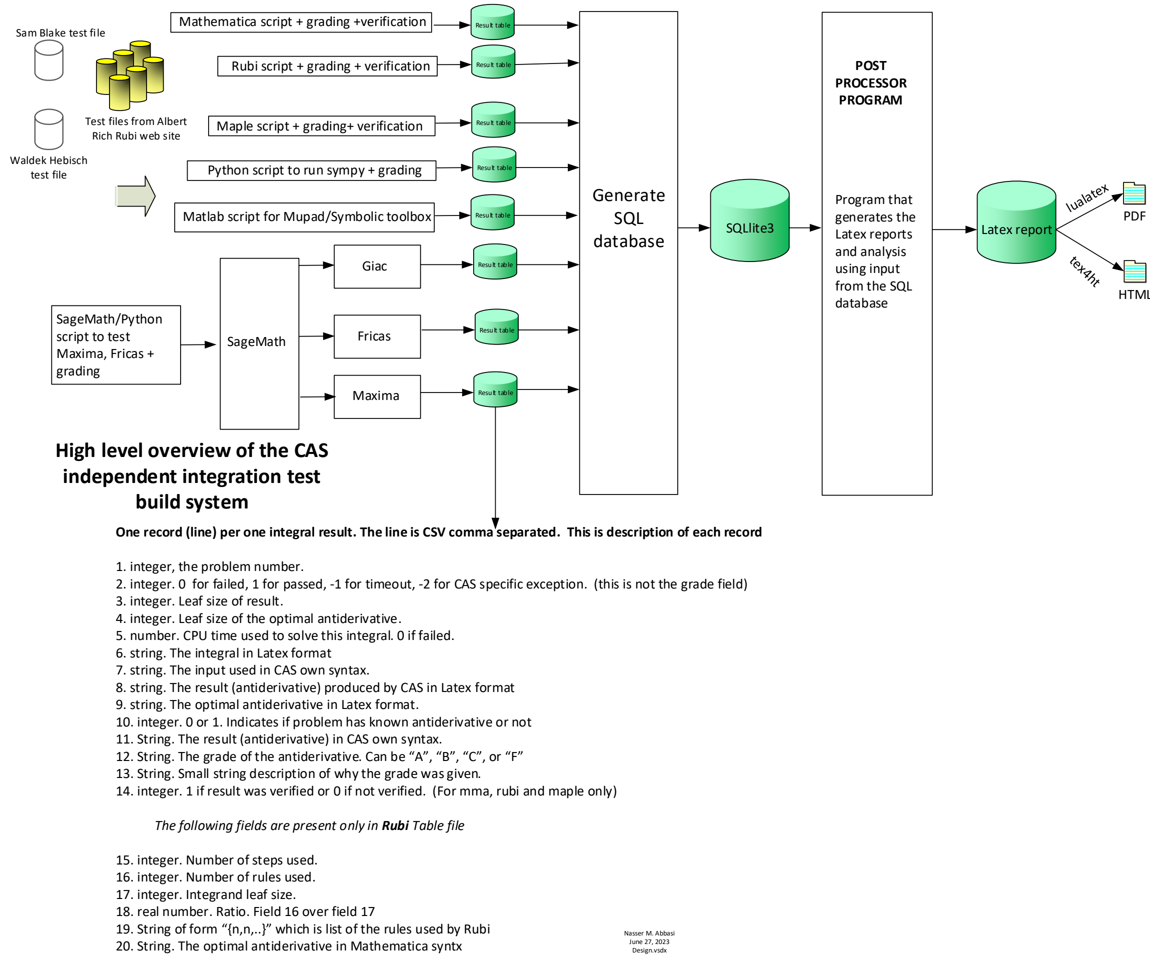
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 108, 109, 111, 113, 115, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade { }

C grade { 105, 107, 110, 112, 114, 116, 117, 119 }

F normal fail { 140 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 84, 86, 87, 88, 89, 91, 93, 95, 97, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 120, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade { 72, 118, 119 }

C grade { 79, 81, 83, 85, 90, 92, 94, 96, 98 }

F normal fail { 104, 121, 122, 140 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 84, 86, 88, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 60, 72, 79, 81, 83, 85, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 118 }

C grade { }

F normal fail { 104, 116, 119, 122 }

F(-1) timedout fail { }

F(-2) exception fail { 140 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 137 }

B grade { }

C grade { }

F normal fail { 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 134, 135, 136, 138, 139, 140 }

F(-1) timedout fail { 34 }

F(-2) exception fail { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 129, 133 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 38, 39, 40, 41, 42, 49, 50, 51, 52, 55, 56, 57, 58, 59, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 84, 86, 88, 89, 91, 93, 95, 97, 99, 101, 103, 106, 113, 115, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade { 60, 65, 72, 79, 81, 83, 85, 87, 90, 92, 94, 96, 98, 100, 102, 109, 111, 118 }

C grade { }

F normal fail { 34, 46, 104, 105, 107, 110, 112, 114, 116, 117, 119, 121, 122, 140 }

F(-1) timedout fail { 61, 62, 63, 64, 120 }

F(-2) exception fail { 33, 35, 36, 37, 43, 44, 45, 47, 48, 53, 54, 108 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 58, 59, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 129, 133, 137 }

C grade { }

F normal fail { }

F(-1) timedout fail { 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 134, 135, 136, 138, 139, 140 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 79, 81, 83, 85, 87, 94 }

B grade { 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 65, 72, 78, 80, 82, 84, 86, 93, 95, 97 }

C grade { }

F normal fail { 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 137, 138, 140 }

F(-1) timedout fail { 16, 17, 18, 24, 25, 26, 27, 28, 88, 89, 90, 91, 92, 96, 98, 99, 100, 101, 102, 103, 109, 131, 132, 135, 136, 139 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.005	0.004	0.054	0.188	0.229	0.018	0.292	0.018

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.005	0.005	0.054	0.199	0.235	0.023	0.297	0.017

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.003	0.000	0.054	0.192	0.237	0.026	0.278	0.017

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.004	0.002	0.062	0.190	0.245	0.018	0.289	0.018

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.005	0.007	0.023	0.192	0.241	0.018	0.324	0.014

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	48	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.89	0.85	0.83
time (sec)	N/A	0.040	0.010	0.084	0.183	0.226	0.021	0.308	0.020

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	49	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.91	0.85	0.83
time (sec)	N/A	0.020	0.008	0.083	0.193	0.231	0.020	0.316	0.014

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	48	44	48	46	45
N.S.	1	1.00	1.00	0.83	0.89	0.81	0.89	0.85	0.83
time (sec)	N/A	0.018	0.009	0.059	0.186	0.232	0.021	0.293	0.013

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	49	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.91	0.85	0.83
time (sec)	N/A	0.023	0.010	0.078	0.196	0.226	0.023	0.313	0.014

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	48	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.89	0.85	0.83
time (sec)	N/A	0.022	0.008	0.078	0.186	0.237	0.023	0.288	0.014

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	84	98	0	297	381	86	112
N.S.	1	1.00	0.94	1.10	0.00	3.34	4.28	0.97	1.26
time (sec)	N/A	0.069	0.074	0.076	0.000	0.255	0.478	0.283	0.079

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	73	75	0	235	306	67	172
N.S.	1	1.00	1.04	1.07	0.00	3.36	4.37	0.96	2.46
time (sec)	N/A	0.044	0.041	0.070	0.000	0.265	0.350	0.295	8.643

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	56	0	185	216	55	112
N.S.	1	1.00	1.02	1.00	0.00	3.30	3.86	0.98	2.00
time (sec)	N/A	0.029	0.024	0.051	0.000	0.252	0.176	0.308	0.075

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	38	35	0	120	124	34	46
N.S.	1	1.00	1.12	1.03	0.00	3.53	3.65	1.00	1.35
time (sec)	N/A	0.017	0.006	0.045	0.000	0.241	0.108	0.309	0.019

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	61	61	0	211	564	62	213
N.S.	1	1.00	0.98	0.98	0.00	3.40	9.10	1.00	3.44
time (sec)	N/A	0.033	0.044	0.061	0.000	0.273	4.494	0.303	8.719

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	77	81	0	269	0	79	339
N.S.	1	1.00	0.95	1.00	0.00	3.32	0.00	0.98	4.19
time (sec)	N/A	0.074	0.056	0.063	0.000	0.271	0.000	0.285	8.882

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	102	128	0	358	0	105	447
N.S.	1	1.00	0.98	1.23	0.00	3.44	0.00	1.01	4.30
time (sec)	N/A	0.106	0.084	0.082	0.000	0.299	0.000	0.326	0.342

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	131	157	0	445	0	136	524
N.S.	1	1.00	0.96	1.15	0.00	3.25	0.00	0.99	3.82
time (sec)	N/A	0.145	0.070	0.124	0.000	0.319	0.000	0.294	8.914

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	132	198	0	837	842	161	261
N.S.	1	1.00	0.88	1.32	0.00	5.58	5.61	1.07	1.74
time (sec)	N/A	0.119	0.143	0.107	0.000	0.278	0.989	0.299	8.869

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	109	169	0	635	729	125	279
N.S.	1	1.00	0.96	1.48	0.00	5.57	6.39	1.10	2.45
time (sec)	N/A	0.073	0.097	0.119	0.000	0.271	0.731	0.302	8.815

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	81	97	0	387	280	88	135
N.S.	1	1.00	1.21	1.45	0.00	5.78	4.18	1.31	2.01
time (sec)	N/A	0.025	0.119	0.071	0.000	0.262	0.307	0.302	8.703

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	69	70	0	338	253	76	110
N.S.	1	1.00	1.05	1.06	0.00	5.12	3.83	1.15	1.67
time (sec)	N/A	0.023	0.045	0.067	0.000	0.274	0.295	0.396	8.685

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	341	265	76	119
N.S.	1	1.00	1.06	1.03	0.00	5.17	4.02	1.15	1.80
time (sec)	N/A	0.023	0.050	0.065	0.000	0.267	0.318	0.305	0.050

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	107	177	0	781	0	126	620
N.S.	1	1.00	0.99	1.64	0.00	7.23	0.00	1.17	5.74
time (sec)	N/A	0.093	0.115	0.089	0.000	0.332	0.000	0.312	9.029

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	131	205	0	975	0	171	775
N.S.	1	1.00	0.89	1.39	0.00	6.59	0.00	1.16	5.24
time (sec)	N/A	0.131	0.384	0.087	0.000	0.420	0.000	0.316	9.183

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	175	255	0	1226	0	229	914
N.S.	1	1.00	0.87	1.26	0.00	6.07	0.00	1.13	4.52
time (sec)	N/A	0.179	0.241	0.140	0.000	0.452	0.000	0.336	9.214

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	218	295	0	1407	0	282	1120
N.S.	1	1.00	0.87	1.17	0.00	5.58	0.00	1.12	4.44
time (sec)	N/A	0.221	0.199	0.139	0.000	0.559	0.000	0.319	9.225

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	272	359	0	1640	0	347	1260
N.S.	1	1.00	0.86	1.13	0.00	5.16	0.00	1.09	3.96
time (sec)	N/A	0.265	0.248	0.270	0.000	0.725	0.000	0.315	9.350

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	184	133	0	390	0	275	0
N.S.	1	1.00	0.72	0.52	0.00	1.52	0.00	1.07	0.00
time (sec)	N/A	0.367	0.419	0.226	0.000	0.288	0.000	0.344	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	150	106	0	326	0	222	0
N.S.	1	1.00	0.73	0.52	0.00	1.59	0.00	1.08	0.00
time (sec)	N/A	0.231	0.291	0.171	0.000	0.272	0.000	0.333	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	121	121	0	260	0	160	0
N.S.	1	1.00	0.74	0.74	0.00	1.60	0.00	0.98	0.00
time (sec)	N/A	0.037	0.265	0.130	0.000	0.275	0.000	0.318	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	95	100	0	220	0	119	0
N.S.	1	1.00	0.80	0.84	0.00	1.85	0.00	1.00	0.00
time (sec)	N/A	0.048	0.453	0.118	0.000	0.288	0.000	0.333	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	133	95	0	638	0	0	0
N.S.	1	1.00	0.77	0.55	0.00	3.69	0.00	0.00	0.00
time (sec)	N/A	0.081	0.186	0.141	0.000	0.295	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	125	101	0	653	0	0	0
N.S.	1	1.00	0.72	0.58	0.00	3.77	0.00	0.00	0.00
time (sec)	N/A	0.080	0.199	0.128	0.000	0.319	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	111	81	0	226	0	0	0
N.S.	1	1.00	0.97	0.71	0.00	1.98	0.00	0.00	0.00
time (sec)	N/A	0.091	0.256	0.169	0.000	0.305	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	130	115	0	272	0	0	0
N.S.	1	1.00	0.84	0.74	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.152	0.440	0.154	0.000	0.328	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	159	149	0	336	0	0	0
N.S.	1	1.00	0.78	0.73	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	0.233	0.590	0.189	0.000	0.340	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	422	304	328	0	664	0	509	0
N.S.	1	1.00	0.72	0.78	0.00	1.57	0.00	1.21	0.00
time (sec)	N/A	0.743	0.948	0.763	0.000	0.313	0.000	0.587	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	239	271	0	558	0	419	0
N.S.	1	1.00	0.66	0.74	0.00	1.53	0.00	1.15	0.00
time (sec)	N/A	0.650	0.705	0.659	0.000	0.319	0.000	0.320	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	211	225	0	474	0	355	0
N.S.	1	1.00	0.73	0.78	0.00	1.65	0.00	1.23	0.00
time (sec)	N/A	0.339	0.543	0.753	0.000	0.311	0.000	0.318	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	161	126	0	384	0	276	0
N.S.	1	1.00	0.81	0.64	0.00	1.94	0.00	1.39	0.00
time (sec)	N/A	0.109	0.378	0.191	0.000	0.302	0.000	0.337	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	133	100	0	320	0	224	0
N.S.	1	1.00	0.81	0.61	0.00	1.94	0.00	1.36	0.00
time (sec)	N/A	0.076	0.785	0.153	0.000	0.278	0.000	0.311	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	164	192	0	791	0	0	0
N.S.	1	1.00	0.72	0.85	0.00	3.48	0.00	0.00	0.00
time (sec)	N/A	0.160	0.495	0.133	0.000	0.344	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	156	180	0	757	0	0	0
N.S.	1	1.00	0.71	0.82	0.00	3.46	0.00	0.00	0.00
time (sec)	N/A	0.165	0.480	0.171	0.000	0.341	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	160	140	0	757	0	0	0
N.S.	1	1.00	0.73	0.64	0.00	3.46	0.00	0.00	0.00
time (sec)	N/A	0.157	0.502	0.179	0.000	0.341	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	172	150	0	815	0	0	0
N.S.	1	1.00	0.67	0.58	0.00	3.17	0.00	0.00	0.00
time (sec)	N/A	0.233	0.646	0.187	0.000	0.404	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	141	131	0	332	0	0	0
N.S.	1	1.00	0.72	0.66	0.00	1.69	0.00	0.00	0.00
time (sec)	N/A	0.250	0.687	0.178	0.000	0.327	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	176	161	0	394	0	0	0
N.S.	1	1.00	0.71	0.65	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.322	0.980	0.199	0.000	0.378	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	107	100	0	226	0	134	0
N.S.	1	1.00	0.75	0.70	0.00	1.58	0.00	0.94	0.00
time (sec)	N/A	0.113	0.229	0.139	0.000	0.290	0.000	0.335	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	89	50	0	188	0	90	0
N.S.	1	1.00	0.86	0.49	0.00	1.83	0.00	0.87	0.00
time (sec)	N/A	0.049	0.045	0.109	0.000	0.287	0.000	0.319	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	67	29	0	129	0	57	0
N.S.	1	1.00	0.94	0.41	0.00	1.82	0.00	0.80	0.00
time (sec)	N/A	0.024	0.079	0.081	0.000	0.273	0.000	0.334	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	68	42	0	130	0	59	0
N.S.	1	1.00	1.51	0.93	0.00	2.89	0.00	1.31	0.00
time (sec)	N/A	0.010	0.081	0.100	0.000	0.277	0.000	0.324	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	87	68	0	194	0	0	0
N.S.	1	1.00	1.13	0.88	0.00	2.52	0.00	0.00	0.00
time (sec)	N/A	0.033	0.141	0.118	0.000	0.276	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	111	117	0	232	0	0	0
N.S.	1	1.00	0.93	0.98	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.094	0.247	0.143	0.000	0.313	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	185	164	0	616	0	318	0
N.S.	1	1.00	0.71	0.63	0.00	2.35	0.00	1.21	0.00
time (sec)	N/A	0.319	0.611	0.224	0.000	0.326	0.000	0.338	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	143	129	0	486	0	231	0
N.S.	1	1.00	0.71	0.64	0.00	2.42	0.00	1.15	0.00
time (sec)	N/A	0.190	0.428	0.189	0.000	0.327	0.000	0.347	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	112	84	0	414	0	164	0
N.S.	1	1.00	0.73	0.55	0.00	2.71	0.00	1.07	0.00
time (sec)	N/A	0.108	0.347	0.153	0.000	0.300	0.000	0.348	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	37	34	0	73	0	69	75
N.S.	1	1.00	0.92	0.85	0.00	1.82	0.00	1.72	1.88
time (sec)	N/A	0.024	0.219	0.104	0.000	0.301	0.000	0.304	8.679

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	36	33	0	72	0	74	75
N.S.	1	1.00	0.92	0.85	0.00	1.85	0.00	1.90	1.92
time (sec)	N/A	0.023	0.255	0.098	0.000	0.277	0.000	0.304	8.526

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	108	107	0	411	0	199	0
N.S.	1	1.00	1.15	1.14	0.00	4.37	0.00	2.12	0.00
time (sec)	N/A	0.041	0.387	0.151	0.000	0.313	0.000	0.342	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	134	137	0	496	0	0	0
N.S.	1	1.00	0.93	0.95	0.00	3.44	0.00	0.00	0.00
time (sec)	N/A	0.102	0.470	0.204	0.000	0.304	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	180	173	0	630	0	0	0
N.S.	1	1.00	0.86	0.83	0.00	3.01	0.00	0.00	0.00
time (sec)	N/A	0.186	0.663	0.224	0.000	0.345	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	224	213	0	716	0	0	0
N.S.	1	1.00	0.83	0.79	0.00	2.64	0.00	0.00	0.00
time (sec)	N/A	0.290	0.933	0.272	0.000	0.407	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	271	258	0	866	0	0	0
N.S.	1	1.00	0.79	0.75	0.00	2.52	0.00	0.00	0.00
time (sec)	N/A	0.388	1.372	0.313	0.000	0.511	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	47	37	71	280	107	89
N.S.	1	1.00	0.92	1.27	1.00	1.92	7.57	2.89	2.41
time (sec)	N/A	0.010	0.045	0.060	0.191	0.282	0.335	0.311	8.552

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.005	0.002	0.058	0.183	0.243	0.018	0.305	0.017

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.005	0.002	0.020	0.183	0.251	0.017	0.271	0.017

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.004	0.000	0.021	0.187	0.252	0.018	0.271	0.016

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.004	0.001	0.023	0.183	0.242	0.018	0.282	0.014

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	17	20	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.81	0.95	0.81
time (sec)	N/A	0.007	0.002	0.053	0.179	0.259	0.035	0.270	0.015

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	20	12	16	16
N.S.	1	1.00	1.00	0.94	0.89	1.11	0.67	0.89	0.89
time (sec)	N/A	0.005	0.002	0.029	0.188	0.256	0.037	0.271	0.017

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	69	300	85	241	1377	399	271
N.S.	1	1.00	0.91	3.95	1.12	3.17	18.12	5.25	3.57
time (sec)	N/A	0.035	0.155	0.135	0.189	0.268	0.646	0.286	8.640

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	51	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.94	0.85	0.83
time (sec)	N/A	0.027	0.006	0.095	0.182	0.234	0.020	0.260	0.019

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	48	45	44	44	46	46	45
N.S.	1	1.00	0.89	0.83	0.81	0.81	0.85	0.85	0.83
time (sec)	N/A	0.036	0.007	0.094	0.182	0.235	0.022	0.262	0.013

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	48	44	51	46	45
N.S.	1	1.00	1.00	0.83	0.89	0.81	0.94	0.85	0.83
time (sec)	N/A	0.019	0.006	0.060	0.180	0.249	0.024	0.269	0.013

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	48	45	44	44	46	46	45
N.S.	1	1.00	0.89	0.83	0.81	0.81	0.85	0.85	0.83
time (sec)	N/A	0.031	0.006	0.081	0.178	0.257	0.022	0.469	0.014

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	41	41	48	43	42
N.S.	1	1.00	1.00	0.86	0.84	0.84	0.98	0.88	0.86
time (sec)	N/A	0.019	0.004	0.030	0.183	0.245	0.022	0.270	0.013

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	93	105	0	313	391	92	842
N.S.	1	1.00	0.93	1.05	0.00	3.13	3.91	0.92	8.42
time (sec)	N/A	0.093	0.063	0.091	0.000	0.273	1.750	0.327	8.576

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	250	73	0	1564	194	2457	4127
N.S.	1	1.00	1.23	0.36	0.00	7.70	0.96	12.10	20.33
time (sec)	N/A	0.343	0.110	0.085	0.000	0.311	11.603	0.605	8.862

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	83	0	254	316	75	655
N.S.	1	1.00	0.96	1.02	0.00	3.14	3.90	0.93	8.09
time (sec)	N/A	0.060	0.030	0.078	0.000	0.258	1.114	0.309	8.751

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	202	57	0	1059	129	2109	3026
N.S.	1	1.00	1.13	0.32	0.00	5.92	0.72	11.78	16.91
time (sec)	N/A	0.139	0.083	0.069	0.000	0.273	1.205	0.561	8.863

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	197	223	59	118
N.S.	1	1.00	0.98	0.95	0.00	3.13	3.54	0.94	1.87
time (sec)	N/A	0.049	0.016	0.053	0.000	0.271	0.496	0.284	0.092

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	165	41	0	559	75	503	416
N.S.	1	1.00	1.10	0.27	0.00	3.73	0.50	3.35	2.77
time (sec)	N/A	0.073	0.057	0.069	0.000	0.277	0.395	0.643	0.153

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	36	0	129	131	35	41
N.S.	1	1.00	1.08	1.00	0.00	3.58	3.64	0.97	1.14
time (sec)	N/A	0.030	0.008	0.040	0.000	0.269	0.257	0.282	8.474

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	129	38	0	613	87	1024	763
N.S.	1	1.00	0.86	0.25	0.00	4.09	0.58	6.83	5.09
time (sec)	N/A	0.050	0.053	0.064	0.000	0.281	0.603	0.589	8.799

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	113	65	0	223	253	68	1014
N.S.	1	1.00	1.64	0.94	0.00	3.23	3.67	0.99	14.70
time (sec)	N/A	0.046	0.046	0.059	0.000	0.272	8.642	0.272	9.041

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	191	159	0	1116	148	1839	2997
N.S.	1	1.00	1.10	0.91	0.00	6.41	0.85	10.57	17.22
time (sec)	N/A	0.142	0.253	0.099	0.000	0.272	1.873	0.730	9.003

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	135	85	0	293	0	94	2033
N.S.	1	1.00	1.52	0.96	0.00	3.29	0.00	1.06	22.84
time (sec)	N/A	0.096	0.084	0.079	0.000	0.282	0.000	0.271	9.569

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	151	209	0	868	0	161	1473
N.S.	1	1.00	0.91	1.26	0.00	5.23	0.00	0.97	8.87
time (sec)	N/A	0.158	0.120	0.116	0.000	0.288	0.000	0.614	0.274

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	327	174	0	2856	0	3335	7599
N.S.	1	1.00	0.99	0.53	0.00	8.63	0.00	10.08	22.96
time (sec)	N/A	0.471	0.409	0.105	0.000	0.482	0.000	0.994	9.461

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	121	179	0	663	0	152	1336
N.S.	1	1.00	0.92	1.36	0.00	5.02	0.00	1.15	10.12
time (sec)	N/A	0.103	0.118	0.100	0.000	0.296	0.000	0.685	9.064

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	282	151	0	2257	0	2736	6293
N.S.	1	1.00	1.04	0.56	0.00	8.33	0.00	10.10	23.22
time (sec)	N/A	0.326	0.350	0.109	0.000	0.343	0.000	0.941	9.614

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	93	104	0	407	282	96	187
N.S.	1	1.00	1.19	1.33	0.00	5.22	3.62	1.23	2.40
time (sec)	N/A	0.049	0.059	0.078	0.000	0.296	0.813	0.692	0.087

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	235	123	0	1668	296	2132	4973
N.S.	1	1.00	0.99	0.52	0.00	7.04	1.25	9.00	20.98
time (sec)	N/A	0.241	0.272	0.106	0.000	0.284	7.364	0.880	9.469

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	79	77	0	360	269	82	178
N.S.	1	1.00	1.05	1.03	0.00	4.80	3.59	1.09	2.37
time (sec)	N/A	0.045	0.046	0.079	0.000	0.274	0.690	0.620	0.082

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	222	122	0	1680	0	1970	4854
N.S.	1	1.00	1.00	0.55	0.00	7.60	0.00	8.91	21.96
time (sec)	N/A	0.169	0.295	0.148	0.000	0.297	0.000	0.878	9.288

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	79	75	0	361	267	82	172
N.S.	1	1.00	1.07	1.01	0.00	4.88	3.61	1.11	2.32
time (sec)	N/A	0.045	0.054	0.084	0.000	0.259	0.675	0.618	0.074

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	243	151	0	2309	0	2682	6404
N.S.	1	1.00	0.96	0.60	0.00	9.16	0.00	10.64	25.41
time (sec)	N/A	0.298	0.283	0.150	0.000	0.357	0.000	0.995	9.615

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	207	185	0	813	0	166	5048
N.S.	1	1.00	1.70	1.52	0.00	6.66	0.00	1.36	41.38
time (sec)	N/A	0.128	0.209	0.126	0.000	0.320	0.000	0.618	10.960

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	302	294	0	2912	0	3087	7555
N.S.	1	1.00	0.98	0.95	0.00	9.45	0.00	10.02	24.53
time (sec)	N/A	0.888	0.391	0.123	0.000	0.447	0.000	0.687	1.488

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	248	213	0	1007	0	182	5491
N.S.	1	1.00	1.53	1.31	0.00	6.22	0.00	1.12	33.90
time (sec)	N/A	0.181	0.177	0.124	0.000	0.371	0.000	0.599	11.262

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	344	334	0	3435	0	3651	8739
N.S.	1	1.00	0.95	0.93	0.00	9.52	0.00	10.11	24.21
time (sec)	N/A	1.899	0.485	0.155	0.000	0.703	0.000	0.981	10.321

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	328	263	0	1242	0	274	5999
N.S.	1	1.00	1.50	1.20	0.00	5.67	0.00	1.25	27.39
time (sec)	N/A	0.222	0.228	0.149	0.000	0.447	0.000	0.650	11.776

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	170	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.107	11.074	0.000	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	486	455	0	368	0	0	0
N.S.	1	1.00	1.28	1.20	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.183	1.958	2.260	0.000	0.088	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	108	123	0	232	0	121	0
N.S.	1	1.00	0.84	0.95	0.00	1.80	0.00	0.94	0.00
time (sec)	N/A	0.061	0.091	0.067	0.000	0.291	0.000	0.319	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	452	430	0	314	0	0	0
N.S.	1	1.00	1.30	1.24	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.140	1.639	1.511	0.000	0.093	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	154	136	0	666	0	0	0
N.S.	1	1.00	0.79	0.70	0.00	3.43	0.00	0.00	0.00
time (sec)	N/A	0.128	0.055	0.053	0.000	0.301	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	181	202	0	396	0	640	0
N.S.	1	1.00	0.74	0.83	0.00	1.62	0.00	2.62	0.00
time (sec)	N/A	0.224	0.086	0.090	0.000	0.275	0.000	0.453	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	487	609	654	0	485	0	0	0
N.S.	1	1.00	1.25	1.34	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.293	2.382	3.134	0.000	0.094	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	151	170	0	332	0	498	0
N.S.	1	1.00	0.85	0.96	0.00	1.88	0.00	2.81	0.00
time (sec)	N/A	0.086	0.159	0.082	0.000	0.274	0.000	0.452	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	425	540	620	0	409	0	0	0
N.S.	1	1.00	1.27	1.46	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.273	2.127	2.135	0.000	0.093	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	80	72	0	135	0	56	0
N.S.	1	1.00	0.98	0.88	0.00	1.65	0.00	0.68	0.00
time (sec)	N/A	0.037	0.013	0.048	0.000	0.270	0.000	0.311	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	193	177	0	123	0	0	0
N.S.	1	1.00	1.60	1.46	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.028	11.081	0.521	0.000	0.084	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	80	72	0	137	0	56	0
N.S.	1	1.00	1.57	1.41	0.00	2.69	0.00	1.10	0.00
time (sec)	N/A	0.018	0.018	0.053	0.000	0.249	0.000	0.434	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	303	258	0	0	0	0	0
N.S.	1	1.00	0.92	0.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	11.314	1.039	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	463	533	0	479	0	0	0
N.S.	1	1.00	1.18	1.36	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.156	11.656	0.515	0.000	0.101	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	123	179	0	424	0	193	0
N.S.	1	1.00	1.19	1.74	0.00	4.12	0.00	1.87	0.00
time (sec)	N/A	0.042	0.047	0.063	0.000	0.311	0.000	0.289	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	468	468	519	1136	0	0	0	0	0
N.S.	1	1.00	1.11	2.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.257	12.413	4.442	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	159	220	0	508	0	0	0
N.S.	1	1.00	1.03	1.43	0.00	3.30	0.00	0.00	0.00
time (sec)	N/A	0.103	0.057	0.123	0.000	0.327	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	46	0	0	83	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	1.63	0.00	0.00	0.00
time (sec)	N/A	0.032	0.048	0.000	0.000	0.268	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	287	239	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.248	11.177	0.000	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	66	34	0	55	0	60	0
N.S.	1	1.00	1.47	0.76	0.00	1.22	0.00	1.33	0.00
time (sec)	N/A	0.006	0.087	0.812	0.000	0.244	0.000	0.316	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	66	34	0	55	0	60	0
N.S.	1	1.00	1.47	0.76	0.00	1.22	0.00	1.33	0.00
time (sec)	N/A	0.008	0.001	0.456	0.000	0.245	0.000	0.315	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	66	34	0	55	0	60	0
N.S.	1	1.00	1.47	0.76	0.00	1.22	0.00	1.33	0.00
time (sec)	N/A	0.009	0.001	0.410	0.000	0.265	0.000	0.308	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	80	62	0	70	0	69	0
N.S.	1	1.00	0.93	0.72	0.00	0.81	0.00	0.80	0.00
time (sec)	N/A	0.028	0.071	0.295	0.000	0.259	0.000	0.314	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	80	62	0	70	0	69	0
N.S.	1	1.00	0.93	0.72	0.00	0.81	0.00	0.80	0.00
time (sec)	N/A	0.029	0.001	0.271	0.000	0.260	0.000	0.311	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	80	62	0	70	0	69	0
N.S.	1	1.00	0.93	0.72	0.00	0.81	0.00	0.80	0.00
time (sec)	N/A	0.028	0.001	0.266	0.000	0.278	0.000	0.347	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	35	0	111	0	35	34
N.S.	1	1.00	0.97	0.92	0.00	2.92	0.00	0.92	0.89
time (sec)	N/A	0.010	0.011	1.014	0.000	0.277	0.000	0.298	0.045

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	68	42	0	130	0	59	0
N.S.	1	1.00	1.51	0.93	0.00	2.89	0.00	1.31	0.00
time (sec)	N/A	0.014	0.013	0.089	0.000	0.257	0.000	0.308	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	70	64	0	131	0	53	0
N.S.	1	1.00	1.49	1.36	0.00	2.79	0.00	1.13	0.00
time (sec)	N/A	0.048	0.021	0.067	0.000	0.337	0.000	0.318	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	72	66	0	139	0	58	0
N.S.	1	1.00	1.47	1.35	0.00	2.84	0.00	1.18	0.00
time (sec)	N/A	0.057	0.018	0.064	0.000	0.288	0.000	0.290	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	39	0	124	0	38	44
N.S.	1	1.00	0.93	0.89	0.00	2.82	0.00	0.86	1.00
time (sec)	N/A	0.024	0.005	0.071	0.000	0.273	0.000	0.296	9.476

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	78	72	0	135	0	62	0
N.S.	1	1.00	1.59	1.47	0.00	2.76	0.00	1.27	0.00
time (sec)	N/A	0.010	0.011	1.708	0.000	0.271	0.000	0.308	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	80	72	0	137	0	56	0
N.S.	1	1.00	1.57	1.41	0.00	2.69	0.00	1.10	0.00
time (sec)	N/A	0.041	0.011	0.052	0.000	0.271	0.000	0.284	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	82	74	0	145	0	61	0
N.S.	1	1.00	1.55	1.40	0.00	2.74	0.00	1.15	0.00
time (sec)	N/A	0.045	0.013	0.074	0.000	0.266	0.000	0.299	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	33	29	20	47	0	55	33
N.S.	1	1.00	0.82	0.72	0.50	1.18	0.00	1.38	0.82
time (sec)	N/A	0.018	0.003	0.392	0.291	0.245	0.000	0.300	0.251

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	66	34	0	55	0	60	0
N.S.	1	1.00	1.47	0.76	0.00	1.22	0.00	1.33	0.00
time (sec)	N/A	0.012	0.002	0.412	0.000	0.237	0.000	0.315	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	61	50	0	49	0	47	0
N.S.	1	1.00	1.42	1.16	0.00	1.14	0.00	1.09	0.00
time (sec)	N/A	0.029	0.074	0.071	0.000	0.249	0.000	0.302	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	0.000	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [86] had the largest ratio of [.5000000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	20	0.050
2	A	2	1	1.00	18	0.056
3	A	1	0	1.00	16	0.000
4	A	2	1	1.00	20	0.050
5	A	2	1	1.00	20	0.050
6	A	3	2	1.00	22	0.091
7	A	3	2	1.00	20	0.100
8	A	3	2	1.00	18	0.111
9	A	3	2	1.00	22	0.091
10	A	3	2	1.00	22	0.091
11	A	7	6	1.00	22	0.273
12	A	6	6	1.00	22	0.273
13	A	5	5	1.00	22	0.227
14	A	3	3	1.00	22	0.136
15	A	7	7	1.00	20	0.350
16	A	8	7	1.00	18	0.389
17	A	8	7	1.00	22	0.318
18	A	8	7	1.00	22	0.318
19	A	8	7	1.00	22	0.318
20	A	7	7	1.00	22	0.318
21	A	4	4	1.00	22	0.182
22	A	4	4	1.00	22	0.182
23	A	4	4	1.00	22	0.182
24	A	8	7	1.00	22	0.318

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	8	7	1.00	22	0.318
26	A	8	7	1.00	20	0.350
27	A	8	7	1.00	18	0.389
28	A	8	7	1.00	22	0.318
29	A	8	6	1.00	24	0.250
30	A	7	6	1.00	22	0.273
31	A	5	5	1.00	20	0.250
32	A	4	4	1.00	24	0.167
33	A	7	6	1.00	24	0.250
34	A	7	6	1.00	24	0.250
35	A	5	5	1.00	24	0.208
36	A	6	5	1.00	24	0.208
37	A	7	5	1.00	24	0.208
38	A	10	7	1.00	22	0.318
39	A	10	7	1.00	20	0.350
40	A	8	7	1.00	24	0.292
41	A	6	5	1.00	24	0.208
42	A	5	4	1.00	24	0.167
43	A	8	7	1.00	24	0.292
44	A	8	7	1.00	24	0.292
45	A	8	7	1.00	24	0.292
46	A	9	8	1.00	24	0.333
47	A	7	6	1.00	24	0.250
48	A	8	6	1.00	24	0.250
49	A	6	6	1.00	24	0.250
50	A	4	4	1.00	24	0.167
51	A	3	3	1.00	22	0.136
52	A	2	2	1.00	20	0.100
53	A	3	3	1.00	24	0.125
54	A	5	5	1.00	24	0.208
55	A	8	6	1.00	24	0.250
56	A	7	6	1.00	24	0.250
57	A	6	6	1.00	24	0.250
58	A	1	1	1.00	24	0.042
59	A	1	1	1.00	24	0.042

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	3	3	1.00	24	0.125
61	A	5	5	1.00	22	0.227
62	A	6	5	1.00	20	0.250
63	A	7	5	1.00	24	0.208
64	A	8	5	1.00	24	0.208
65	A	2	1	1.00	18	0.056
66	A	2	1	1.00	18	0.056
67	A	2	1	1.00	16	0.062
68	A	1	0	1.00	14	0.000
69	A	2	1	1.00	18	0.056
70	A	2	1	1.00	18	0.056
71	A	2	1	1.00	18	0.056
72	A	3	2	1.00	20	0.100
73	A	3	2	1.00	20	0.100
74	A	4	3	1.00	18	0.167
75	A	3	2	1.00	16	0.125
76	A	4	3	1.00	20	0.150
77	A	3	2	1.00	20	0.100
78	A	8	7	1.00	20	0.350
79	A	6	5	1.00	20	0.250
80	A	7	7	1.00	20	0.350
81	A	5	4	1.00	20	0.200
82	A	6	6	1.00	20	0.300
83	A	4	3	1.00	20	0.150
84	A	4	4	1.00	20	0.200
85	A	4	3	1.00	18	0.167
86	A	8	8	1.00	16	0.500
87	A	5	4	1.00	20	0.200
88	A	9	8	1.00	20	0.400
89	A	9	8	1.00	20	0.400
90	A	7	5	1.00	20	0.250
91	A	8	8	1.00	20	0.400
92	A	6	5	1.00	20	0.250
93	A	5	5	1.00	20	0.250
94	A	5	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	5	5	1.00	20	0.250
96	A	5	4	1.00	20	0.200
97	A	5	5	1.00	20	0.250
98	A	5	4	1.00	20	0.200
99	A	9	8	1.00	18	0.444
100	A	6	5	1.00	16	0.312
101	A	9	8	1.00	20	0.400
102	A	7	5	1.00	20	0.250
103	A	9	8	1.00	20	0.400
104	A	3	3	1.00	20	0.150
105	A	5	5	1.00	24	0.208
106	A	5	5	1.00	24	0.208
107	A	5	5	1.00	24	0.208
108	A	8	7	1.00	24	0.292
109	A	8	8	1.00	24	0.333
110	A	6	6	1.00	24	0.250
111	A	6	5	1.00	24	0.208
112	A	6	6	1.00	24	0.250
113	A	4	4	1.00	24	0.167
114	A	2	2	1.00	24	0.083
115	A	2	2	1.00	24	0.083
116	A	6	6	1.00	24	0.250
117	A	5	5	1.00	24	0.208
118	A	3	3	1.00	24	0.125
119	A	6	6	1.00	24	0.250
120	A	5	5	1.00	24	0.208
121	A	1	1	1.00	34	0.029
122	A	7	4	1.00	27	0.148
123	A	2	2	1.00	18	0.111
124	A	3	3	1.00	18	0.167
125	A	3	3	1.00	17	0.176
126	A	5	5	1.00	18	0.278
127	A	6	6	1.00	18	0.333
128	A	6	6	1.00	17	0.353
129	A	2	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	3	3	1.00	18	0.167
131	A	3	3	1.00	22	0.136
132	A	3	3	1.00	24	0.125
133	A	3	3	1.00	20	0.150
134	A	3	3	1.00	20	0.150
135	A	3	3	1.00	24	0.125
136	A	3	3	1.00	26	0.115
137	A	3	3	1.00	18	0.167
138	A	3	3	1.00	18	0.167
139	A	3	3	1.00	20	0.150
140	A	2	2	1.00	36	0.056

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^2(ax^2+bx^3+cx^4)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 25

$$\int x^2\left(ax^2+bx^3+cx^4\right)\,dx=\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

```
[Out] 1/5*a*x^5+1/6*b*x^6+1/7*c*x^7
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {14}

$$\int x^2\left(ax^2+bx^3+cx^4\right)\,dx=\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

```
[In] Int[x^2*(a*x^2 + b*x^3 + c*x^4),x]
[Out] (a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(ax^4+bx^5+cx^6\right)\,dx\\&=\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^2(ax^2+bx^3+cx^4)\ dx=\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

```
[In] Integrate[x^2*(a*x^2 + b*x^3 + c*x^4),x]
[Out] (a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{x^5(30c\,x^2+35bx+42a)}{210}$	20
default	$\frac{1}{5}a\,x^5+\frac{1}{6}b\,x^6+\frac{1}{7}c\,x^7$	20
norman	$\frac{1}{5}a\,x^5+\frac{1}{6}b\,x^6+\frac{1}{7}c\,x^7$	20
risch	$\frac{1}{5}a\,x^5+\frac{1}{6}b\,x^6+\frac{1}{7}c\,x^7$	20
parallelrisch	$\frac{1}{5}a\,x^5+\frac{1}{6}b\,x^6+\frac{1}{7}c\,x^7$	20

```
[In] int(x^2*(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] 1/210*x^5*(30*c*x^2+35*b*x+42*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(ax^2+bx^3+cx^4)\ dx=\frac{1}{7}\,cx^7+\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
[Out] 1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2\left(ax^2+bx^3+cx^4\right)\,dx=\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

```
[In] integrate(x**2*(c*x**4+b*x**3+a*x**2),x)
[Out] a*x**5/5 + b*x**6/6 + c*x**7/7
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2\left(ax^2+bx^3+cx^4\right)\,dx=\frac{1}{7}\,cx^7+\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
[Out] 1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2\left(ax^2+bx^3+cx^4\right)\,dx=\frac{1}{7}\,cx^7+\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
[Out] 1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2\left(ax^2+bx^3+cx^4\right)\,dx=\frac{x^5\,\left(30\,c\,x^2+35\,b\,x+42\,a\right)}{210}$$

```
[In] int(x^2*(a*x^2 + b*x^3 + c*x^4),x)
[Out] (x^5*(42*a + 35*b*x + 30*c*x^2))/210
```

3.2 $\int x(ax^2 + bx^3 + cx^4)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 25

$$\int x\left(ax^2 + bx^3 + cx^4\right) dx = \frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6}$$

```
[Out] 1/4*a*x^4+1/5*b*x^5+1/6*c*x^6
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{14\}$

$$\int x\left(ax^2 + bx^3 + cx^4\right) dx = \frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6}$$

```
[In] Int[x*(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(ax^3 + bx^4 + cx^5\right) dx \\ &= \frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\left(ax^2+bx^3+cx^4\right)\,dx=\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

```
[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4),x]
[Out] (a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{x^4(10c\,x^2+12bx+15a)}{60}$	20
default	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5+\frac{1}{6}c\,x^6$	20
norman	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5+\frac{1}{6}c\,x^6$	20
risch	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5+\frac{1}{6}c\,x^6$	20
parallelrisch	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5+\frac{1}{6}c\,x^6$	20

```
[In] int(x*(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] 1/60*x^4*(10*c*x^2+12*b*x+15*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\left(ax^2+bx^3+cx^4\right)\,dx=\frac{1}{6}\,cx^6+\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
[Out] 1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax^2+bx^3+cx^4)\, dx = \frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2),x)
[Out] a*x**4/4 + b*x**5/5 + c*x**6/6
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax^2+bx^3+cx^4)\, dx = \frac{1}{6}\,cx^6+\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
[Out] 1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax^2+bx^3+cx^4)\, dx = \frac{1}{6}\,cx^6+\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
[Out] 1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax^2+bx^3+cx^4)\, dx = \frac{x^4\,\left(10\,c\,x^2+12\,b\,x+15\,a\right)}{60}$$

```
[In] int(x*(a*x^2 + b*x^3 + c*x^4),x)
[Out] (x^4*(15*a + 12*b*x + 10*c*x^2))/60
```

3.3 $\int (ax^2 + bx^3 + cx^4)\ dx$

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int (ax^2 + bx^3 + cx^4)\ dx = \frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

```
[Out] 1/3*a*x^3+1/4*b*x^4+1/5*c*x^5
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ax^2 + bx^3 + cx^4)\ dx = \frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

```
[In] Int[a*x^2 + b*x^3 + c*x^4,x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

Rubi steps

$$\text{integral} = \frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left(ax^2+bx^3+cx^4\right)\,dx=\frac{ax^3}{3}+\frac{bx^4}{4}+\frac{cx^5}{5}$$

```
[In] Integrate[a*x^2 + b*x^3 + c*x^4,x]
[Out] (a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{x^3\left(12c\,x^2+15bx+20a\right)}{60}$	20
default	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4+\frac{1}{5}c\,x^5$	20
norman	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4+\frac{1}{5}c\,x^5$	20
risch	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4+\frac{1}{5}c\,x^5$	20
parallelrisch	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4+\frac{1}{5}c\,x^5$	20
parts	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4+\frac{1}{5}c\,x^5$	20

```
[In] int(c*x^4+b*x^3+a*x^2,x,method=_RETURNVERBOSE)
[Out] 1/60*x^3*(12*c*x^2+15*b*x+20*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3+cx^4\right)\,dx=\frac{1}{5}\,cx^5+\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

```
[In] integrate(c*x^4+b*x^3+a*x^2,x, algorithm="fricas")
[Out] 1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3+cx^4\right)\,dx=\frac{ax^3}{3}+\frac{bx^4}{4}+\frac{cx^5}{5}$$

```
[In] integrate(c*x**4+b*x**3+a*x**2,x)
[Out] a*x**3/3 + b*x**4/4 + c*x**5/5
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3+cx^4\right)\,dx=\frac{1}{5}\,cx^5+\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

```
[In] integrate(c*x^4+b*x^3+a*x^2,x, algorithm="maxima")
[Out] 1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3+cx^4\right)\,dx=\frac{1}{5}\,cx^5+\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

```
[In] integrate(c*x^4+b*x^3+a*x^2,x, algorithm="giac")
[Out] 1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3+cx^4\right)\,dx=\frac{x^3\,\left(12\,c\,x^2+15\,b\,x+20\,a\right)}{60}$$

```
[In] int(a*x^2 + b*x^3 + c*x^4,x)
[Out] (x^3*(20*a + 15*b*x + 12*c*x^2))/60
```

3.4 $\int \frac{ax^2+bx^3+cx^4}{x} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 25

$$\int \frac{ax^2+bx^3+cx^4}{x} \, dx = \frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

```
[Out] 1/2*a*x^2+1/3*b*x^3+1/4*c*x^4
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {14}

$$\int \frac{ax^2+bx^3+cx^4}{x} \, dx = \frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)/x,x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(ax+bx^2+cx^3\right) \, dx \\ &= \frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{ax^2 + bx^3 + cx^4}{x} \, dx = \frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)/x,x]
[Out] (a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{x^2 (3c x^2+4bx+6a)}{12}$	20
default	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3+\frac{1}{4}c\,x^4$	20
norman	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3+\frac{1}{4}c\,x^4$	20
risch	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3+\frac{1}{4}c\,x^4$	20
parallelrisch	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3+\frac{1}{4}c\,x^4$	20

```
[In] int((c*x^4+b*x^3+a*x^2)/x,x,method=_RETURNVERBOSE)
[Out] 1/12*x^2*(3*c*x^2+4*b*x+6*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3 + cx^4}{x} \, dx = \frac{1}{4} \, cx^4 + \frac{1}{3} \, bx^3 + \frac{1}{2} \, ax^2$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="fricas")
[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3 + cx^4}{x} \, dx = \frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)/x,x)
[Out] a*x**2/2 + b*x**3/3 + c*x**4/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3 + cx^4}{x} \, dx = \frac{1}{4} cx^4 + \frac{1}{3} bx^3 + \frac{1}{2} ax^2$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="maxima")
[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3 + cx^4}{x} \, dx = \frac{1}{4} cx^4 + \frac{1}{3} bx^3 + \frac{1}{2} ax^2$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="giac")
[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3 + cx^4}{x} \, dx = \frac{x^2 \left(3 \, c \, x^2 + 4 \, b \, x + 6 \, a\right)}{12}$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)/x,x)
[Out] (x^2*(6*a + 4*b*x + 3*c*x^2))/12
```

3.5 $\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {14}

$$\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)/x^2,x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a+bx+cx^2\right)\,dx\\ &= ax+\frac{bx^2}{2}+\frac{cx^3}{3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{ax^2 + bx^3 + cx^4}{x^2} \, dx = ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)/x^2,x]
[Out] a*x + (b*x^2)/2 + (c*x^3)/3
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
default	$ax+\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3$	17
risch	$ax+\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3$	17
parallelrisch	$ax+\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3$	17
parts	$ax+\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3$	17
gosper	$\frac{x\left(2c\,x^2+3bx+6a\right)}{6}$	18
norman	$\frac{a\,x^2+\frac{1}{2}b\,x^3+\frac{1}{3}c\,x^4}{x}$	23

```
[In] int((c*x^4+b*x^3+a*x^2)/x^2,x,method=_RETURNVERBOSE)
[Out] a*x+1/2*b*x^2+1/3*c*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax^2 + bx^3 + cx^4}{x^2} \, dx = \frac{1}{3} \, cx^3 + \frac{1}{2} \, bx^2 + ax$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="fricas")
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{ax^2 + bx^3 + cx^4}{x^2} \, dx = ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)/x**2,x)
[Out] a*x + b*x**2/2 + c*x**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax^2 + bx^3 + cx^4}{x^2} \, dx = \frac{1}{3} cx^3 + \frac{1}{2} bx^2 + ax$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="maxima")
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax^2 + bx^3 + cx^4}{x^2} \, dx = \frac{1}{3} cx^3 + \frac{1}{2} bx^2 + ax$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="giac")
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax^2 + bx^3 + cx^4}{x^2} \, dx = \frac{c\,x^3}{3} + \frac{b\,x^2}{2} + a\,x$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)/x^2,x)
[Out] a*x + (b*x^2)/2 + (c*x^3)/3
```

3.6 $\int x^2(ax^2+bx^3+cx^4)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int x^2\left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

```
[Out] 1/7*a^2*x^7+1/4*a*b*x^8+1/9*(2*a*c+b^2)*x^9+1/5*b*c*x^10+1/11*c^2*x^11
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1599, 712}

$$\int x^2\left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{a^2x^7}{7}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{1}{4}abx^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

```
[In] Int[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^1
1)/11
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^6\left(a+bx+cx^2\right)^2\,dx\\ &= \int \left(a^2x^6+2abx^7+\left(b^2+2ac\right)x^8+2bcx^9+c^2x^{10}\right)\,dx\\ &= \frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x^2\left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

```
[In] Integrate[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^1
1)/11
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^7}{7}+\frac{ab\,x^8}{4}+\frac{(2ac+b^2)x^9}{9}+\frac{bc\,x^{10}}{5}+\frac{c^2x^{11}}{11}$	45
norman	$\frac{c^2x^{11}}{11}+\frac{bc\,x^{10}}{5}+\left(\frac{2ac}{9}+\frac{b^2}{9}\right)x^9+\frac{ab\,x^8}{4}+\frac{a^2x^7}{7}$	46
gosper	$\frac{x^7\left(1260c^2x^4+2772bc\,x^3+3080ac\,x^2+1540b^2x^2+3465abx+1980a^2\right)}{13860}$	47
risch	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{2}{9}x^9ac+\frac{1}{9}b^2x^9+\frac{1}{5}bc\,x^{10}+\frac{1}{11}c^2x^{11}$	47
parallelrisch	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{2}{9}x^9ac+\frac{1}{9}b^2x^9+\frac{1}{5}bc\,x^{10}+\frac{1}{11}c^2x^{11}$	47

```
[In] int(x^2*(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/7*a^2*x^7+1/4*a*b*x^8+1/9*(2*a*c+b^2)*x^9+1/5*b*c*x^10+1/11*c^2*x^11
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^2 (ax^2 + bx^3 + cx^4)^2 \, dx = \frac{1}{11} c^2 x^{11} + \frac{1}{5} bcx^{10} + \frac{1}{4} abx^8 + \frac{1}{9} (b^2 + 2\,ac) x^9 + \frac{1}{7} a^2 x^7$$

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/4*a*b*x^8 + 1/9*(b^2 + 2*a*c)*x^9 + 1/7*a^
2*x^7
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int x^2 (ax^2 + bx^3 + cx^4)^2 \, dx = \frac{a^2 x^7}{7} + \frac{abx^8}{4} + \frac{bcx^{10}}{5} + \frac{c^2 x^{11}}{11} + x^9 \cdot \left(\frac{2ac}{9} + \frac{b^2}{9}\right)$$

```
[In] integrate(x**2*(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**7/7 + a*b*x**8/4 + b*c*x**10/5 + c**2*x**11/11 + x**9*(2*a*c/9 + b*
*2/9)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^2 (ax^2 + bx^3 + cx^4)^2 \, dx = \frac{1}{11} c^2 x^{11} + \frac{1}{5} bcx^{10} + \frac{1}{4} abx^8 + \frac{1}{9} (b^2 + 2\,ac) x^9 + \frac{1}{7} a^2 x^7$$

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/4*a*b*x^8 + 1/9*(b^2 + 2*a*c)*x^9 + 1/7*a^
2*x^7
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x^2(ax^2+bx^3+cx^4)^2\,dx=\frac{1}{11}\,c^2x^{11}+\frac{1}{5}\,bcx^{10}+\frac{1}{9}\,b^2x^9+\frac{2}{9}\,acx^9+\frac{1}{4}\,abx^8+\frac{1}{7}\,a^2x^7$$

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/9*b^2*x^9 + 2/9*a*c*x^9 + 1/4*a*b*x^8 + 1/
7*a^2*x^7
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int x^2(ax^2+bx^3+cx^4)^2\,dx=x^9\left(\frac{b^2}{9}+\frac{2\,a\,c}{9}\right)+\frac{a^2\,x^7}{7}+\frac{c^2\,x^{11}}{11}+\frac{a\,b\,x^8}{4}+\frac{b\,c\,x^{10}}{5}$$

```
[In] int(x^2*(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] x^9*((2*a*c)/9 + b^2/9) + (a^2*x^7)/7 + (c^2*x^11)/11 + (a*b*x^8)/4 + (b*c*
x^10)/5
```

3.7 $\int x(ax^2+bx^3+cx^4)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 54

$$\int x(ax^2+bx^3+cx^4)^2\,dx=\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{1}{8}(b^2+2ac)\,x^8+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

```
[Out] 1/6*a^2*x^6+2/7*a*b*x^7+1/8*(2*a*c+b^2)*x^8+2/9*b*c*x^9+1/10*c^2*x^10
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1599, 712}

$$\int x(ax^2+bx^3+cx^4)^2\,dx=\frac{a^2x^6}{6}+\frac{1}{8}x^8(2ac+b^2)+\frac{2}{7}abx^7+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

```
[In] Int[x*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2*
x^10)/10
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int x^5 \left(a+bx+cx^2\right)^2 \, dx \\ &= \int \left(a^2x^5+2abx^6+\left(b^2+2ac\right)x^7+2bcx^8+c^2x^9\right) \, dx \\ &= \frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{1}{8}\left(b^2+2ac\right)x^8+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x\left(ax^2+bx^3+cx^4\right)^2 \, dx = \frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{1}{8}\left(b^2+2ac\right)x^8+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

```
[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4)^2,x]
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2*
x^10)/10
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^6}{6}+\frac{2ab\,x^7}{7}+\frac{(2ac+b^2)x^8}{8}+\frac{2bc\,x^9}{9}+\frac{c^2x^{10}}{10}$	45
norman	$\frac{c^2x^{10}}{10}+\frac{2bc\,x^9}{9}+\left(\frac{ac}{4}+\frac{b^2}{8}\right)x^8+\frac{2ab\,x^7}{7}+\frac{a^2x^6}{6}$	46
gosper	$\frac{x^6(252c^2x^4+560bc\,x^3+630ac\,x^2+315b^2x^2+720abx+420a^2)}{2520}$	47
risch	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{4}x^8ac+\frac{1}{8}b^2x^8+\frac{2}{9}bc\,x^9+\frac{1}{10}c^2x^{10}$	47
parallelrisch	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{4}x^8ac+\frac{1}{8}b^2x^8+\frac{2}{9}bc\,x^9+\frac{1}{10}c^2x^{10}$	47

```
[In] int(x*(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/6*a^2*x^6+2/7*a*b*x^7+1/8*(2*a*c+b^2)*x^8+2/9*b*c*x^9+1/10*c^2*x^10
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x(ax^2 + bx^3 + cx^4)^2 \, dx = \frac{1}{10} c^2 x^{10} + \frac{2}{9} bcx^9 + \frac{2}{7} abx^7 + \frac{1}{8} (b^2 + 2 \, ac)x^8 + \frac{1}{6} a^2 x^6$$

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] 1/10*c^2*x^10 + 2/9*b*c*x^9 + 2/7*a*b*x^7 + 1/8*(b^2 + 2*a*c)*x^8 + 1/6*a^2
*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int x(ax^2 + bx^3 + cx^4)^2 \, dx = \frac{a^2x^6}{6} + \frac{2abx^7}{7} + \frac{2bcx^9}{9} + \frac{c^2x^{10}}{10} + x^8\left(\frac{ac}{4} + \frac{b^2}{8}\right)$$

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2)**2,x)
[Out] a**2*x**6/6 + 2*a*b*x**7/7 + 2*b*c*x**9/9 + c**2*x**10/10 + x**8*(a*c/4 + b
**2/8)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x(ax^2 + bx^3 + cx^4)^2 \, dx = \frac{1}{10} c^2 x^{10} + \frac{2}{9} bcx^9 + \frac{2}{7} abx^7 + \frac{1}{8} (b^2 + 2 \, ac)x^8 + \frac{1}{6} a^2 x^6$$

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] 1/10*c^2*x^10 + 2/9*b*c*x^9 + 2/7*a*b*x^7 + 1/8*(b^2 + 2*a*c)*x^8 + 1/6*a^2
*x^6
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x(ax^2+bx^3+cx^4)^2\,dx=\frac{1}{10}\,c^2x^{10}+\frac{2}{9}\,bcx^9+\frac{1}{8}\,b^2x^8+\frac{1}{4}\,acx^8+\frac{2}{7}\,abx^7+\frac{1}{6}\,a^2x^6$$

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10 + 2/9*b*c*x^9 + 1/8*b^2*x^8 + 1/4*a*c*x^8 + 2/7*a*b*x^7 + 1/6
*a^2*x^6
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int x(ax^2+bx^3+cx^4)^2\,dx=x^8\left(\frac{b^2}{8}+\frac{a\,c}{4}\right)+\frac{a^2\,x^6}{6}+\frac{c^2\,x^{10}}{10}+\frac{2\,a\,b\,x^7}{7}+\frac{2\,b\,c\,x^9}{9}$$

```
[In] int(x*(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] x^8*((a*c)/4 + b^2/8) + (a^2*x^6)/6 + (c^2*x^10)/10 + (2*a*b*x^7)/7 + (2*b*
c*x^9)/9
```

3.8 $\int (ax^2 + bx^3 + cx^4)^2 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 54

$$\int (ax^2 + bx^3 + cx^4)^2 \, dx = \frac{a^2x^5}{5} + \frac{1}{3}abx^6 + \frac{1}{7}(b^2 + 2ac)\, x^7 + \frac{1}{4}bcx^8 + \frac{c^2x^9}{9}$$

```
[Out] 1/5*a^2*x^5+1/3*a*b*x^6+1/7*(2*a*c+b^2)*x^7+1/4*b*c*x^8+1/9*c^2*x^9
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1608, 712}

$$\int (ax^2 + bx^3 + cx^4)^2 \, dx = \frac{a^2x^5}{5} + \frac{1}{7}x^7(2ac + b^2) + \frac{1}{3}abx^6 + \frac{1}{4}bcx^8 + \frac{c^2x^9}{9}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9)
/9
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^4 \left(a+bx+cx^2\right)^2 \, dx \\
&= \int \left(a^2x^4+2abx^5+\left(b^2+2ac\right)x^6+2bcx^7+c^2x^8\right) \, dx \\
&= \frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \left(ax^2+bx^3+cx^4\right)^2 \, dx=\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2,x]
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9)
/9
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^5}{5}+\frac{ab\,x^6}{3}+\frac{(2ac+b^2)x^7}{7}+\frac{bc\,x^8}{4}+\frac{c^2x^9}{9}$	45
norman	$\frac{c^2x^9}{9}+\frac{bc\,x^8}{4}+\left(\frac{2ac}{7}+\frac{b^2}{7}\right)x^7+\frac{ab\,x^6}{3}+\frac{a^2x^5}{5}$	46
gosper	$\frac{x^5(140c^2x^4+315bc\,x^3+360ac\,x^2+180b^2x^2+420abx+252a^2)}{1260}$	47
risch	$\frac{1}{5}a^2x^5+\frac{1}{3}ab\,x^6+\frac{2}{7}x^7ac+\frac{1}{7}b^2x^7+\frac{1}{4}bc\,x^8+\frac{1}{9}c^2x^9$	47
parallelrisch	$\frac{1}{5}a^2x^5+\frac{1}{3}ab\,x^6+\frac{2}{7}x^7ac+\frac{1}{7}b^2x^7+\frac{1}{4}bc\,x^8+\frac{1}{9}c^2x^9$	47

```
[In] int((c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/5*a^2*x^5+1/3*a*b*x^6+1/7*(2*a*c+b^2)*x^7+1/4*b*c*x^8+1/9*c^2*x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(ax^2 + bx^3 + cx^4\right)^2 \, dx = \frac{1}{9}\, c^2 x^9 + \frac{1}{4}\, bcx^8 + \frac{1}{3}\, abx^6 + \frac{1}{7}\, \left(b^2 + 2\, ac\right)x^7 + \frac{1}{5}\, a^2 x^5$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/3*a*b*x^6 + 1/7*(b^2 + 2*a*c)*x^7 + 1/5*a^2*x
^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \left(ax^2 + bx^3 + cx^4\right)^2 \, dx = \frac{a^2 x^5}{5} + \frac{abx^6}{3} + \frac{bcx^8}{4} + \frac{c^2 x^9}{9} + x^7 \cdot \left(\frac{2ac}{7} + \frac{b^2}{7}\right)$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**5/5 + a*b*x**6/3 + b*c*x**8/4 + c**2*x**9/9 + x**7*(2*a*c/7 + b**2/
7)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \left(ax^2 + bx^3 + cx^4\right)^2 \, dx = \frac{1}{9}\, c^2 x^9 + \frac{1}{4}\, bcx^8 + \frac{1}{7}\, b^2 x^7 + \frac{1}{5}\, a^2 x^5 + \frac{1}{21}\, \left(6\, cx^7 + 7\, bx^6\right)a$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/7*b^2*x^7 + 1/5*a^2*x^5 + 1/21*(6*c*x^7 + 7*b
*x^6)*a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{1}{4}\,bcx^8+\frac{1}{7}\,b^2x^7+\frac{2}{7}\,acx^7+\frac{1}{3}\,abx^6+\frac{1}{5}\,a^2x^5$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] 1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 1/3*a*b*x^6 + 1/5*a
^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \left(ax^2+bx^3+cx^4\right)^2\,dx=x^7\left(\frac{b^2}{7}+\frac{2\,a\,c}{7}\right)+\frac{a^2\,x^5}{5}+\frac{c^2\,x^9}{9}+\frac{a\,b\,x^6}{3}+\frac{b\,c\,x^8}{4}$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^2,x)
[Out] x^7*((2*a*c)/7 + b^2/7) + (a^2*x^5)/5 + (c^2*x^9)/9 + (a*b*x^6)/3 + (b*c*x^
8)/4
```

3.9 $\int \frac{(ax^2+bx^3+cx^4)^2}{x} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x} \, dx = \frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{1}{6}\left(b^2+2ac\right)x^6 + \frac{2}{7}bcx^7 + \frac{c^2x^8}{8}$$

```
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*(2*a*c+b^2)*x^6+2/7*b*c*x^7+1/8*c^2*x^8
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1599, 712}

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x} \, dx = \frac{a^2x^4}{4} + \frac{1}{6}x^6\left(2ac+b^2\right) + \frac{2}{5}abx^5 + \frac{2}{7}bcx^7 + \frac{c^2x^8}{8}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^2/x,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2*
x^8)/8
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^3\left(a+bx+cx^2\right)^2\,dx\\ &= \int \left(a^2x^3+2abx^4+\left(b^2+2ac\right)x^5+2bcx^6+c^2x^7\right)\,dx\\ &= \frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{1}{6}\left(b^2+2ac\right)x^6+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^2}{x}\,dx = \frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{1}{6}\left(b^2+2ac\right)x^6+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}$$

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x,x]
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2*
x^8)/8
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^4}{4}+\frac{2ab\,x^5}{5}+\frac{(2ac+b^2)x^6}{6}+\frac{2bc\,x^7}{7}+\frac{c^2x^8}{8}$	45
norman	$\frac{c^2x^8}{8}+\frac{2bc\,x^7}{7}+\left(\frac{ac}{3}+\frac{b^2}{6}\right)x^6+\frac{2ab\,x^5}{5}+\frac{a^2x^4}{4}$	46
gosper	$\frac{x^4\left(105c^2x^4+240bc\,x^3+280ac\,x^2+140b^2x^2+336abx+210a^2\right)}{840}$	47
risch	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{3}x^6ac+\frac{1}{6}b^2x^6+\frac{2}{7}bc\,x^7+\frac{1}{8}c^2x^8$	47
parallelrisch	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{3}x^6ac+\frac{1}{6}b^2x^6+\frac{2}{7}bc\,x^7+\frac{1}{8}c^2x^8$	47

```
[In] int((c*x^4+b*x^3+a*x^2)^2/x,x,method=_RETURNVERBOSE)
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*(2*a*c+b^2)*x^6+2/7*b*c*x^7+1/8*c^2*x^8
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x} dx = \frac{1}{8} c^2 x^8 + \frac{2}{7} bcx^7 + \frac{2}{5} abx^5 + \frac{1}{6} (b^2 + 2 ac)x^6 + \frac{1}{4} a^2 x^4$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 2/5*a*b*x^5 + 1/6*(b^2 + 2*a*c)*x^6 + 1/4*a^2*x
^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x} dx = \frac{a^2 x^4}{4} + \frac{2abx^5}{5} + \frac{2bcx^7}{7} + \frac{c^2 x^8}{8} + x^6 \left(\frac{ac}{3} + \frac{b^2}{6}\right)$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**2/x,x)
```

```
[Out] a**2*x**4/4 + 2*a*b*x**5/5 + 2*b*c*x**7/7 + c**2*x**8/8 + x**6*(a*c/3 + b**
2/6)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x} dx = \frac{1}{8} c^2 x^8 + \frac{2}{7} bcx^7 + \frac{2}{5} abx^5 + \frac{1}{6} (b^2 + 2 ac)x^6 + \frac{1}{4} a^2 x^4$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 2/5*a*b*x^5 + 1/6*(b^2 + 2*a*c)*x^6 + 1/4*a^2*x
^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x} dx = \frac{1}{8}c^2x^8 + \frac{2}{7}bcx^7 + \frac{1}{6}b^2x^6 + \frac{1}{3}acx^6 + \frac{2}{5}abx^5 + \frac{1}{4}a^2x^4$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 2/5*a*b*x^5 + 1/4*a
^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x} dx = x^6 \left(\frac{b^2}{6} + \frac{a\,c}{3}\right) + \frac{a^2\,x^4}{4} + \frac{c^2\,x^8}{8} + \frac{2\,a\,b\,x^5}{5} + \frac{2\,b\,c\,x^7}{7}$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^2/x,x)
```

```
[Out] x^6*((a*c)/3 + b^2/6) + (a^2*x^4)/4 + (c^2*x^8)/8 + (2*a*b*x^5)/5 + (2*b*c*
x^7)/7
```

3.10 $\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx = \frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}$$

```
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*(2*a*c+b^2)*x^5+1/3*b*c*x^6+1/7*c^2*x^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1599, 712}

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx = \frac{a^2x^3}{3}+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7)
/7
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^2\left(a+bx+cx^2\right)^2\,dx\\ &= \int \left(a^2x^2+2abx^3+\left(b^2+2ac\right)x^4+2bcx^5+c^2x^6\right)\,dx\\ &= \frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^2}{x^2}\,dx=\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}$$

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7)
/7
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^3}{3}+\frac{ab\,x^4}{2}+\frac{(2ac+b^2)x^5}{5}+\frac{bc\,x^6}{3}+\frac{c^2x^7}{7}$	45
gosper	$\frac{x^3(30c^2x^4+70bc\,x^3+84ac\,x^2+42b^2x^2+105abx+70a^2)}{210}$	47
risch	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{2}{5}x^5ac+\frac{1}{5}b^2x^5+\frac{1}{3}bc\,x^6+\frac{1}{7}c^2x^7$	47
parallelrisch	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{2}{5}x^5ac+\frac{1}{5}b^2x^5+\frac{1}{3}bc\,x^6+\frac{1}{7}c^2x^7$	47
norman	$\frac{\left(\frac{2ac}{5}+\frac{b^2}{5}\right)x^6+\frac{a^2x^4}{3}+\frac{c^2x^8}{7}+\frac{ab\,x^5}{2}+\frac{bc\,x^7}{3}}{x}$	50

```
[In] int((c*x^4+b*x^3+a*x^2)^2/x^2,x,method=_RETURNVERBOSE)
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*(2*a*c+b^2)*x^5+1/3*b*c*x^6+1/7*c^2*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx = \frac{1}{7}\,c^2x^7 + \frac{1}{3}\,bcx^6 + \frac{1}{2}\,abx^4 + \frac{1}{5}\,(b^2+2\,ac)x^5 + \frac{1}{3}\,a^2x^3$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 + 1/3*a^2*x
^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx = \frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{bcx^6}{3} + \frac{c^2x^7}{7} + x^5\cdot\left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**2/x**2,x)
```

```
[Out] a**2*x**3/3 + a*b*x**4/2 + b*c*x**6/3 + c**2*x**7/7 + x**5*(2*a*c/5 + b**2/
5)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx = \frac{1}{7}\,c^2x^7 + \frac{1}{3}\,bcx^6 + \frac{1}{2}\,abx^4 + \frac{1}{5}\,(b^2+2\,ac)x^5 + \frac{1}{3}\,a^2x^3$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 + 1/3*a^2*x
^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x^2} dx = \frac{1}{7} c^2 x^7 + \frac{1}{3} bcx^6 + \frac{1}{5} b^2 x^5 + \frac{2}{5} acx^5 + \frac{1}{2} abx^4 + \frac{1}{3} a^2 x^3$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 1/2*a*b*x^4 + 1/3*a
^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x^2} dx = x^5 \left(\frac{b^2}{5} + \frac{2\,a\,c}{5}\right) + \frac{a^2\,x^3}{3} + \frac{c^2\,x^7}{7} + \frac{a\,b\,x^4}{2} + \frac{b\,c\,x^6}{3}$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^2/x^2,x)
```

```
[Out] x^5*((2*a*c)/5 + b^2/5) + (a^2*x^3)/3 + (c^2*x^7)/7 + (a*b*x^4)/2 + (b*c*x^
6)/3
```

3.11 $\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx=-\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{b(b^2-3ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2c^3}$$

```
[Out] -b*x/c^2+1/2*x^2/c+1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/c^3+b*(-3*a*c+b^2)*arctan
h((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1599, 715, 648, 632, 212, 642}

$$\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx=\frac{b(b^2-3ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2c^3}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

```
[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*c^
3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^3}{a+bx+cx^2}\,dx$$

$$= \int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+(b^2-ac)\,x}{c^2\,(a+bx+cx^2)}\right)\,dx$$

$$= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{\int \frac{ab+(b^2-ac)x}{a+bx+cx^2}\,dx}{c^2}$$

$$
= -\frac{bx}{c^2} + \frac{x^2}{2c} - \frac{(b(b^2-3ac)) \int \frac{1}{a+bx+cx^2} \, dx}{2c^3} + \frac{(b^2-ac) \int \frac{b+2cx}{a+bx+cx^2} \, dx}{2c^3}
$$

$$
= -\frac{bx}{c^2} + \frac{x^2}{2c} + \frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3} + \frac{(b(b^2-3ac))\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2} \, dx, x, b+2cx\right)}{c^3}
$$

$$
= -\frac{bx}{c^2} + \frac{x^2}{2c} + \frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$
\int \frac{x^5}{ax^2+bx^3+cx^4} \, dx = \frac{cx(-2b+cx) - \frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + (b^2-ac)\log(a+x(b+cx))}{2c^3}
$$

```
[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c
]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x)])/(2*c^3)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10

method	result
default	$-\frac{-\frac{1}{2}c\,x^2+bx}{c^2}+\frac{\frac{(-ac+b^2)\ln(c\,x^2+bx+a)}{2c}+\frac{2\left(ab-\frac{(-ac+b^2)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c^2}$
risch	$\frac{x^2}{2c}-\frac{bx}{c^2}-\frac{2\ln\left(12a^2b\,c^2-7a\,b^3c+b^5-2\sqrt{-b^2(4ac-b^2)(3ac-b^2)^2}\,cx-\sqrt{-b^2(4ac-b^2)(3ac-b^2)^2}\,b\right)a^2}{c(4ac-b^2)}+\frac{5\ln\left(12a^2b\,c^2-7a\,b^3c-\right.}{}$

```
[In] int(x^5/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2*(-1/2*c*x^2+b*x)+1/c^2*(1/2*(-a*c+b^2)/c*ln(c*x^2+b*x+a)+2*(a*b-1/2*
(-a*c+b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.34

$$\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx$$
$$= \left[\frac{(b^2c^2-4\,ac^3)x^2-(b^3-3\,abc)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^3c-4\,abc^2)x+(b^4}{2\,(b^2c^3-4\,ac^4)} \right.$$

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2 - 4*a*c^3)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^
2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2
+ b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*x^2 + 2*(b^3 - 3*
a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2 +
b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(83) = 166.

Time = 0.48 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.28

$$\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx = -\frac{bx}{c^2}+\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)\log\left(x+\frac{2a^2c-ab^2+4ac^3\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right) + \left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)\log\left(x+\frac{2a^2c-ab^2+4ac^3\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right) + \frac{x^2}{2c}$$

```
[In] integrate(x**5/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] -b*x/c**2 + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
- (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4*a*c**3*(-b*sqrt(-4*
```

```
a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))
 - b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
 - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*(3*a*
c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*
c - a*b**2 + 4*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c
- b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*sqrt(-4*a*c + b**2)*(3*a*c
 - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)
) + x**2/(2*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{ax^2 + bx^3 + cx^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{ax^2 + bx^3 + cx^4}\,dx = \frac{cx^2 - 2\,bx}{2\,c^2} + \frac{(b^2 - ac)\log\left(cx^2 + bx + a\right)}{2\,c^3} - \frac{(b^3 - 3\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}c^3}$$

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/c^3 - (b^3 -
 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx = \frac{x^2}{2\,c} - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(4\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{2\,\left(4\,a\,c^4-b^2\,c^3\right)} - \frac{b\,x}{c^2} + \frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(3\,a\,c-b^2\right)}{c^3\,\sqrt{4\,a\,c-b^2}}$$

```
[In] int(x^5/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] x^2/(2*c) - (log(a + b*x + c*x^2)*(b^4 + 4*a^2*c^2 - 5*a*b^2*c))/(2*(4*a*c^
4 - b^2*c^3)) - (b*x)/c^2 + (b*atan((b + 2*c*x)/(4*a*c - b^2)^(1/2))*(3*a*c
 - b^2))/(c^3*(4*a*c - b^2)^(1/2))
```

3.12 $\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx=\frac{x}{c}-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}$$

```
[Out] x/c-1/2*b*ln(c*x^2+b*x+a)/c^2-(-2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(
1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1599, 717, 648, 632, 212, 642}

$$\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx=-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

```
[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2
- 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2}{a+bx+cx^2}\,dx \\
&= \frac{x}{c}+\frac{\int \frac{-a-bx}{a+bx+cx^2}\,dx}{c} \\
&= \frac{x}{c}-\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{(b^2-2ac)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2} \\
&= \frac{x}{c}-\frac{b\log(a+bx+cx^2)}{2c^2}-\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^2} \\
&= \frac{x}{c}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log(a+bx+cx^2)}{2c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx=\frac{x}{c}+\frac{(b^2-2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}$$

```
[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2
 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07

method	result
default	$\frac{x}{c}+\frac{-\frac{b\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-a+\frac{b^2}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c}$
risch	$\frac{x}{c}-\frac{2\ln\left(-8a^2c^2+6a\,b^2c-b^4-2\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,cx-\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,b\right)ab}{c(4ac-b^2)}+\frac{\ln\left(-8a^2c^2+6a\,b^2c-b^4-2\sqrt{-(4ac-\right.}{}$

```
[In] int(x^4/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] x/c+1/c*(-1/2*b/c*ln(c*x^2+b*x+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.36

$$\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx$$
$$=\left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^2c-4\,ac^2)x+(b^3-4\,abc)\log\left(cx^2\right.}{2\,(b^2c^2-4\,ac^3)}\right.$$
$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)-2\,(b^2c-4\,ac^2)x+(b^3-4\,abc)\log\left(cx^2+bx+a\right.}{2\,(b^2c^2-4\,ac^3)}$$

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a
*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2*c - 4*a*c^2
)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), -1/2*(2*(b
^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(
b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(65) = 130$.

Time = 0.35 (sec) , antiderivative size = 306, normalized size of antiderivative = 4.37

$$\int \frac{x^4}{ax^2 + bx^3 + cx^4}\,dx = \left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x + \frac{-ab - 4ac^2\left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right) + b^2c\left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right) + \left(-\frac{b}{2c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x + \frac{-ab - 4ac^2\left(-\frac{b}{2c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right) + b^2c\left(-\frac{b}{2c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right) + \frac{x}{c}$$

```
[In] integrate(x**4/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*
log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/
(2*c**2*(4*a*c - b**2))) + b**2*c*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c
 - b**2)/(2*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(2*c**2) + sqrt(-4
*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4*a*c*
*2*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)
)) + b**2*c*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*
c - b**2))))/(2*a*c - b**2)) + x/c
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{ax^2 + bx^3 + cx^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{ax^2 + bx^3 + cx^4}\,dx = \frac{x}{c} - \frac{b\log\left(cx^2 + bx + a\right)}{2\,c^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}c^2}$$

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] x/c - 1/2*b*log(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.46

$$\int \frac{x^4}{ax^2 + bx^3 + cx^4}\,dx = \frac{x}{c} + \frac{b^3\,\ln\left(c\,x^2 + b\,x + a\right)}{2\,\left(4\,a\,c^3 - b^2\,c^2\right)} - \frac{2\,a\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}} + \frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c - b^2}} + \frac{b^2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}} + \frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c^2\,\sqrt{4\,a\,c - b^2}} - \frac{2\,a\,b\,c\,\ln\left(c\,x^2 + b\,x + a\right)}{4\,a\,c^3 - b^2\,c^2}$$

```
[In] int(x^4/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] x/c + (b^3*log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) - (2*a*atan(b/(4*a
*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) + (
b^2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c^2*(4*a*c
- b^2)^(1/2)) - (2*a*b*c*log(a + b*x + c*x^2))/(4*a*c^3 - b^2*c^2)
```

3.13 $\int \frac{x^3}{ax^2+bx^3+cx^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 56

$$\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx=\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

```
[Out] 1/2*ln(c*x^2+b*x+a)/c+b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1599, 648, 632, 212, 642}

$$\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx=\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

```
[In] Int[x^3/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a +
b*x + c*x^2]/(2*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x}{a+bx+cx^2}\,dx \\
&= \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c} - \frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2c} \\
&= \frac{\log\left(a+bx+cx^2\right)}{2c} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c} \\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx+cx^2\right)}{2c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx = \frac{-\frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log(a+x(b+cx))}{2c}$$

```
[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4),x]
[Out] ((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a +
 x*(b + c*x)])/(2*c)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

method	result
default	$\frac{\ln\left(c\,x^2+bx+a\right)}{2c}-\frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}}$
risch	$\frac{2\ln\left(-2\sqrt{-b^2(4ac-b^2)}\,cx-4abc+b^3-\sqrt{-b^2(4ac-b^2)}\,b\right)a}{4ac-b^2}-\frac{\ln\left(-2\sqrt{-b^2(4ac-b^2)}\,cx-4abc+b^3-\sqrt{-b^2(4ac-b^2)}\,b\right)b^2}{2c(4ac-b^2)}+\frac{\ln\left(-2\sqrt{\right.}}{}$

```
[In] int(x^3/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(c*x^2+b*x+a)/c-b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.30

$$\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx$$
$$= \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+(b^2-4\,ac)\log\left(cx^2+bx+a\right)}{2\,(b^2c-4\,ac^2)}, \frac{2\sqrt{-b^2+4\,ac}b\,\mathrm{arctan}}{}\right.$$

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^2 - 4*a*c)*log(c*x^2 + b*x +
 a))/(b^2*c - 4*a*c^2), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 + 4*a
*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b^2*c
 - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(49) = 98.

Time = 0.18 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.86

$$\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)} + \frac{1}{2c}\right)\log\left(x + \frac{-4ac\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)} + \frac{1}{2c}\right)\log\left(x + \frac{-4ac\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right)$$

```
[In] integrate(x**3/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(-b
*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(-b*sqrt(
-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b) + (b*sqrt(-4*a*c + b**2)
/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)/(2*
c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*
c - b**2)) + 1/(2*c)))/b)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{ax^2 + bx^3 + cx^4} \, dx = -\frac{b \arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,acc}} + \frac{\log\left(cx^2 + bx + a\right)}{2\,c}$$

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*log(
c*x^2 + b*x + a)/c
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.00

$$\int \frac{x^3}{ax^2 + bx^3 + cx^4} \, dx = \frac{2\,a\,c\,\ln\left(c\,x^2 + b\,x + a\right)}{4\,a\,c^2 - b^2\,c} - \frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}} + \frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c - b^2}} - \frac{b^2\,\ln\left(c\,x^2 + b\,x + a\right)}{2\,\left(4\,a\,c^2 - b^2\,c\right)}$$

```
[In] int(x^3/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] (2*a*c*log(a + b*x + c*x^2))/(4*a*c^2 - b^2*c) - (b*atan(b/(4*a*c - b^2)^(1
/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) - (b^2*log(a +
b*x + c*x^2))/(2*(4*a*c^2 - b^2*c))
```

3.14 $\int \frac{x^2}{ax^2+bx^3+cx^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 34

$$\int \frac{x^2}{ax^2+bx^3+cx^4}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1599, 632, 212}

$$\int \frac{x^2}{ax^2+bx^3+cx^4}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[In] Int[x^2/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{a+bx+cx^2}\,dx \\ &= -\left(2\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)\right) \\ &= -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{ax^2+bx^3+cx^4}\,dx = \frac{2\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

```
[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4),x]
[Out] (2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$	35
risch	$-\frac{\ln\left(2cx+\sqrt{-4ac+b^2}+b\right)}{\sqrt{-4ac+b^2}}+\frac{\ln\left(-2cx+\sqrt{-4ac+b^2}-b\right)}{\sqrt{-4ac+b^2}}$	61

```
[In] int(x^2/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.53

$$\int \frac{x^2}{ax^2+bx^3+cx^4}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4\,ac}},\right.$$
$$\left.-\frac{2\,\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(34) = 68.

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.65

$$\int \frac{x^2}{ax^2+bx^3+cx^4}\,dx = -\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$
$$+\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$

```
[In] integrate(x**2/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqr
t(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c*s
qrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{ax^2 + bx^3 + cx^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax^2 + bx^3 + cx^4}\,dx = \frac{2\,\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}}$$

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{x^2}{ax^2 + bx^3 + cx^4}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}} + \frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c - b^2}}$$

```
[In] int(x^2/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] (2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)
^(1/2)
```

3.15 $\int \frac{x}{ax^2+bx^3+cx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{x}{ax^2+bx^3+cx^4}\,dx=\frac{b\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}$$

```
[Out] ln(x)/a-1/2*ln(c*x^2+b*x+a)/a+b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a/(-4
*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1599, 719, 29, 648, 632, 212, 642}

$$\int \frac{x}{ax^2+bx^3+cx^4}\,dx=\frac{b\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

```
[In] Int[x/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + Log[x]/a
 - Log[a + b*x + c*x^2]/(2*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x\,(a+bx+cx^2)}\,dx\\
&= \frac{\int \frac{1}{x}\,dx}{a}+\frac{\int \frac{-b-cx}{a+bx+cx^2}\,dx}{a}\\
&= \frac{\log(x)}{a}-\frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a}-\frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2a}
\end{aligned}$$

$$= \frac{\log(x)}{a} - \frac{\log(a+bx+cx^2)}{2a} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{a}$$

$$= \frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log(a+bx+cx^2)}{2a}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{x}{ax^2+bx^3+cx^4}\,dx = -\frac{\frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 2\log(x) + \log(a+x(b+cx))}{2a}$$

```
[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4),x]
[Out] -1/2*((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*L
og[x] + Log[a + x*(b + c*x)])/a
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result
default	$\frac{\ln(x)}{a} + \frac{-\frac{\ln\left(c\,x^2+bx+a\right)}{2} - \frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a}$
risch	$-\frac{2\ln\left(\left(8a\,b^2c-2b^4+6\sqrt{-b^2(4ac-b^2)}\,ac-2\sqrt{-b^2(4ac-b^2)}\,b^2\right)x+12cb\,a^2-3a\,b^3-\sqrt{-b^2(4ac-b^2)}\,ab\right)c}{4ac-b^2} + \frac{\ln\left(\left(8a\,b^2c-2b^4+6\sqrt{-\right.\right.}{}$

```
[In] int(x/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] ln(x)/a+1/a*(-1/2*ln(c*x^2+b*x+a)-b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.40

$$\int \frac{x}{ax^2 + bx^3 + cx^4}\,dx = \left[\frac{\sqrt{b^2 - 4\,ac}b\log\left(\frac{2\,c^2x^2 + 2\,bcx + b^2 - 2\,ac + \sqrt{b^2 - 4\,ac}(2\,cx + b)}{cx^2 + bx + a}\right) - (b^2 - 4\,ac)\log\left(cx^2 + bx + a\right) + 2\,(b^2 - 4\,ac)\log\left(x\right)}{2\,(ab^2 - 4\,a^2c)},\right.$$

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^2 - 4*a*c)*log(c*x^2 + b*x +
 a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/2*(2*sqrt(-b^2 + 4*a*c)*
b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log
(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(54) = 108$.

Time = 4.49 (sec) , antiderivative size = 564, normalized size of antiderivative = 9.10

$$\int \frac{x}{ax^2 + bx^3 + cx^4}\,dx = \left(-\frac{b\sqrt{-4ac + b^2}}{2a\,(4ac - b^2)} - \frac{1}{2a}\right)\log\left(x + \frac{24a^4c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right)^2 - 14a^3b^2c\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right)^2 - 12a^3c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right) + \cdots}{9abc^2 - 2b^3 \cdots}\right) + \left(\frac{b\sqrt{-4ac + b^2}}{2a\,(4ac - b^2)} - \frac{1}{2a}\right)\log\left(x + \frac{24a^4c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right)^2 - 14a^3b^2c\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right)^2 - 12a^3c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right) + 2a^2b^4 \cdots}{9abc^2 - 2b^3c}\right) + \frac{\log\left(x\right)}{a}$$

```
[In] integrate(x/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c*
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**
2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c*
```

```
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a
*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
 b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b*
*3*c)) + log(x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{ax^2 + bx^3 + cx^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax^2 + bx^3 + cx^4}\,dx = -\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a} - \frac{\log\left(cx^2+bx+a\right)}{2\,a} + \frac{\log\left(|x|\right)}{a}$$

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/2*log(
c*x^2 + b*x + a)/a + log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 8.72 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.44

$$\int \frac{x}{ax^2+bx^3+cx^4}\,dx=\frac{\ln(x)}{a}-\ln\left(bc-\left(x\left(6\,a\,c^2-2\,b^2\,c\right)-a\,b\,c\right)\left(\frac{1}{2\,a}-\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)+3\,c^2\,x\right)\left(\frac{1}{2\,a}-\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)-\ln\left(\left(x\left(6\,a\,c^2-2\,b^2\,c\right)-a\,b\,c\right)\left(\frac{1}{2\,a}+\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)-b\,c-3\,c^2\,x\right)\left(\frac{1}{2\,a}+\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)$$

```
[In] int(x/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] log(x)/a - log(b*c - (x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a) - (b*(b^2 - 4
*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) + 3*c^2*x)*(1/(2*a) - (b*(b^2 - 4*a*c)^
(1/2))/(2*(a*b^2 - 4*a^2*c))) - log((x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a
) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) - b*c - 3*c^2*x)*(1/(2*a
) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c)))
```

3.16 $\int \frac{1}{ax^2+bx^3+cx^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{1}{ax^2+bx^3+cx^4} dx = -\frac{1}{ax} - \frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx+cx^2)}{2a^2}$$

```
[Out] -1/a/x-b*ln(x)/a^2+1/2*b*ln(c*x^2+b*x+a)/a^2-(-2*a*c+b^2)*arctanh((2*c*x+b)
/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1608, 723, 814, 648, 632, 212, 642}

$$\int \frac{1}{ax^2+bx^3+cx^4} dx = -\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}} + \frac{b\log(a+bx+cx^2)}{2a^2} - \frac{b\log(x)}{a^2} - \frac{1}{ax}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(-1),x]
```

```
[Out] -(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sq
rt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*x^2])/(2*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2 (a+bx+cx^2)} \, dx$$

$$= -\frac{1}{ax} + \frac{\int \frac{-b-cx}{x(a+bx+cx^2)} \, dx}{a}$$

$$= -\frac{1}{ax} + \frac{\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right) dx}{a}$$

$$= -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{\int \frac{b^2-ac+bcx}{a+bx+cx^2} dx}{a^2}$$

$$= -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{b\int \frac{b+2cx}{a+bx+cx^2} dx}{2a^2} + \frac{(b^2-2ac)\int \frac{1}{a+bx+cx^2} dx}{2a^2}$$

$$= -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx+cx^2)}{2a^2} - \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b+2cx\right)}{a^2}$$

$$= -\frac{1}{ax} - \frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx+cx^2)}{2a^2}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{1}{ax^2+bx^3+cx^4} dx = \frac{-\frac{2a}{x} + \frac{2(b^2-2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 2b\log(x) + b\log(a+x(b+cx))}{2a^2}$$

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-1),x]
```

```
[Out] ((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-
b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)])/(2*a^2)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

method	result
default	$-\frac{1}{ax} - \frac{b\ln(x)}{a^2} + \frac{\frac{b\ln(cx^2+bx+a)}{2} + \frac{2\left(-ac+\frac{b^2}{2}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a^2}$
risch	$-\frac{1}{ax} - \frac{b\ln(x)}{a^2} + \left(\sum_{\_R=\text{RootOf}\left((4a^3c-a^2b^2)\_Z^2+(-4abc+b^3)\_Z+c^2\right)} \_R\ln\left(\left(\left(6a^3c-2a^2b^2\right)\_R^2 - 2\_Rabc\right.\right.\right.$

```
[In] int(1/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/x-b*ln(x)/a^2+1/a^2*(1/2*b*ln(c*x^2+b*x+a)+2*(-a*c+1/2*b^2)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.32

$$\int \frac{1}{ax^2+bx^3+cx^4}\,dx = \left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}x\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,ab^2-8\,a^2c-(b^3-4\,abc)x\log\left(cx^2+\right.}{2\,(a^2b^2-4\,a^3c)x}\right.$$
$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}x\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,ab^2-8\,a^2c-(b^3-4\,abc)x\log\left(cx^2+bx+a\right)}{2\,(a^2b^2-4\,a^3c)x}$$

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2
*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*a*b^2 - 8*a^2*
c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((
a^2*b^2 - 4*a^3*c)*x), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4
*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^2 - 4*
a^3*c)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{ax^2+bx^3+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{ax^2+bx^3+cx^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{1}{ax^2+bx^3+cx^4}\,dx = \frac{b\log\left(cx^2+bx+a\right)}{2\,a^2} - \frac{b\log\left(|x|\right)}{a^2} + \frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^2} - \frac{1}{ax}$$

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*b*log(c*x^2 + b*x + a)/a^2 - b*log(abs(x))/a^2 + (b^2 - 2*a*c)*arctan((
2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 339, normalized size of antiderivative = 4.19

$$\int \frac{1}{ax^2+bx^3+cx^4}\,dx = \frac{\ln\left(2\,a\,b^3+2\,b^4\,x-2\,a\,b^2\,\sqrt{b^2-4\,a\,c}+a^2\,c\,\sqrt{b^2-4\,a\,c}-2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b^2\,c\,x\right.}{4\,a^3\,c-a^2\,b^2} - \frac{1}{a\,x} - \frac{\ln\left(2\,a\,b^3+2\,b^4\,x+2\,a\,b^2\,\sqrt{b^2-4\,a\,c}-a^2\,c\,\sqrt{b^2-4\,a\,c}+2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b^2\right.}{4\,a^3\,c-a^2\,b^2} - \frac{b\,\ln\left(x\right)}{a^2}$$

```
[In] int(1/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] (log(2*a*b^3 + 2*b^4*x - 2*a*b^2*(b^2 - 4*a*c)^(1/2) + a^2*c*(b^2 - 4*a*c)^
(1/2) - 2*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x
 + 4*a*b*c*x*(b^2 - 4*a*c)^(1/2))*(a*(2*b*c - c*(b^2 - 4*a*c)^(1/2)) - b^3/
2 + (b^2*(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - 1/(a*x) - (log(2*a*
b^3 + 2*b^4*x + 2*a*b^2*(b^2 - 4*a*c)^(1/2) - a^2*c*(b^2 - 4*a*c)^(1/2) + 2
*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x - 4*a*b*
c*x*(b^2 - 4*a*c)^(1/2))*(b^3/2 - a*(2*b*c + c*(b^2 - 4*a*c)^(1/2)) + (b^2*
(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - (b*log(x))/a^2
```

3.17 $\int \frac{1}{x(ax^2+bx^3+cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{1}{x(ax^2+bx^3+cx^4)}\,dx = -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b(b^2-3ac)\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2a^3}$$

```
[Out] -1/2/a/x^2+b/a^2/x+(-a*c+b^2)*ln(x)/a^3-1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/a^3+
b*(-3*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1599, 723, 814, 648, 632, 212, 642}

$$\int \frac{1}{x(ax^2+bx^3+cx^4)}\,dx = \frac{b(b^2-3ac)\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}-\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

```
[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2
- 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2 - a*c
)*Log[a + b*x + c*x^2])/(2*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^3 (a+bx+cx^2)} \, dx \\
&= -\frac{1}{2ax^2} + \frac{\int \frac{-b-cx}{x^2(a+bx+cx^2)} \, dx}{a} \\
&= -\frac{1}{2ax^2} + \frac{\int \left(-\frac{b}{ax^2} + \frac{b^2-ac}{a^2x} + \frac{-b(b^2-2ac)-c(b^2-ac)x}{a^2(a+bx+cx^2)}\right) dx}{a} \\
&= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} + \frac{\int \frac{-b(b^2-2ac)-c(b^2-ac)x}{a+bx+cx^2} \, dx}{a^3} \\
&= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b(b^2-3ac)) \int \frac{1}{a+bx+cx^2} \, dx}{2a^3} - \frac{(b^2-ac) \int \frac{b+2cx}{a+bx+cx^2} \, dx}{2a^3} \\
&= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3} \\
&\quad + \frac{(b(b^2-3ac)) \operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2} \, dx, x, b+2cx\right)}{a^3} \\
&= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} \\
&\quad + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{1}{x(ax^2+bx^3+cx^4)} \, dx = \frac{-\frac{a^2}{x^2} + \frac{2ab}{x} - \frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + 2(b^2-ac)\log(x) + (-b^2+ac)\log(a+x(b+cx))}{2a^3}$$

```
[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]
[Out] (-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x] + (-b^2 + a*c)*Log[a +
x*(b + c*x)])/(2*a^3)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.23

method	result
default	$-\frac{1}{2a\,x^{2}}+\frac{\left(-ac+b^{2}\right)\ln\left(x\right)}{a^{3}}+\frac{b}{a^{2}x}+\frac{\frac{\left(a\,c^{2}-b^{2}c\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(2abc-b^{3}-\frac{\left(a\,c^{2}-b^{2}c\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{a^{3}}$
risch	$\frac{\frac{bx}{a^{2}}-\frac{1}{2a}}{x^{2}}-\frac{\ln(x)c}{a^{2}}+\frac{\ln(x)b^{2}}{a^{3}}+\left(\sum_{\_R=\text{RootOf}\left(\left(4c\,a^{4}-a^{3}b^{2}\right)\_Z^{2}+\left(-4a^{2}c^{2}+5a\,b^{2}c-b^{4}\right)\_Z+c^{3}\right)}\_R\ln\left(\left(\left(6c\,a^{5}-2b^{2}a^{4}\right.\right.\right.\right.$

[In] int(1/x/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)

[Out] -1/2/a/x^2+(-a*c+b^2)*ln(x)/a^3+b/a^2/x+1/a^3*(1/2*(a*c^2-b^2*c)/c*ln(c*x^2
+b*x+a)+2*(2*a*b*c-b^3-1/2*(a*c^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.44

$$\int \frac{1}{x\left(ax^{2}+bx^{3}+cx^{4}\right)}\,dx = \left[-\frac{(b^{3}-3\,abc)\sqrt{b^{2}-4\,ac}x^{2}\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac-\sqrt{b^{2}-4\,ac}(2\,cx+b)}{cx^{2}+bx+a}\right)+a^{2}b^{2}-4\,a^{3}c+(b^{4}-5\,ab^{2}c+4\,a^{2}c^{2})}{2\,(a^{3}b^{2}-4\,a^{4}c)x^{2}}\right.$$

[In] integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")

[Out] [-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^2 + 2*b*c*x + b^2
 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + a^2*b^2 - 4*
a^3*c + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) - 2*(b^4 - 5
*a*b^2*c + 4*a^2*c^2)*x^2*log(x) - 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a
^4*c)*x^2), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - a^2*b^2 + 4*a^3*c - (b^4 - 5*a*b^2*c
 + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) + 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^
2*log(x) + 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2)]

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\,(ax^2+bx^3+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\,(ax^2+bx^3+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \frac{1}{x\,(ax^2+bx^3+cx^4)}\,dx = -\frac{(b^2-ac)\log(cx^2+bx+a)}{2\,a^3} + \frac{(b^2-ac)\log(|x|)}{a^3} - \frac{(b^3-3\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^3} + \frac{2\,abx-a^2}{2\,a^3x^2}$$

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*log(abs(x))/a^3 - (
b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a
^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 447, normalized size of antiderivative = 4.30

$$\int \frac{1}{x\,(ax^2+bx^3+cx^4)}\,dx$$

$$= \frac{\ln\left(2\,a\,b^4+2\,b^5\,x+6\,a^3\,c^2+2\,a\,b^3\,\sqrt{b^2-4\,a\,c}+2\,b^4\,x\,\sqrt{b^2-4\,a\,c}-9\,a^2\,b^2\,c-10\,a\,b^3\,c\,x-3\,a^2\,b\,c\,\sqrt{b^2-\right.}{4}$$

$$-\frac{\ln\left(2\,a\,b^4+2\,b^5\,x+6\,a^3\,c^2-2\,a\,b^3\,\sqrt{b^2-4\,a\,c}-2\,b^4\,x\,\sqrt{b^2-4\,a\,c}-9\,a^2\,b^2\,c-10\,a\,b^3\,c\,x+3\,a^2\,b\,c\,\sqrt{b^2\right.}{}$$

$$-\frac{\frac{1}{2\,a}-\frac{b\,x}{a^2}}{x^2}-\frac{\ln\left(x\right)\,\left(a\,c-b^2\right)}{a^3}$$

```
[In] int(1/(x*(a*x^2 + b*x^3 + c*x^4)),x)
```

```
[Out] (log(2*a*b^4 + 2*b^5*x + 6*a^3*c^2 + 2*a*b^3*(b^2 - 4*a*c)^(1/2) + 2*b^4*x*
(b^2 - 4*a*c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x - 3*a^2*b*c*(b^2 - 4*a*c)^
(1/2) + 9*a^2*b*c^2*x + 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c*x*(b^2
- 4*a*c)^(1/2))*(b^4/2 - a*((5*b^2*c)/2 + (3*b*c*(b^2 - 4*a*c)^(1/2))/2) +
(b^3*(b^2 - 4*a*c)^(1/2))/2 + 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (log(2*a*b^
4 + 2*b^5*x + 6*a^3*c^2 - 2*a*b^3*(b^2 - 4*a*c)^(1/2) - 2*b^4*x*(b^2 - 4*a*
c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x + 3*a^2*b*c*(b^2 - 4*a*c)^(1/2) + 9*a
^2*b*c^2*x - 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c*x*(b^2 - 4*a*c)^(1
/2))*(a*((5*b^2*c)/2 - (3*b*c*(b^2 - 4*a*c)^(1/2))/2) - b^4/2 + (b^3*(b^2 -
 4*a*c)^(1/2))/2 - 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (1/(2*a) - (b*x)/a^2)/
x^2 - (log(x)*(a*c - b^2))/a^3
```

3.18 $\int \frac{1}{x^2(ax^2+bx^3+cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 137

$$\int \frac{1}{x^2(ax^2+bx^3+cx^4)}\,dx = -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x} - \frac{(b^4-4ab^2c+2a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} - \frac{b(b^2-2ac)\log(x)}{a^4}+\frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^4}$$

```
[Out] -1/3/a/x^3+1/2*b/a^2/x^2+(a*c-b^2)/a^3/x-b*(-2*a*c+b^2)*ln(x)/a^4+1/2*b*(-2
*a*c+b^2)*ln(c*x^2+b*x+a)/a^4-(2*a^2*c^2-4*a*b^2*c+b^4)*arctanh((2*c*x+b)/(
-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1599, 723, 814, 648, 632, 212, 642}

$$\int \frac{1}{x^2(ax^2+bx^3+cx^4)}\,dx = \frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^4}-\frac{b\log(x)(b^2-2ac)}{a^4}-\frac{b^2-ac}{a^3x}+\frac{b}{2a^2x^2}-\frac{(2a^2c^2-4ab^2c+b^4)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-\frac{1}{3ax^3}$$

```
[In] Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*b^2*c +
2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c])
```

```
- (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*
a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
```

```
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^4\left(a+bx+cx^2\right)}\,dx \\
&= -\frac{1}{3ax^3}+\frac{\int \frac{-b-cx}{x^3(a+bx+cx^2)}\,dx}{a} \\
&= -\frac{1}{3ax^3}+\frac{\int \left(-\frac{b}{ax^3}+\frac{b^2-ac}{a^2x^2}+\frac{-b^3+2abc}{a^3x}+\frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a^3(a+bx+cx^2)}\right)dx}{a} \\
&= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{b(b^2-2ac)\log(x)}{a^4}+\frac{\int \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a+bx+cx^2}\,dx}{a^4} \\
&= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{b(b^2-2ac)\log(x)}{a^4} \\
&\quad+\frac{(b(b^2-2ac))\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^4}+\frac{(b^4-4ab^2c+2a^2c^2)\int \frac{1}{a+bx+cx^2}\,dx}{2a^4} \\
&= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{b(b^2-2ac)\log(x)}{a^4}+\frac{b(b^2-2ac)\log\left(a+bx+cx^2\right)}{2a^4} \\
&\quad-\frac{(b^4-4ab^2c+2a^2c^2)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{a^4} \\
&= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{(b^4-4ab^2c+2a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} \\
&\quad-\frac{b(b^2-2ac)\log(x)}{a^4}+\frac{b(b^2-2ac)\log\left(a+bx+cx^2\right)}{2a^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$
\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)}\,dx
= \frac{-\frac{2a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{6a(-b^2+ac)}{x}+\frac{6(b^4-4ab^2c+2a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-6(b^3-2abc)\log(x)+3(b^3-2abc)\log(a+x(b}{6a^4}
$$

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] ((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 - 4*a*b^2*c
+ 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 6
*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)])/(6*a^4)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

method	result
default	$-\frac{1}{3a\,x^{3}}-\frac{-ac+b^{2}}{x\,a^{3}}+\frac{b(2ac-b^{2})\ln(x)}{a^{4}}+\frac{b}{2a^{2}x^{2}}+\frac{\frac{(-2ab\,c^{2}+b^{3}c)\ln(c\,x^{2}+bx+a)}{2c}+\frac{2\left(a^{2}c^{2}-3a\,b^{2}c+b^{4}-\frac{(-2ab\,c^{2}+b^{3}c)b}{2c}\right)\arctan\left(\right.}{\sqrt{4ac-b^{2}}}}{a^{4}}$
risch	Expression too large to display

```
[In] int(1/x^2/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/x^3-(-a*c+b^2)/x/a^3+b*(2*a*c-b^2)/a^4*ln(x)+1/2*b/a^2/x^2+1/a^4*(1/
2*(-2*a*b*c^2+b^3*c)/c*ln(c*x^2+b*x+a)+2*(a^2*c^2-3*a*b^2*c+b^4-1/2*(-2*a*b
*c^2+b^3*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 445, normalized size of antiderivative = 3.25

$$\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)}\,dx = \left[\frac{3\left(b^4-4\,ab^2c+2\,a^2c^2\right)\sqrt{b^2-4\,ac}x^3\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,a^3b^2+8\,a^4c+3\left(b^5-6\,a\right.}{}\right.$$
$$-\frac{6\left(b^4-4\,ab^2c+2\,a^2c^2\right)\sqrt{-b^2+4\,ac}x^3\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,a^3b^2-8\,a^4c-3\left(b^5-6\,ab^3c+8\right.}{6\left(a\right.}$$

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^2
+ 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a))
 - 2*a^3*b^2 + 8*a^4*c + 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 +
b*x + a) - 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) - 6*(a*b^4 - 5*a^2*
b^2*c + 4*a^3*c^2)*x^2 + 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^
3), -1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^3*arctan(-sq
rt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a^3*b^2 - 8*a^4*c - 3*(b^5
- 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) + 6*(b^5 - 6*a*b^3*c +
8*a^2*b*c^2)*x^3*log(x) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2 - 3*(a^2*
b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (ax^2 + bx^3 + cx^4)} \, dx = \text{Timed out}$$

```
[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (ax^2 + bx^3 + cx^4)} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2 (ax^2 + bx^3 + cx^4)} \, dx = \frac{(b^3 - 2\,abc)\log\left(cx^2 + bx + a\right)}{2\,a^4} - \frac{(b^3 - 2\,abc)\log\left(|x|\right)}{a^4} + \frac{(b^4 - 4\,ab^2c + 2\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}a^4} + \frac{3\,a^2bx - 2\,a^3 - 6\,(ab^2 - a^2c)x^2}{6\,a^4x^3}$$

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(b^3 - 2*a*b*c)*log(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*log(abs(x))/
a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
(sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a^3 - 6*(a*b^2 - a^2*c)*x^2)/
(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.82

$$\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)}\,dx=\ln\Big(2\,a\,b^4\,\sqrt{b^2-4\,a\,c}-2\,b^6\,x-2\,a\,b^5+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}+11\,a^2\,b^3\,c-13\,a^3\,b\,c^2+2\,a^3\,c^3\,x+a^3\,c^2\,\sqrt{b^2-4\,a\,c}-17\,a^2\,b^2\,c^2\,x+12\,a\,b^4\,c\,x-5\,a^2\,b^2\,c\,\sqrt{b^2-4\,a\,c}-8\,a\,b^3\,c\,x\,\sqrt{b^2-4\,a\,c}+7\,a^2\,b\,c^2\,x\,\sqrt{b^2-4\,a\,c}\Big)\left(\frac{b^3}{2\,a^4}-\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2\,a^4}-\frac{b\,c}{a^3}+\frac{a^2\,c^2\,\sqrt{b^2-4\,a\,c}}{4\,a^5\,c-a^4\,b^2}\right)+\ln\Big(2\,a\,b^5+2\,b^6\,x+2\,a\,b^4\,\sqrt{b^2-4\,a\,c}+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}-11\,a^2\,b^3\,c+13\,a^3\,b\,c^2-2\,a^3\,c^3\,x+a^3\,c^2\,\sqrt{b^2-4\,a\,c}+17\,a^2\,b^2\,c^2\,x-12\,a\,b^4\,c\,x-5\,a^2\,b^2\,c\,\sqrt{b^2-4\,a\,c}-8\,a\,b^3\,c\,x\,\sqrt{b^2-4\,a\,c}+7\,a^2\,b\,c^2\,x\,\sqrt{b^2-4\,a\,c}\Big)\left(\frac{b^3}{2\,a^4}+\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2\,a^4}-\frac{b\,c}{a^3}-\frac{a^2\,c^2\,\sqrt{b^2-4\,a\,c}}{4\,a^5\,c-a^4\,b^2}\right)+\frac{\frac{x^2\,\left(a\,c-b^2\right)}{a^3}-\frac{1}{3\,a}+\frac{b\,x}{2\,a^2}}{x^3}+\frac{b\,\ln\left(x\right)\,\left(2\,a\,c-b^2\right)}{a^4}$$

```
[In] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x)
```

```
[Out] log(2*a*b^4*(b^2 - 4*a*c)^(1/2) - 2*b^6*x - 2*a*b^5 + 2*b^5*x*(b^2 - 4*a*c)
^(1/2) + 11*a^2*b^3*c - 13*a^3*b*c^2 + 2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^
(1/2) - 17*a^2*b^2*c^2*x + 12*a*b^4*c*x - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) -
 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/
(2*a^4) - (b^2*(b^2 - 4*a*c)^(1/2))/(2*a^4) - (b*c)/a^3 + (a^2*c^2*(b^2 - 4
*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + log(2*a*b^5 + 2*b^6*x + 2*a*b^4*(b^2 -
4*a*c)^(1/2) + 2*b^5*x*(b^2 - 4*a*c)^(1/2) - 11*a^2*b^3*c + 13*a^3*b*c^2 -
2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^(1/2) + 17*a^2*b^2*c^2*x - 12*a*b^4*c*x
 - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) - 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^
2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/(2*a^4) + (b^2*(b^2 - 4*a*c)^(1/2))/(2*
a^4) - (b*c)/a^3 - (a^2*c^2*(b^2 - 4*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + ((x
^2*(a*c - b^2))/a^3 - 1/(3*a) + (b*x)/(2*a^2))/x^3 + (b*log(x)*(2*a*c - b^2
))/a^4
```

3.19 $\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{2(b^2-3ac)\,x}{c^2\,(b^2-4ac)} - \frac{bx^2}{c\,(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} - \frac{2(b^4-6ab^2c+6a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{3/2}} - \frac{b\log(a+bx+cx^2)}{c^3}$$

```
[Out] 2*(-3*a*c+b^2)*x/c^2/(-4*a*c+b^2)-b*x^2/c/(-4*a*c+b^2)+x^3*(b*x+2*a)/(-4*a*
c+b^2)/(c*x^2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+
b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)-b*ln(c*x^2+b*x+a)/c^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1599, 752, 814, 648, 632, 212, 642}

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{2(6a^2c^2-6ab^2c+b^4)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{3/2}} + \frac{2x(b^2-3ac)}{c^2\,(b^2-4ac)} - \frac{bx^2}{c\,(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} - \frac{b\log(a+bx+cx^2)}{c^3}$$

```
[In] Int[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c)) + (x^3*
(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^
```

```
2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) -
(b*Log[a + b*x + c*x^2])/c^3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^4}{(a+bx+cx^2)^2}\,dx\\
&= \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{\int \frac{x^2(6a+2bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}\\
&= \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{\int \left(-\frac{2(b^2-3ac)}{c^2}+\frac{2bx}{c}+\frac{2(a(b^2-3ac)+b(b^2-4ac)x)}{c^2(a+bx+cx^2)}\right)dx}{-b^2+4ac}\\
&= \frac{2(b^2-3ac)\,x}{c^2(b^2-4ac)} - \frac{bx^2}{c(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{2\int \frac{a(b^2-3ac)+b(b^2-4ac)x}{a+bx+cx^2}\,dx}{c^2(b^2-4ac)}\\
&= \frac{2(b^2-3ac)\,x}{c^2(b^2-4ac)} - \frac{bx^2}{c(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}\\
&\quad - \frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{c^3} + \frac{(b^4-6ab^2c+6a^2c^2)\int \frac{1}{a+bx+cx^2}\,dx}{c^3(b^2-4ac)}\\
&= \frac{2(b^2-3ac)\,x}{c^2(b^2-4ac)} - \frac{bx^2}{c(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{b\log(a+bx+cx^2)}{c^3}\\
&\quad - \frac{(2(b^4-6ab^2c+6a^2c^2))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c^3(b^2-4ac)}\\
&= \frac{2(b^2-3ac)\,x}{c^2(b^2-4ac)} - \frac{bx^2}{c(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}\\
&\quad - \frac{2(b^4-6ab^2c+6a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}} - \frac{b\log(a+bx+cx^2)}{c^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.88

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{cx + \frac{-b^4x-ab^2(b-4cx)+a^2c(3b-2cx)}{(b^2-4ac)(a+x(b+cx))} - \frac{2(b^4-6ab^2c+6a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} - b\log(a+x(b+cx))}{c^3}$$

```
[In] Integrate[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2 - 4*a*c)*
(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + x*(b + c*x)])/c^3
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{x}{c^2}-\frac{\frac{-\frac{\left(2a^2c^2-4a\,b^2c+b^4\right)x}{c\left(4ac-b^2\right)}+\frac{ba\left(3ac-b^2\right)}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{\frac{\left(4abc-b^3\right)\ln\left(c\,x^2+bx+a\right)}{c}+\frac{4\left(3c\,a^2-b^2a-\frac{\left(4abc-b^3\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{c^2}$	198
risch	Expression too large to display	1176

```
[In] int(x^8/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^2-1/c^2*((-(2*a^2*c^2-4*a*b^2*c+b^4)/c/(4*a*c-b^2)*x+b*a/c*(3*a*c-b^2)/
(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(4*a*b*c-b^3)/c*ln(c*x^2+b*x+
a)+2*(3*c*a^2-b^2*a-1/2*(4*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(146) = 292$.

Time = 0.28 (sec) , antiderivative size = 837, normalized size of antiderivative = 5.58

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \left[-\frac{ab^5-7\,a^2b^3c+12\,a^3bc^2-(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^3-(b^5c-8\,ab^3c^2+16\,a^2bc^3)x^2+(ab^4-6\,a^2b^2c\right.$$
$$-\frac{ab^5-7\,a^2b^3c+12\,a^3bc^2-(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^3-(b^5c-8\,ab^3c^2+16\,a^2bc^3)x^2+2\,(ab^4-6\,a^2b^2}{}$$

```
[In] integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6
*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2
*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(
b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^6 - 9*a*b^4*c + 26*a^2*b^
```

```
2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*log(c*x
^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2
*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x), -(a*b^5
 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^3 -
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c
^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2
)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c
)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4
+ 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^
3*c^4 + 16*a^2*b*c^5)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(141) = 282$.

Time = 0.99 (sec) , antiderivative size = 842, normalized size of antiderivative = 5.61

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx = \left(-\frac{b}{c^3} - \frac{\sqrt{-(4ac-b^2)^3}\cdot(6a^2c^2-6ab^2c+b^4)}{c^3\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}\right)\log\left(x + \frac{-10a^2bc - 16a^2c^4\left(-\frac{b}{c^3} - \frac{\sqrt{-(4ac-b^2)^3}\cdot(6a^2c^2-6ab^2c+b^4)}{c^3\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}\right)}{}\right.$$

$$+\left(-\frac{b}{c^3} + \frac{\sqrt{-(4ac-b^2)^3}\cdot(6a^2c^2-6ab^2c+b^4)}{c^3\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}\right)\log\left(x + \frac{-10a^2bc - 16a^2c^4\left(-\frac{b}{c^3} + \frac{\sqrt{-(4ac-b^2)^3}\cdot(6a^2c^2-6ab^2c+b^4)}{c^3\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}\right)}{}\right.$$

$$+\frac{-3a^2bc+ab^3+x(2a^2c^2-4ab^2c+b^4)}{4a^2c^4-ab^2c^3+x^2\cdot(4ac^5-b^2c^4)+x\,(4abc^4-b^3c^3)} + \frac{x}{c^2}$$

```
[In] integrate(x**8/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**
2*b*c - 16*a**2*c**4*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a
*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**
6))) + 2*a*b**3 + 8*a*b**2*c**3*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2
```

```
*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6))) - b**4*c**2*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**
2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*
c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-b/c**3 + sqrt(-(4*a
*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a
**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**2*b*c - 16*a**2*c**4*
(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a*b**3 + 8*a
*b**2*c**3*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c +
b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - b**
4*c**2*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4
)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))))/(12*a**2
*c**2 - 12*a*b**2*c + 2*b**4)) + (-3*a**2*b*c + a*b**3 + x*(2*a**2*c**2 - 4
*a*b**2*c + b**4))/(4*a**2*c**4 - a*b**2*c**3 + x**2*(4*a*c**5 - b**2*c**4)
 + x*(4*a*b*c**4 - b**3*c**3)) + x/c**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(ax^2 + bx^3 + cx^4)^2}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07

$$\int \frac{x^8}{(ax^2 + bx^3 + cx^4)^2}\, dx = \frac{2\,(b^4 - 6\,ab^2c + 6\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^3 - 4\,ac^4)\sqrt{-b^2 + 4\,ac}} + \frac{x}{c^2} - \frac{b\log\left(cx^2 + bx + a\right)}{c^3} - \frac{\frac{(b^4-4\,ab^2c+2\,a^2c^2)x}{c} + \frac{ab^3-3\,a^2bc}{c}}{(cx^2 + bx + a)(b^2 - 4\,ac)c^2}$$

```
[In] integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^
2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + x/c^2 - b*log(c*x^2 + b*x + a)/c^3 -
 ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 - 3*a^2*b*c)/c)/((c*x^2 + b*x
+ a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.74

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{x}{c^2} + \frac{\frac{a\,(b^3-3\,a\,b\,c)}{c\,(4\,a\,c-b^2)} + \frac{x\,(2\,a^2\,c^2-4\,a\,b^2\,c+b^4)}{c\,(4\,a\,c-b^2)}}{c^3\,x^2+b\,c^2\,x+a\,c^2} + \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-128\,a^3\,b\,c^3+96\,a^2\,b^3\,c^2-24\,a\,b^5\,c+2\,b^7\right)}{2\,\left(64\,a^3\,c^6-48\,a^2\,b^2\,c^5+12\,a\,b^4\,c^4-b^6\,c^3\right)} - \frac{2\,\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c^2-4\,a\,b\,c^3}{c^2\,\left(4\,a\,c-b^2\right)^{3/2}}\right)\,\left(6\,a^2\,c^2-6\,a\,b^2\,c+b^4\right)}{c^3\,\left(4\,a\,c-b^2\right)^{3/2}}$$

```
[In] int(x^8/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] x/c^2 + ((a*(b^3 - 3*a*b*c))/(c*(4*a*c - b^2)) + (x*(b^4 + 2*a^2*c^2 - 4*a*
b^2*c))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x) + (log(a + b*x + c*x
^2)*(2*b^7 - 128*a^3*b*c^3 + 96*a^2*b^3*c^2 - 24*a*b^5*c))/(2*(64*a^3*c^6 -
 b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (2*atan((2*c*x)/(4*a*c - b^2)^
(1/2) - (b^3*c^2 - 4*a*b*c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(b^4 + 6*a^2*c^2 -
 6*a*b^2*c))/(c^3*(4*a*c - b^2)^(3/2))
```

3.20 $\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 114

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{bx}{c(b^2-4ac)}+\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}+\frac{b(b^2-6ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)^{3/2}}+\frac{\log(a+bx+cx^2)}{2c^2}$$

```
[Out] -b*x/c/(-4*a*c+b^2)+x^2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2)
*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/2*ln(c*x^2+
b*x+a)/c^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1599, 752, 787, 648, 632, 212, 642}

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{b(b^2-6ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)^{3/2}}+\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{bx}{c(b^2-4ac)}+\frac{\log(a+bx+cx^2)}{2c^2}$$

```
[In] Int[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*
x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*(b^2
- 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
```

```
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^3}{(a+bx+cx^2)^2}\,dx$$

$$= \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{\int \frac{x(4a+bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= -\frac{bx}{c(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{\int \frac{-ab+(-b^2+4ac)x}{a+bx+cx^2}\,dx}{c(b^2-4ac)}$$

$$= -\frac{bx}{c(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2} - \frac{(b(b^2-6ac))\int \frac{1}{a+bx+cx^2}\,dx}{2c^2(b^2-4ac)}$$

$$= -\frac{bx}{c(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{\log(a+bx+cx^2)}{2c^2} + \frac{(b(b^2-6ac))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c^2(b^2-4ac)}$$

$$= -\frac{bx}{c(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)^{3/2}} + \frac{\log(a+bx+cx^2)}{2c^2}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{\frac{2(-2a^2c+b^3x+ab(b-3cx))}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(b^2-6ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + \log(a+x(b+cx))}{2c^2}$$

```
[In] Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]
[Out] ((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x)))
 + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c
)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.48

method	result
default	$\frac{\frac{b(3ac-b^2)x}{c^2(4ac-b^2)}+\frac{a(2ac-b^2)}{(4ac-b^2)c^2}}{cx^2+bx+a}+\frac{\frac{(4ac-b^2)\ln(cx^2+bx+a)}{2c}+\frac{2\left(-ab-\frac{(4ac-b^2)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c(4ac-b^2)}$
risch	$\frac{\frac{b(3ac-b^2)x}{c^2(4ac-b^2)}+\frac{a(2ac-b^2)}{(4ac-b^2)c^2}}{cx^2+bx+a}+\frac{8\ln\left(-24a^2bc^2+10ab^3c-b^5-2\sqrt{-b^2(4ac-b^2)(6ac-b^2)^2}\,cx-\sqrt{-b^2(4ac-b^2)(6ac-b^2)^2}\,b\right)a^2}{(4ac-b^2)^2}-\frac{4\ln(}{}$

[In] int(x^7/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)

[Out] (b/c^2*(3*a*c-b^2)/(4*a*c-b^2)*x+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^2+b*x+a)+1/c/(4*a*c-b^2)*(1/2*(4*a*c-b^2)/c*ln(c*x^2+b*x+a)+2*(-a*b-1/2*(4*a*c-b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(108) = 216$.

Time = 0.27 (sec) , antiderivative size = 635, normalized size of antiderivative = 5.57

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx = \left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+(ab^3-6\,a^2bc+(b^3c-6\,abc^2)x^2+(b^4-6\,ab^2c)x)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+}{}\right.}{2\,(ab^4c^2-8\,a^2b^2c^3+}\right.$$

[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(104) = 208$.

Time = 0.73 (sec) , antiderivative size = 729, normalized size of antiderivative = 6.39

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx = \left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}\right.$$

$$\left.+\frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2c^2\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12}\right.}{6abc-b^3}\right.$$

$$+\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}\right.$$

$$\left.+\frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2c^2\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-}\right.}{6abc-b^3}\right.$$

$$+\frac{2a^2c-ab^2+x(3abc-b^3)}{4a^2c^3-ab^2c^2+x^2\cdot(4ac^4-b^2c^3)+x\,(4abc^3-b^3c^2)}$$

```
[In] integrate(x**7/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*sqrt(-(4*
a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4*a*c - b
**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(b*sqrt(-(4*a*c - b**2)*
*3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a*c - b**2)**
3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c
- b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) +
1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x*(3*a*b*c - b**3))/(
4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) + x*(4*a*b*c**3 - b
**3*c**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}} + \frac{\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{ab^2-2\,a^2c+(b^3-3\,abc)x}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3
)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 + b*x + a)/c^2 + (a*b^2 - 2*a^2*c + (
b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.45

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{\frac{a\,(2\,a\,c-b^2)}{c^2\,(4\,a\,c-b^2)}+\frac{b\,x\,(3\,a\,c-b^2)}{c^2\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6\right)}{2\,\left(64\,a^3\,c^5-48\,a^2\,b^2\,c^4+12\,a\,b^4\,c^3-b^6\,c^2\right)} + \frac{b\,\mathrm{atan}\left(\frac{c^2\,{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(\frac{2\,b\,x\,\left(6\,a\,c-b^2\right)}{c\,{\left(4\,a\,c-b^2\right)}^3}+\frac{b^2\,\left(4\,a\,c^2-b^2\,c\right)\,\left(6\,a\,c-b^2\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^4}\right)}{b^3-6\,a\,b\,c}\right)\,\left(6\,a\,c-b^2\right)}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

```
[In] int(x^7/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] ((a*(2*a*c - b^2))/(c^2*(4*a*c - b^2)) + (b*x*(3*a*c - b^2))/(c^2*(4*a*c -
b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6 - 64*a^3*c^3 + 48*a^2
*b^2*c^2 - 12*a*b^4*c))/(2*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^
2*c^4)) + (b*atan((c^2*(4*a*c - b^2)^(5/2)*((2*b*x*(6*a*c - b^2))/(c*(4*a*c
 - b^2)^3) + (b^2*(4*a*c^2 - b^2*c)*(6*a*c - b^2))/(c^3*(4*a*c - b^2)^4)))/
(b^3 - 6*a*b*c))*(6*a*c - b^2))/(c^2*(4*a*c - b^2)^(3/2))
```

3.21 $\int \frac{x^6}{(ax^2+bx^3+cx^4)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2} \, dx = \frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{4a\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] x*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*a*arctanh((2*c*x+b)/(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1599, 736, 632, 212}

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2} \, dx = \frac{4a\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} + \frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}$$

```
[In] Int[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (x*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c))), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx\\
&= \frac{x(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2a)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}\\
&= \frac{x(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{(4a)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{b^2-4ac}\\
&= \frac{x(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4a\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{b^2x+a(b-2cx)}{c(-b^2+4ac)(a+x(b+cx))} + \frac{4a\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

```
[In] Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4*a*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.45

method	result
default	$\frac{-\frac{(2ac-b^2)x}{c(4ac-b^2)}+\frac{ab}{c(4ac-b^2)}}{c\,x^2+bx+a}+\frac{4a\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{(2ac-b^2)x}{c(4ac-b^2)}+\frac{ab}{c(4ac-b^2)}}{c\,x^2+bx+a}+\frac{2a\ln\left((-8a\,c^2+2b^2c)x+(-4ac+b^2)^{\frac{3}{2}}-4abc+b^3\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{2a\ln\left((8a\,c^2-2b^2c)x+(-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)}{(-4ac+b^2)^{\frac{3}{2}}}$

```
[In] int(x^6/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-(2*a*c-b^2)/c/(4*a*c-b^2)*x+a*b/c/(4*a*c-b^2))/(c*x^2+b*x+a)+4*a/(4*a*c-b
^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(63) = 126.

Time = 0.26 (sec) , antiderivative size = 387, normalized size of antiderivative = 5.78

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \left[-\frac{ab^3-4\,a^2bc+2\,(ac^2x^2+abcx+a^2c)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+(b^4-6\,ab^2c+}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^2+(b^5c-8\,ab^3c^2+16\,a^2bc^3)x}\right.$$
$$\left.-\frac{ab^3-4\,a^2bc-4\,(ac^2x^2+abcx+a^2c)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+(b^4-6\,ab^2c+8\,a^2c^2)x}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^2+(b^5c-8\,ab^3c^2+16\,a^2bc^3)x}\right]$$

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(a*b^3 - 4*a^2*b*c + 2*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(b^2 - 4*a*c)*lo
g((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^
2 + b*x + a)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 +
 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*x), -(a*b^3 - 4*a^2*b*c - 4*(a*c^2*x^2 + a*b*c*x + a^2*c
)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
+ (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 +
(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(61) = 122$.

Time = 0.31 (sec) , antiderivative size = 280, normalized size of antiderivative = 4.18

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx = -2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}-2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right) +2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right) + \frac{ab+x(-2ac+b^2)}{4a^2c^2-ab^2c+x^2\cdot(4ac^3-b^2c^2)+x\,(4abc^2-b^3c)}$$

```
[In] integrate(x**6/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] -2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**3*c**2*sqrt(-1/(4*a*c - b**
2)**3) + 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - 2*a*b**4*sqrt(-1/(4*a*
c - b**2)**3) + 2*a*b)/(4*a*c)) + 2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (3
2*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 16*a**2*b**2*c*sqrt(-1/(4*a*c - b*
*2)**3) + 2*a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) + (a*b + x*
(-2*a*c + b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*
(4*a*b*c**2 - b**3*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{4\,a\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{b^2x-2\,acx+ab}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.01

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{\frac{x\,(2\,a\,c-b^2)}{c\,(4\,a\,c-b^2)}-\frac{a\,b}{c\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} - \frac{4\,a\,\mathrm{atan}\left(\frac{\left(\frac{2\,a\,(b^3-4\,a\,b\,c)}{(4\,a\,c-b^2)^{5/2}}-\frac{4\,a\,c\,x}{(4\,a\,c-b^2)^{3/2}}\right)(4\,a\,c-b^2)}{2\,a}\right)}{(4\,a\,c-b^2)^{3/2}}$$

```
[In] int(x^6/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] - ((x*(2*a*c - b^2))/(c*(4*a*c - b^2)) - (a*b)/(c*(4*a*c - b^2)))/(a + b*x
+ c*x^2) - (4*a*atan((((2*a*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*a*c*x
)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a)))/(4*a*c - b^2)^(3/2)
```

3.22 $\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{2a+bx}{(b^2-4ac)\,(a+bx+cx^2)} - \frac{2b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)-2*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1599, 652, 632, 212}

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{2a+bx}{(b^2-4ac)\,(a+bx+cx^2)} - \frac{2b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x}{(a+bx+cx^2)^2}\,dx \\
&= \frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)} + \frac{b\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac} \\
&= \frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)} - \frac{(2b)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{b^2-4ac} \\
&= \frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)} - \frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{2a+bx}{(b^2-4ac)(a+x(b+cx))} - \frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

```
[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{-bx-2a}{(4ac-b^2)(c\,x^2+bx+a)}-\frac{2b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	70
risch	$\frac{-\frac{bx}{4ac-b^2}-\frac{2a}{4ac-b^2}}{c\,x^2+bx+a}+\frac{b\ln\left((-8a\,c^2+2b^2c)x-(-4ac+b^2)^{\frac{3}{2}}-4abc+b^3\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{b\ln\left((8a\,c^2-2b^2c)x-(-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)}{(-4ac+b^2)^{\frac{3}{2}}}$	148

```
[In] int(x^5/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] (-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(62) = 124.

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.12

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx = \left[\frac{2\,ab^2-8\,a^2c-(bcx^2+b^2x+ab)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+(b^3-4\,abc)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x}, \frac{2\,ab}{}\right.$$

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] [(2*a*b^2 - 8*a^2*c - (b*c*x^2 + b^2*x + a*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
 a)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), (2*a*b^2 -
 8*a^2*c - 2*(b*c*x^2 + b^2*x + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*
c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c
+ 16*a^2*b*c^2)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.

Time = 0.30 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.83

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)$$
$$-b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)$$
$$+\frac{-2a-bx}{4a^2c-ab^2+x^2\cdot(4ac^2-b^2c)+x(4abc-b^3)}$$

```
[In] integrate(x**5/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2
)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b**2)
**3) + b**2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (16*a**2*b*c**
2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5
*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) + (-2*a - b*x)/(4*a**2*c - a*b
**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{2\,b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} + \frac{bx+2\,a}{(cx^2+bx+a)(b^2-4\,ac)}$$

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c
)) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.67

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{\frac{2\,a}{4\,a\,c-b^2}+\frac{b\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a} - \frac{2\,b\,\mathrm{atan}\left(\frac{\left(\frac{b^2}{\left(4\,a\,c-b^2\right)^{3/2}}+\frac{2\,b\,c\,x}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\left(4\,a\,c-b^2\right)}{b}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

```
[In] int(x^5/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] - ((2*a)/(4*a*c - b^2) + (b*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (2*b*atan
(((b^2/(4*a*c - b^2)^(3/2) + (2*b*c*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/
b))/(4*a*c - b^2)^(3/2)
```

3.23 $\int \frac{x^4}{(ax^2+bx^3+cx^4)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2} dx = -\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)} + \frac{4c\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (-2*c*x-b)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1599, 628, 632, 212}

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2} dx = \frac{4c\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

```
[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{(a+bx+cx^2)^2}\,dx \\
&= -\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)} - \frac{(2c)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac} \\
&= -\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)} + \frac{(4c)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{b^2-4ac} \\
&= -\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)} + \frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{\frac{b+2cx}{a+x(b+cx)} + \frac{4c\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{b^2-4ac}$$

```
[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]
[Out] -(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2cx+b}{(4ac-b^2)(c\,x^2+bx+a)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	68
risch	$\frac{\frac{2cx}{4ac-b^2}+\frac{b}{4ac-b^2}}{c\,x^2+bx+a}+\frac{2c\ln\left((-8a\,c^2+2b^2c)x+(-4ac+b^2)^{\frac{3}{2}}-4abc+b^3\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{2c\ln\left((8a\,c^2-2b^2c)x+(-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)}{(-4ac+b^2)^{\frac{3}{2}}}$	144

```
[In] int(x^4/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(62) = 124.

Time = 0.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.17

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$=\left[-\frac{b^3-4\,abc+2\,(c^2x^2+bcx+ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x},\right.$$
$$\left.-\frac{b^3-4\,abc-4\,(c^2x^2+bcx+ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x}\right]$$

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x
^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b^3 -
 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(61) = 122.

Time = 0.32 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.02

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx = -2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right) +2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right) +\frac{b+2cx}{4a^2c-ab^2+x^2\cdot(4ac^2-b^2c)+x(4abc-b^3)}$$

```
[In] integrate(x**4/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] -2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b**
2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*a*
c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (
32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b + 2*
c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{4\,c\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{2\,cx+b}{(cx^2+bx+a)(b^2-4\,ac)}$$

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.80

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{\frac{b}{4\,a\,c-b^2}+\frac{2\,c\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a} - \frac{4\,c\,\mathrm{atan}\left(\frac{\left(\frac{2\,c\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{4\,c^2\,x}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,c}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

```
[In] int(x^4/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] (b/(4*a*c - b^2) + (2*c*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (4*c*atan((((
2*c*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*c^2*x)/(4*a*c - b^2)^(3/2))*(
4*a*c - b^2))/(2*c)))/(4*a*c - b^2)^(3/2)
```

3.24 $\int \frac{x^3}{\left(ax^2+bx^3+cx^4\right)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 108

$$\int \frac{x^3}{\left(ax^2+bx^3+cx^4\right)^2} \, dx = \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{b\left(b^2-6ac\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}} +\frac{\log(x)}{a^2}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}$$

```
[Out] (b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2)*arctanh((2*c*
x+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1/2*ln(c*x^2+b*x+
a)/a^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1599, 754, 814, 648, 632, 212, 642}

$$\int \frac{x^3}{\left(ax^2+bx^3+cx^4\right)^2} \, dx = \frac{b\left(b^2-6ac\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^2} +\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[In] Int[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c
)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4*a*c)^(3/2)) + Log[x
]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \frac{-b^2+4ac-bcx}{x(a+bx+cx^2)}\,dx}{a\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \left(\frac{-b^2+4ac}{ax}+\frac{b(b^2-5ac)+c(b^2-4ac)x}{a(a+bx+cx^2)}\right)dx}{a\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\log(x)}{a^2}-\frac{\int \frac{b(b^2-5ac)+c(b^2-4ac)x}{a+bx+cx^2}\,dx}{a^2\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\log(x)}{a^2}-\frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^2}-\frac{\left(b(b^2-6ac)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a^2\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}\\
&\quad+\frac{\left(b(b^2-6ac)\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{a^2\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \frac{\frac{2a(b^2-2ac+bcx)}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(b^2-6ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+2\log(x)-\log(a+x(b+cx))}{2a^2}$$

```
[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2
- 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*L
og[x] - Log[a + x*(b + c*x)])/(2*a^2)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.64

method	result
default	$\frac{\ln(x)}{a^2} - \frac{\frac{\frac{abcx}{4ac-b^2} - \frac{a\left(2ac-b^2\right)}{4ac-b^2}}{c\,x^2+bx+a} + \frac{\frac{\left(4a\,c^2-b^2c\right)\ln\left(c\,x^2+bx+a\right)}{2c} + \frac{2\left(5abc-b^3-\frac{\left(4a\,c^2-b^2c\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^2}$
risch	$\frac{-\frac{bxc}{a\left(4ac-b^2\right)} + \frac{2ac-b^2}{a\left(4ac-b^2\right)}}{c\,x^2+bx+a} + \frac{\ln(x)}{a^2} + \left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^5c^3-48a^4b^2c^2+12a^3b^4c-a^2b^6\right)\_Z^2+\left(64c^3a^3-48a^2b^2c^2+12a\,b^4c-b^6\right)\_Z\right.}\right.$

[In] int(x^3/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)

[Out] ln(x)/a^2-1/a^2*((a*b*c/(4*a*c-b^2)*x-a*(2*a*c-b^2)/(4*a*c-b^2))/(c*x^2+b*x
+a)+1/(4*a*c-b^2)*(1/2*(4*a*c^2-b^2*c)/c*ln(c*x^2+b*x+a)+2*(5*a*b*c-b^3-1/2
*(4*a*c^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(102) = 204.

Time = 0.33 (sec) , antiderivative size = 781, normalized size of antiderivative = 7.23

$$\int \frac{x^3}{\left(ax^2+bx^3+cx^4\right)^2}\,dx$$
$$= \left[\frac{2\,ab^4 - 12\,a^2b^2c + 16\,a^3c^2 + \left(ab^3 - 6\,a^2bc + \left(b^3c - 6\,abc^2\right)x^2 + \left(b^4 - 6\,ab^2c\right)x\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+\cdots}{\cdots}\right.}{\cdots}\right.$$

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6
*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2
 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4 - 8*
a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2 + (a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^
3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c

```
 - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(
c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4
 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2
+ (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.17

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^2b^2-4\,a^3c)\sqrt{-b^2+4\,ac}} - \frac{\log\left(cx^2+bx+a\right)}{2\,a^2} + \frac{\log\left(|x|\right)}{a^2} + \frac{abcx+ab^2-2\,a^2c}{(cx^2+bx+a)(b^2-4\,ac)a^2}$$

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*c
)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 + b*x + a)/a^2 + log(abs(x))/a^2 + (a
*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 620, normalized size of antiderivative = 5.74

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{\ln(x)}{a^2} + \frac{\frac{2\,a\,c-b^2}{a\,(4\,a\,c-b^2)} - \frac{b\,c\,x}{a\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a}$$
$$+ \frac{\ln\Big(2\,a\,b^6 + 2\,b^7\,x - 96\,a^4\,c^3 + 2\,a\,b^3\,\sqrt{-(4\,a\,c-b^2)^3} - 23\,a^2\,b^4\,c + 2\,b^4\,x\,\sqrt{-(4\,a\,c-b^2)^3} + 84\,a^3\,b^2\,c^2 +}{}$$
$$+ \frac{\ln\Big(96\,a^4\,c^3 - 2\,b^7\,x - 2\,a\,b^6 + 2\,a\,b^3\,\sqrt{-(4\,a\,c-b^2)^3} + 23\,a^2\,b^4\,c + 2\,b^4\,x\,\sqrt{-(4\,a\,c-b^2)^3} - 84\,a^3\,b^2\,c^2 -}{}$$

```
[In] int(x^3/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] log(x)/a^2 + ((2*a*c - b^2)/(a*(4*a*c - b^2)) - (b*c*x)/(a*(4*a*c - b^2)))/
(a + b*x + c*x^2) + (log(2*a*b^6 + 2*b^7*x - 96*a^4*c^3 + 2*a*b^3*(-(4*a*c
- b^2)^3)^(1/2) - 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) + 84*a^3*
b^2*c^2 + 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b
^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) - 120*a^3*b*c^3*x - 12*a*b^2*c*
x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 + b^3*(-(4*a*c - b^2)^3)^(1/2
) + 48*a^2*b^2*c^2 - 12*a*b^4*c - 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^2
*(4*a*c - b^2)^3) + (log(96*a^4*c^3 - 2*b^7*x - 2*a*b^6 + 2*a*b^3*(-(4*a*c
- b^2)^3)^(1/2) + 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) - 84*a^3*
b^2*c^2 - 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b
^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) + 120*a^3*b*c^3*x - 12*a*b^2*c*
x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^(1/2
) + 48*a^2*b^2*c^2 - 12*a*b^4*c + 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^2
*(4*a*c - b^2)^3)
```

3.25 $\int \frac{x^2}{(ax^2+bx^3+cx^4)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 148

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2} \, dx = -\frac{2(b^2-3ac)}{a^2(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x(a+bx+cx^2)} - \frac{2(b^4-6ab^2c+6a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} - \frac{2b\log(x)}{a^3} + \frac{b\log(a+bx+cx^2)}{a^3}$$

```
[Out] -2*(-3*a*c+b^2)/a^2/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^
2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/
a^3/(-4*a*c+b^2)^(3/2)-2*b*ln(x)/a^3+b*ln(c*x^2+b*x+a)/a^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1599, 754, 814, 648, 632, 212, 642}

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2} \, dx = \frac{b\log(a+bx+cx^2)}{a^3} - \frac{2b\log(x)}{a^3} - \frac{2(b^2-3ac)}{a^2x(b^2-4ac)} - \frac{2(6a^2c^2-6ab^2c+b^4)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} + \frac{-2ac+b^2+bcx}{ax(b^2-4ac)(a+bx+cx^2)}$$

```
[In] Int[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 -
4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 +
(b*Log[a + b*x + c*x^2])/a^3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
```

```
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx \\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)}-\frac{\int \frac{-2\left(b^2-3ac\right)-2bcx}{x^2\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)} \\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)}-\frac{\int \left(\frac{2\left(-b^2+3ac\right)}{ax^2}-\frac{2b\left(-b^2+4ac\right)}{a^2x}+\frac{2\left(-b^4+5ab^2c-3a^2c^2-bc\left(b^2-4ac\right)x\right)}{a^2\left(a+bx+cx^2\right)}\right)dx}{a\left(b^2-4ac\right)} \\
&= -\frac{2(b^2-3ac)}{a^2\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} \\
&\quad -\frac{2b\log(x)}{a^3}-\frac{2\int \frac{-b^4+5ab^2c-3a^2c^2-bc\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx}{a^3\left(b^2-4ac\right)} \\
&= -\frac{2(b^2-3ac)}{a^2\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)}-\frac{2b\log(x)}{a^3} \\
&\quad +\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{a^3}+\frac{\left(b^4-6ab^2c+6a^2c^2\right)\int \frac{1}{a+bx+cx^2}\,dx}{a^3\left(b^2-4ac\right)} \\
&= -\frac{2(b^2-3ac)}{a^2\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)}-\frac{2b\log(x)}{a^3} \\
&\quad +\frac{b\log\left(a+bx+cx^2\right)}{a^3}-\frac{\left(2\left(b^4-6ab^2c+6a^2c^2\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{a^3\left(b^2-4ac\right)} \\
&= -\frac{2(b^2-3ac)}{a^2\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} \\
&\quad -\frac{2\left(b^4-6ab^2c+6a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(a+bx+cx^2\right)}{a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{\frac{a}{x}+\frac{a(b^3-3abc+b^2cx-2ac^2x)}{(b^2-4ac)(a+x(b+cx))}+\frac{2(b^4-6ab^2c+6a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+2b\log(x)-b\log(a+x(b+cx))}{a^3}$$

```
[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)*(a + x*(b
 + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] - b*Log[a + x*(b + c*x)])/a^3)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.39

method	result
default	$-\frac{1}{a^2x}-\frac{2b\ln(x)}{a^3}-\frac{\frac{\frac{ac(2ac-b^2)x}{4ac-b^2}+\frac{ab(3ac-b^2)}{4ac-b^2}}{cx^2+bx+a}+\frac{\frac{(-4abc^2+b^3c)\ln(cx^2+bx+a)}{c}+\frac{4\left(3a^2c^2-5ab^2c+b^4-\frac{(-4abc^2+b^3c)b}{2c}\right)\arctan\left(\frac{2cx\cdots}{\sqrt{4ac\cdots}}\right.}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^3}$
risch	$\frac{-\frac{2c(3ac-b^2)x^2}{a^2(4ac-b^2)}-\frac{b(7ac-2b^2)x}{a^2(4ac-b^2)}-\frac{1}{a}}{x(cx^2+bx+a)}-\frac{2b\ln(x)}{a^3}+2\left(\sum_{\_R=\operatorname{RootOf}((64a^6c^3-48a^5b^2c^2+12a^4b^4c-a^3b^6)\_Z^2+(-64bc^3a^3+48b^3c^2a^2\cdots}\right.$

```
[In] int(x^2/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2/x-2*b*ln(x)/a^3-1/a^3*((a*c*(2*a*c-b^2)/(4*a*c-b^2)*x+a*b*(3*a*c-b^2
)/(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(-4*a*b*c^2+b^3*c)/c*ln(c*x
^2+b*x+a)+2*(3*a^2*c^2-5*a*b^2*c+b^4-1/2*(-4*a*b*c^2+b^3*c)*b/c)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(144) = 288$.

Time = 0.42 (sec) , antiderivative size = 975, normalized size of antiderivative = 6.59

$$\int \frac{x^2}{(ax^2 + bx^3 + cx^4)^2}\, dx$$
$$= \left[-\frac{a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + 2\,(ab^4c - 7\,a^2b^2c^2 + 12\,a^3c^3)x^2 + ((b^4c - 6\,ab^2c^2 + 6\,a^2c^3)x^3 + (b^5 - 6\,ab^3c}{} \right.$$
$$-\frac{a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + 2\,(ab^4c - 7\,a^2b^2c^2 + 12\,a^3c^3)x^2 + 2\,((b^4c - 6\,ab^2c^2 + 6\,a^2c^3)x^3 + (b^5 - 6\,ab^3c}{}$$

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3
*c^3)*x^2 + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a
^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*sqrt(b^2 - 4*a*c)*log(
(2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2
+ b*x + a)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a
^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5
*b^2*c + 16*a^6*c^2)*x), -(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c
- 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^
3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)
*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)
) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^
3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5*b^2*c + 1
6*a^6*c^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(c*x**4+b*x**3+a*x**2)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{2\,(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^2-4\,a^4c)\sqrt{-b^2+4\,ac}} - \frac{2\,b^2cx^2-6\,ac^2x^2+2\,b^3x-7\,abcx+ab^2-4\,a^2c}{(a^2b^2-4\,a^3c)(cx^3+bx^2+ax)} + \frac{b\log\left(cx^2+bx+a\right)}{a^3} - \frac{2\,b\log\left(|x|\right)}{a^3}$$

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^
3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2*b^2*c*x^2 - 6*a*c^2*x^2 + 2*b^3*x
 - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*x^3 + b*x^2 + a*x))
 + b*log(c*x^2 + b*x + a)/a^3 - 2*b*log(abs(x))/a^3
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 775, normalized size of antiderivative = 5.24

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \ln\Bigg(2\,a\,b^7+2\,b^8\,x+2\,a\,b^4\,\sqrt{-(4\,a\,c-b^2)^3}-23\,a^2\,b^5\,c-108\,a^4\,b\,c^3+24\,a^4\,c^4\,x+2\,b^5\,x\,\sqrt{-(4\,a\,c-b^2)^3}+87\,a^3\,b^3\,c^2+3\,a^3\,c^2\,\sqrt{-(4\,a\,c-b^2)^3}-9\,a^2\,b^2\,c\,\sqrt{-(4\,a\,c-b^2)^3}+97\,a^2\,b^4\,c^2\,x-138\,a^3\,b^2\,c^3\,x-24\,a\,b^6\,c\,x-12\,a\,b^3\,c\,x\,\sqrt{-(4\,a\,c-b^2)^3}+15\,a^2\,b\,c^2\,x\,\sqrt{-(4\,a\,c-b^2)^3}\Bigg)\left(\frac{b^4\,\sqrt{-(4\,a\,c-b^2)^3}+6\,a^2\,c^2\,\sqrt{-(4\,a\,c-b^2)^3}-6\,a\,b^2\,c\,\sqrt{-(4\,a\,c-b^2)^3}}{-64\,a^6\,c^3+48\,a^5\,b^2\,c^2-12\,a^4\,b^4\,c+a^3\,b^6}+\frac{b}{a^3}\right)-\frac{\frac{1}{a}-\frac{x\,(2\,b^3-7\,a\,b\,c)}{a^2\,(4\,a\,c-b^2)}+\frac{2\,c\,x^2\,(3\,a\,c-b^2)}{a^2\,(4\,a\,c-b^2)}}{c\,x^3+b\,x^2+a\,x}$$
$$-\ln\Bigg(2\,a\,b^4\,\sqrt{-(4\,a\,c-b^2)^3}-2\,b^8\,x-2\,a\,b^7+23\,a^2\,b^5\,c+108\,a^4\,b\,c^3-24\,a^4\,c^4\,x+2\,b^5\,x\,\sqrt{-(4\,a\,c-b^2)^3}-87\,a^3\,b^3\,c^2+3\,a^3\,c^2\,\sqrt{-(4\,a\,c-b^2)^3}-9\,a^2\,b^2\,c\,\sqrt{-(4\,a\,c-b^2)^3}-97\,a^2\,b^4\,c^2\,x+138\,a^3\,b^2\,c^3\,x+24\,a\,b^6\,c\,x-12\,a\,b^3\,c\,x\,\sqrt{-(4\,a\,c-b^2)^3}+15\,a^2\,b\,c^2\,x\,\sqrt{-(4\,a\,c-b^2)^3}\Bigg)\left(\frac{b^4\,\sqrt{-(4\,a\,c-b^2)^3}+6\,a^2\,c^2\,\sqrt{-(4\,a\,c-b^2)^3}-6\,a\,b^2\,c\,\sqrt{-(4\,a\,c-b^2)^3}}{-64\,a^6\,c^3+48\,a^5\,b^2\,c^2-12\,a^4\,b^4\,c+a^3\,b^6}-\frac{b}{a^3}\right)-\frac{2\,b\,\ln(x)}{a^3}$$

```
[In] int(x^2/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] log(2*a*b^7 + 2*b^8*x + 2*a*b^4*(-(4*a*c - b^2)^3)^(1/2) - 23*a^2*b^5*c - 1
08*a^4*b*c^3 + 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) + 87*a^3*b^3
*c^2 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^
(1/2) + 97*a^2*b^4*c^2*x - 138*a^3*b^2*c^3*x - 24*a*b^6*c*x - 12*a*b^3*c*x*
(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(
-(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2) + b/a^3) - (1/a - (x*(2*b^3 - 7*a*b*c))/(a^2*(4*a*c - b^2)) + (2*c*x^2*
(3*a*c - b^2))/(a^2*(4*a*c - b^2)))/(a*x + b*x^2 + c*x^3) - log(2*a*b^4*(-(
4*a*c - b^2)^3)^(1/2) - 2*b^8*x - 2*a*b^7 + 23*a^2*b^5*c + 108*a^4*b*c^3 -
24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) - 87*a^3*b^3*c^2 + 3*a^3*c^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 97*a^2*
```

```
b^4*c^2*x + 138*a^3*b^2*c^3*x + 24*a*b^6*c*x - 12*a*b^3*c*x*(-(4*a*c - b^2)
^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(-(4*a*c - b^2)^
3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - b/a^3) -
(2*b*log(x))/a^3
```

3.26 $\int \frac{x}{(ax^2+bx^3+cx^4)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 202

$$\begin{aligned}\int \frac{x}{(ax^2+bx^3+cx^4)^2} dx = &-\frac{3b^2-8ac}{2a^2(b^2-4ac)x^2}+\frac{b(3b^2-11ac)}{a^3(b^2-4ac)x} \\ &+\frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)} \\ &+\frac{b(3b^4-20ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}} \\ &+\frac{(3b^2-2ac)\log(x)}{a^4}-\frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4}\end{aligned}$$

```
[Out] 1/2*(8*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x^2+b*(-11*a*c+3*b^2)/a^3/(-4*a*c+b^2)/x
+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^2+b*x+a)+b*(30*a^2*c^2-20*a*b^2*
c+3*b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(3/2)+(-2*a
*c+3*b^2)*ln(x)/a^4-1/2*(-2*a*c+3*b^2)*ln(c*x^2+b*x+a)/a^4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used

= {1599, 754, 814, 648, 632, 212, 642}

$$\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4} + \frac{\log(x)(3b^2-2ac)}{a^4} + \frac{b(3b^2-11ac)}{a^3x(b^2-4ac)} - \frac{3b^2-8ac}{2a^2x^2(b^2-4ac)} + \frac{b(30a^2c^2-20ab^2c+3b^4)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}} + \frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

```
[In] Int[x/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -1/2*(3*b^2 - 8*a*c)/(a^2*(b^2 - 4*a*c)*x^2) + (b*(3*b^2 - 11*a*c))/(a^3*(b
^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^
2)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4
*a*c]])/(a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2
- 2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^3 (a+bx+cx^2)^2} \, dx$$

$$= \frac{b^2-2ac+bcx}{a (b^2-4ac) x^2 (a+bx+cx^2)} - \frac{\int \frac{-3b^2+8ac-3bcx}{x^3(a+bx+cx^2)} \, dx}{a (b^2-4ac)}$$

$$= \frac{b^2-2ac+bcx}{a (b^2-4ac) x^2 (a+bx+cx^2)} - \frac{\int \left(\frac{-3b^2+8ac}{ax^3} + \frac{3b^3-11abc}{a^2x^2} + \frac{(b^2-4ac)(-3b^2+2ac)}{a^3x} + \frac{b(3b^4-17ab^2c+19a^2c^2)+c(b^2-4ac)(3b^2-2ac)x}{a^3(a+bx+cx^2)}\right) dx}{a (b^2-4ac)}$$

$$= -\frac{3b^2-8ac}{2a^2 (b^2-4ac) x^2} + \frac{b(3b^2-11ac)}{a^3 (b^2-4ac) x} + \frac{b^2-2ac+bcx}{a (b^2-4ac) x^2 (a+bx+cx^2)} + \frac{(3b^2-2ac)\log(x)}{a^4} - \frac{\int \frac{b(3b^4-17ab^2c+19a^2c^2)+c(b^2-4ac)(3b^2-2ac)x}{a+bx+cx^2} \, dx}{a^4 (b^2-4ac)}$$

$$= -\frac{3b^2-8ac}{2a^2(b^2-4ac)x^2} + \frac{b(3b^2-11ac)}{a^3(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)} + \frac{(3b^2-2ac)\log(x)}{a^4} - \frac{(3b^2-2ac)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{2a^4} - \frac{(b(3b^4-20ab^2c+30a^2c^2))\int\frac{1}{a+bx+cx^2}\,dx}{2a^4(b^2-4ac)}$$

$$= -\frac{3b^2-8ac}{2a^2(b^2-4ac)x^2} + \frac{b(3b^2-11ac)}{a^3(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)} + \frac{(3b^2-2ac)\log(x)}{a^4} - \frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4} + \frac{(b(3b^4-20ab^2c+30a^2c^2))\,\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{a^4(b^2-4ac)}$$

$$= -\frac{3b^2-8ac}{2a^2(b^2-4ac)x^2} + \frac{b(3b^2-11ac)}{a^3(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)} + \frac{b(3b^4-20ab^2c+30a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}} + \frac{(3b^2-2ac)\log(x)}{a^4} - \frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx$$

$$= \frac{-\frac{a^2}{x^2} + \frac{4ab}{x} + \frac{2a(b^4-4ab^2c+2a^2c^2+b^3cx-3abc^2x)}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(3b^4-20ab^2c+30a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + 2(3b^2-2ac)\log(x) + (-3b}{2a^4}$$

```
[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*a
*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c + 3
0*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2
*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 + 2*a*c)*Log[a + x*(b + c*x)])/(2*a^4)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.26

method	result
default	$-\frac{1}{2a^2x^2}+\frac{(-2ac+3b^2)\ln(x)}{a^4}+\frac{2b}{a^3x}+\frac{\frac{\frac{acb(3ac-b^2)x}{4ac-b^2}-\frac{a(2a^2c^2-4ab^2c+b^4)}{4ac-b^2}}{cx^2+bx+a}+\frac{(8a^2c^3-14b^2ac^2+3b^4c)\ln(cx^2+bx+a)}{2c}+\frac{2(19a^2bc^2\cdots}{\cdots}}{a^4}$
risch	$\frac{\frac{bc(11ac-3b^2)x^3}{a^3(4ac-b^2)}-\frac{(8a^2c^2-25ab^2c+6b^4)x^2}{2a^3(4ac-b^2)}+\frac{3bx}{2a^2}-\frac{1}{2a}}{(cx^2+bx+a)x^2}-\frac{2\ln(x)c}{a^3}+\frac{3b^2\ln(x)}{a^4}+\left(\sum_{\_R=\mathrm{RootOf}((64a^7c^3-48a^6b^2c^2+12a^5b^4c-a^4b\cdots}\right.$

[In] int(x/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)

[Out] -1/2/a^2/x^2+(-2*a*c+3*b^2)*ln(x)/a^4+2/a^3*b/x+1/a^4*((a*c*b*(3*a*c-b^2)/(4*a*c-b^2)*x-a*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)/c*ln(c*x^2+b*x+a)+2*(19*a^2*b*c^2-17*a*b^3*c+3*b^5-1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(194) = 388.

Time = 0.45 (sec) , antiderivative size = 1226, normalized size of antiderivative = 6.07

$$\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Too large to display}$$

[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] [-1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3)*x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^2), -1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*

```
c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^
4*c^3)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a
*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)
*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) -
 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64
*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*
a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2
)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a
^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3
*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(x))/((a^4*b^4
*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^
2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.13

$$\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{(3\,b^5-20\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^4b^2-4\,a^5c)\sqrt{-b^2+4\,ac}} - \frac{(3\,b^2-2\,ac)\log(cx^2+bx+a)}{2\,a^4} + \frac{(3\,b^2-2\,ac)\log(|x|)}{a^4} - \frac{a^3b^2-4\,a^4c-2\,(3\,ab^3c-11\,a^2bc^2)x^3-(6\,ab^4-25\,a^2b^2c+8\,a^3c^2)x^2-3\,(a^2b^3-4\,a^3bc)x}{2\,(cx^2+bx+a)(b^2-4\,ac)a^4x^2}$$

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))
/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(3*b^2 - 2*a*c)*log(c*x^2 +
 b*x + a)/a^4 + (3*b^2 - 2*a*c)*log(abs(x))/a^4 - 1/2*(a^3*b^2 - 4*a^4*c -
2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2
 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 914, normalized size of antiderivative = 4.52

$$\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{\ln\left(6\,a\,b^8+6\,b^9\,x+192\,a^5\,c^4-6\,a\,b^5\,\sqrt{-(4\,a\,c-b^2)^3}-73\,a^2\,b^6\,c-6\,b^6\,x\,\sqrt{-(4\,a\,c-b^2)^3}+307\,a^3\,b^4\,c^2\right.}{} - \frac{\ln(x)\,(2\,a\,c-3\,b^2)}{a^4} - \frac{\frac{1}{2\,a}-\frac{3\,b\,x}{2\,a^2}+\frac{x^2\,(8\,a^2\,c^2-25\,a\,b^2\,c+6\,b^4)}{2\,a^3\,(4\,a\,c-b^2)}-\frac{b\,c\,x^3\,(11\,a\,c-3\,b^2)}{a^3\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^3+a\,x^2} + \frac{\ln\left(6\,a\,b^8+6\,b^9\,x+192\,a^5\,c^4+6\,a\,b^5\,\sqrt{-(4\,a\,c-b^2)^3}-73\,a^2\,b^6\,c+6\,b^6\,x\,\sqrt{-(4\,a\,c-b^2)^3}+307\,a^3\,b^4\,c\right.}{}$$

```
[In] int(x/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 - 6*a*b^5*(-(4*a*c - b^2)^3)^(1/2) - 7
3*a^2*b^6*c - 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b^4*c^2 - 492*a^4*
b^2*c^3 + 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 27*a^3*b*c^2*(-(4*a*c - b
^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x + 24*a^3*c^3*x*(-(4*a*
c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x + 40*a*b^4*c*x*(-(4*a*c
- b^2)^3)^(1/2) - 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(3*b^8 + 128*a
```

```
^4*c^4 - 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2 - 288*a^3*b^2*c^3
 - 38*a*b^6*c - 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b^3*c*(-(4*a*c
 - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3) - (log(x)*(2*a*c - 3*b^2))/a^4 -
 (1/(2*a) - (3*b*x)/(2*a^2) + (x^2*(6*b^4 + 8*a^2*c^2 - 25*a*b^2*c))/(2*a^3
*(4*a*c - b^2)) - (b*c*x^3*(11*a*c - 3*b^2))/(a^3*(4*a*c - b^2)))/(a*x^2 +
b*x^3 + c*x^4) + (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 + 6*a*b^5*(-(4*a*c -
b^2)^3)^(1/2) - 73*a^2*b^6*c + 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b
^4*c^2 - 492*a^4*b^2*c^3 - 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 27*a^3*b
*c^2*(-(4*a*c - b^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x - 24*
a^3*c^3*x*(-(4*a*c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x - 40*a*
b^4*c*x*(-(4*a*c - b^2)^3)^(1/2) + 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2
))*(3*b^8 + 128*a^4*c^4 + 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2
- 288*a^3*b^2*c^3 - 38*a*b^6*c + 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 20
*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3)
```

3.27 $\int \frac{1}{(ax^2+bx^3+cx^4)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 252

$$\int \frac{1}{(ax^2+bx^3+cx^4)^2} dx = -\frac{2(2b^2-5ac)}{3a^2(b^2-4ac)x^3}+\frac{b(2b^2-7ac)}{a^3(b^2-4ac)x^2} -\frac{2(2b^4-9ab^2c+5a^2c^2)}{a^4(b^2-4ac)x}+\frac{b^2-2ac+bcx}{a(b^2-4ac)x^3(a+bx+cx^2)} -\frac{2(2b^6-15ab^4c+30a^2b^2c^2-10a^3c^3)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^5(b^2-4ac)^{3/2}} -\frac{2b(2b^2-3ac)\log(x)}{a^5}+\frac{b(2b^2-3ac)\log(a+bx+cx^2)}{a^5}$$

```
[Out] -2/3*(-5*a*c+2*b^2)/a^2/(-4*a*c+b^2)/x^3+b*(-7*a*c+2*b^2)/a^3/(-4*a*c+b^2)/
x^2-2*(5*a^2*c^2-9*a*b^2*c+2*b^4)/a^4/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-
4*a*c+b^2)/x^3/(c*x^2+b*x+a)-2*(-10*a^3*c^3+30*a^2*b^2*c^2-15*a*b^4*c+2*b^6
)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^5/(-4*a*c+b^2)^(3/2)-2*b*(-3*a*c+
2*b^2)*ln(x)/a^5+b*(-3*a*c+2*b^2)*ln(c*x^2+b*x+a)/a^5
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used

$= \{1608, 754, 814, 648, 632, 212, 642\}$

$$\int \frac{1}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{b(2b^2-3ac)\log(a+bx+cx^2)}{a^5} - \frac{2b\log(x)(2b^2-3ac)}{a^5} + \frac{b(2b^2-7ac)}{a^3x^2(b^2-4ac)} - \frac{2(2b^2-5ac)}{3a^2x^3(b^2-4ac)} - \frac{2(5a^2c^2-9ab^2c+2b^4)}{a^4x(b^2-4ac)} - \frac{2(-10a^3c^3+30a^2b^2c^2-15ab^4c+2b^6)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^5(b^2-4ac)^{3/2}} + \frac{-2ac+b^2+bcx}{ax^3(b^2-4ac)(a+bx+cx^2)}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(-2),x]
```

```
[Out] (-2*(2*b^2 - 5*a*c))/(3*a^2*(b^2 - 4*a*c)*x^3) + (b*(2*b^2 - 7*a*c))/(a^3*(
b^2 - 4*a*c)*x^2) - (2*(2*b^4 - 9*a*b^2*c + 5*a^2*c^2))/(a^4*(b^2 - 4*a*c)*
x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^3*(a + b*x + c*x^2)) - (2*(2*
b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]])/(a^5*(b^2 - 4*a*c)^(3/2)) - (2*b*(2*b^2 - 3*a*c)*Log[x])/a^5 +
 (b*(2*b^2 - 3*a*c)*Log[a + b*x + c*x^2])/a^5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^4 \left(a+bx+cx^2\right)^2} \, dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^3\left(a+bx+cx^2\right)} - \frac{\int \frac{-2\left(2b^2-5ac\right)-4bcx}{x^4\left(a+bx+cx^2\right)} \, dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^3\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{2\left(-2b^2+5ac\right)}{ax^4} - \frac{2\left(-2b^3+7abc\right)}{a^2x^3} - \frac{2\left(2b^4-9ab^2c+5a^2c^2\right)}{a^3x^2} + \frac{2b\left(b^2-4ac\right)\left(2b^2-3ac\right)}{a^4x} + \frac{2\left(-2b^6+13ab^4c-21a^2b^2c^2+5a^3c^3 \cdots\right)}{a^4\left(a+bx+ \cdots\right)} \cdots\right)}{a\left(b^2-4ac\right)}$$

$$= -\frac{2(2b^2-5ac)}{3a^2\left(b^2-4ac\right)x^3} + \frac{b(2b^2-7ac)}{a^3\left(b^2-4ac\right)x^2} - \frac{2(2b^4-9ab^2c+5a^2c^2)}{a^4\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^3\left(a+bx+cx^2\right)} - \frac{2b(2b^2-3ac)\log(x)}{a^5} - \frac{2\int \frac{-2b^6+13ab^4c-21a^2b^2c^2+5a^3c^3-bc\left(b^2-4ac\right)\left(2b^2-3ac\right)x}{a+bx+cx^2} \, dx}{a^5\left(b^2-4ac\right)}$$

$$
\begin{aligned}
&= -\frac{2(2b^2-5ac)}{3a^2(b^2-4ac)x^3} + \frac{b(2b^2-7ac)}{a^3(b^2-4ac)x^2} - \frac{2(2b^4-9ab^2c+5a^2c^2)}{a^4(b^2-4ac)x} \\
&\quad + \frac{b^2-2ac+bcx}{a(b^2-4ac)x^3(a+bx+cx^2)} - \frac{2b(2b^2-3ac)\log(x)}{a^5} \\
&\quad + \frac{(b(2b^2-3ac))\int \frac{b+2cx}{a+bx+cx^2}\,dx}{a^5} + \frac{(2b^6-15ab^4c+30a^2b^2c^2-10a^3c^3)\int \frac{1}{a+bx+cx^2}\,dx}{a^5(b^2-4ac)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2(2b^2-5ac)}{3a^2(b^2-4ac)x^3} + \frac{b(2b^2-7ac)}{a^3(b^2-4ac)x^2} \\
&\quad - \frac{2(2b^4-9ab^2c+5a^2c^2)}{a^4(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x^3(a+bx+cx^2)} \\
&\quad - \frac{2b(2b^2-3ac)\log(x)}{a^5} + \frac{b(2b^2-3ac)\log(a+bx+cx^2)}{a^5} \\
&\quad - \frac{(2(2b^6-15ab^4c+30a^2b^2c^2-10a^3c^3))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{a^5(b^2-4ac)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2(2b^2-5ac)}{3a^2(b^2-4ac)x^3} + \frac{b(2b^2-7ac)}{a^3(b^2-4ac)x^2} \\
&\quad - \frac{2(2b^4-9ab^2c+5a^2c^2)}{a^4(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x^3(a+bx+cx^2)} \\
&\quad - \frac{2(2b^6-15ab^4c+30a^2b^2c^2-10a^3c^3)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^5(b^2-4ac)^{3/2}} \\
&\quad - \frac{2b(2b^2-3ac)\log(x)}{a^5} + \frac{b(2b^2-3ac)\log(a+bx+cx^2)}{a^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.87

$$
\int \frac{1}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{-\frac{a^3}{x^3} + \frac{3a^2b}{x^2} + \frac{3a(-3b^2+2ac)}{x} - \frac{3a(b^5-5ab^3c+5a^2bc^2+b^4cx-4ab^2c^2x+2a^2c^3x)}{(b^2-4ac)(a+x(b+cx))} - \frac{6(2b^6-15ab^4c+30a^2b^2c^2-10a^3c^3)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}}{3a^5}
$$

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-2),x]
```

```
[Out] (-(a^3/x^3) + (3*a^2*b)/x^2 + (3*a*(-3*b^2 + 2*a*c))/x - (3*a*(b^5 - 5*a*b^
3*c + 5*a^2*b*c^2 + b^4*c*x - 4*a*b^2*c^2*x + 2*a^2*c^3*x))/((b^2 - 4*a*c)*
(a + x*(b + c*x))) - (6*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*
ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 6*(-2*b^3 +
3*a*b*c)*Log[x] + 3*(2*b^3 - 3*a*b*c)*Log[a + x*(b + c*x)])/(3*a^5)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.17

method	result
default	$-\frac{1}{3a^2x^3}-\frac{-2ac+3b^2}{x\,a^4}+\frac{b}{a^3x^2}+\frac{2b(3ac-2b^2)\ln(x)}{a^5}+\frac{\frac{ac\left(2a^2c^2-4a\,b^2c+b^4\right)x}{4ac-b^2}+\frac{ab\left(5a^2c^2-5a\,b^2c+b^4\right)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\left(-12a^2b\,c^3+11a\,b^3c^2-2b\right.}{c}$
risch	$\frac{\frac{2c\left(5a^2c^2-9a\,b^2c+2b^4\right)x^4}{\left(4ac-b^2\right)a^4}+\frac{b\left(17a^2c^2-20a\,b^2c+4b^4\right)x^3}{a^4\left(4ac-b^2\right)}+\frac{\left(5ac-6b^2\right)x^2}{3a^3}+\frac{2bx}{3a^2}-\frac{1}{3a}}{x^3(c\,x^2+bx+a)}+\frac{6b\ln(x)c}{a^4}-\frac{4b^3\ln(x)}{a^5}+2\left(\sum_{\_R=\text{RootOf}\left((64c^3\right.}\right.$

```
[In] int(1/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^2/x^3-(-2*a*c+3*b^2)/x/a^4+1/a^3*b/x^2+2*b*(3*a*c-2*b^2)/a^5*ln(x)+1
/a^5*((a*c*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2)*x+a*b*(5*a^2*c^2-5*a*b^2*c
+b^4)/(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(-12*a^2*b*c^3+11*a*b^3
*c^2-2*b^5*c)/c*ln(c*x^2+b*x+a)+2*(5*c^3*a^3-21*a^2*b^2*c^2+13*a*b^4*c-2*b^
6-1/2*(-12*a^2*b*c^3+11*a*b^3*c^2-2*b^5*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 693 vs. $2(246) = 492$.

Time = 0.56 (sec) , antiderivative size = 1407, normalized size of antiderivative = 5.58

$$\int \frac{1}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/3*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 6*(2*a*b^6*c - 17*a^2*b^4*c^2 +
 41*a^3*b^2*c^3 - 20*a^4*c^4)*x^4 + 3*(4*a*b^7 - 36*a^2*b^5*c + 97*a^3*b^3*
c^2 - 68*a^4*b*c^3)*x^3 + (6*a^2*b^6 - 53*a^3*b^4*c + 136*a^4*b^2*c^2 - 80*
a^5*c^3)*x^2 - 3*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 10*a^3*c^4)*x^
5 + (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 - 10*a^3*b*c^3)*x^4 + (2*a*b^6 - 1
5*a^2*b^4*c + 30*a^3*b^2*c^2 - 10*a^4*c^3)*x^3)*sqrt(b^2 - 4*a*c)*log((2*c^
2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) - 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x - 3*((2*b^7*c - 19*a*b^
5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b
^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 4
8*a^4*b*c^3)*x^3)*log(c*x^2 + b*x + a) + 6*((2*b^7*c - 19*a*b^5*c^2 + 56*a^
2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a
^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*
```

```
x^3)*log(x))/((a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*x^5 + (a^5*b^5 - 8*a
^6*b^3*c + 16*a^7*b*c^2)*x^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^3), -
1/3*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 6*(2*a*b^6*c - 17*a^2*b^4*c^2 + 4
1*a^3*b^2*c^3 - 20*a^4*c^4)*x^4 + 3*(4*a*b^7 - 36*a^2*b^5*c + 97*a^3*b^3*c^
2 - 68*a^4*b*c^3)*x^3 + (6*a^2*b^6 - 53*a^3*b^4*c + 136*a^4*b^2*c^2 - 80*a^
5*c^3)*x^2 + 6*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 10*a^3*c^4)*x^5
+ (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 - 10*a^3*b*c^3)*x^4 + (2*a*b^6 - 15*
a^2*b^4*c + 30*a^3*b^2*c^2 - 10*a^4*c^3)*x^3)*sqrt(-b^2 + 4*a*c)*arctan(-sq
rt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(a^3*b^5 - 8*a^4*b^3*c + 16
*a^5*b*c^2)*x - 3*((2*b^7*c - 19*a*b^5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)
*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^
7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*x^3)*log(c*x^2 + b*x + a)
 + 6*((2*b^7*c - 19*a*b^5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8
 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^
5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*x^3)*log(x))/((a^5*b^4*c - 8*a^6*b^2*c
^2 + 16*a^7*c^3)*x^5 + (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*x^4 + (a^6*b^
4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ax^2 + bx^3 + cx^4)^2} dx = \text{Timed out}$$

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(ax^2 + bx^3 + cx^4)^2} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.12

$$\int \frac{1}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{2\,(2\,b^6-15\,ab^4c+30\,a^2b^2c^2-10\,a^3c^3)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^5b^2-4\,a^6c)\sqrt{-b^2+4\,ac}} + \frac{(2\,b^3-3\,abc)\log\left(cx^2+bx+a\right)}{a^5} - \frac{2\,(2\,b^3-3\,abc)\log\left(|x|\right)}{a^5} - \frac{a^4b^2-4\,a^5c+6\,(2\,ab^4c-9\,a^2b^2c^2+5\,a^3c^3)x^4+3\,(4\,ab^5-20\,a^2b^3c+17\,a^3bc^2)x^3+(6\,a^2b^4-29\,a^3b^2c+}{3\,(cx^2+bx+a)(b^2-4\,ac)a^5x^3}$$

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*arctan((2*c*x + b)/sqr
t(-b^2 + 4*a*c))/((a^5*b^2 - 4*a^6*c)*sqrt(-b^2 + 4*a*c)) + (2*b^3 - 3*a*b*
c)*log(c*x^2 + b*x + a)/a^5 - 2*(2*b^3 - 3*a*b*c)*log(abs(x))/a^5 - 1/3*(a^
4*b^2 - 4*a^5*c + 6*(2*a*b^4*c - 9*a^2*b^2*c^2 + 5*a^3*c^3)*x^4 + 3*(4*a*b^
5 - 20*a^2*b^3*c + 17*a^3*b*c^2)*x^3 + (6*a^2*b^4 - 29*a^3*b^2*c + 20*a^4*c
^2)*x^2 - 2*(a^3*b^3 - 4*a^4*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^5*x
^3)
```

Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 1120, normalized size of antiderivative = 4.44

$$\int \frac{1}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{\frac{x^2\,(5\,a\,c-6\,b^2)}{3\,a^3}-\frac{1}{3\,a}+\frac{2\,b\,x}{3\,a^2}+\frac{x^3\,(17\,a^2\,b\,c^2-20\,a\,b^3\,c+4\,b^5)}{a^4\,(4\,a\,c-b^2)}+\frac{2\,c\,x^4\,(5\,a^2\,c^2-9\,a\,b^2\,c+2\,b^4)}{a^4\,(4\,a\,c-b^2)}}{c\,x^5+b\,x^4+a\,x^3} + \frac{\ln\left(4\,a\,b^9+4\,b^{10}\,x-4\,a\,b^6\,\sqrt{-{(4\,a\,c-b^2)}^3}-52\,a^2\,b^7\,c+308\,a^5\,b\,c^4-40\,a^5\,c^5\,x-4\,b^7\,x\,\sqrt{-(4\,a\,c-b^2}\right.}{} + \frac{\ln\left(4\,a\,b^9+4\,b^{10}\,x+4\,a\,b^6\,\sqrt{-{(4\,a\,c-b^2)}^3}-52\,a^2\,b^7\,c+308\,a^5\,b\,c^4-40\,a^5\,c^5\,x+4\,b^7\,x\,\sqrt{-(4\,a\,c-b^2}\right.}{} + \frac{2\,b\,\ln\left(x\right)\,\left(3\,a\,c-2\,b^2\right)}{a^5}$$

```
[In] int(1/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] ((x^2*(5*a*c - 6*b^2))/(3*a^3) - 1/(3*a) + (2*b*x)/(3*a^2) + (x^3*(4*b^5 +
17*a^2*b*c^2 - 20*a*b^3*c))/(a^4*(4*a*c - b^2)) + (2*c*x^4*(2*b^4 + 5*a^2*c
```

```
^2 - 9*a*b^2*c))/(a^4*(4*a*c - b^2)))/(a*x^3 + b*x^4 + c*x^5) + (log(4*a*b^
9 + 4*b^10*x - 4*a*b^6*(-(4*a*c - b^2)^3)^(1/2) - 52*a^2*b^7*c + 308*a^5*b*
c^4 - 40*a^5*c^5*x - 4*b^7*x*(-(4*a*c - b^2)^3)^(1/2) + 243*a^3*b^5*c^2 - 4
73*a^4*b^3*c^3 + 5*a^4*c^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^4*c*(-(4*a*c
 - b^2)^3)^(1/2) + 266*a^2*b^6*c^2*x - 563*a^3*b^4*c^3*x + 438*a^4*b^2*c^4*
x - 54*a*b^8*c*x - 33*a^3*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a*b^5*c*x*(
-(4*a*c - b^2)^3)^(1/2) + 41*a^3*b*c^3*x*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*
b^3*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(a^2*(132*b^5*c^2 - 30*b^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2)) - a^3*(272*b^3*c^3 - 10*c^3*(-(4*a*c - b^2)^3)^(1/2)) + 2
*b^9 - 2*b^6*(-(4*a*c - b^2)^3)^(1/2) - a*(27*b^7*c - 15*b^4*c*(-(4*a*c - b
^2)^3)^(1/2)) + 192*a^4*b*c^4))/(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a
^7*b^2*c^2) + (log(4*a*b^9 + 4*b^10*x + 4*a*b^6*(-(4*a*c - b^2)^3)^(1/2) -
52*a^2*b^7*c + 308*a^5*b*c^4 - 40*a^5*c^5*x + 4*b^7*x*(-(4*a*c - b^2)^3)^(1
/2) + 243*a^3*b^5*c^2 - 473*a^4*b^3*c^3 - 5*a^4*c^3*(-(4*a*c - b^2)^3)^(1/2
) - 24*a^2*b^4*c*(-(4*a*c - b^2)^3)^(1/2) + 266*a^2*b^6*c^2*x - 563*a^3*b^4
*c^3*x + 438*a^4*b^2*c^4*x - 54*a*b^8*c*x + 33*a^3*b^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) - 30*a*b^5*c*x*(-(4*a*c - b^2)^3)^(1/2) - 41*a^3*b*c^3*x*(-(4*a*c
- b^2)^3)^(1/2) + 66*a^2*b^3*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(a^2*(132*b^5*
c^2 + 30*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2)) - a^3*(272*b^3*c^3 + 10*c^3*(-(4
*a*c - b^2)^3)^(1/2)) + 2*b^9 + 2*b^6*(-(4*a*c - b^2)^3)^(1/2) - a*(27*b^7*
c + 15*b^4*c*(-(4*a*c - b^2)^3)^(1/2)) + 192*a^4*b*c^4))/(a^5*b^6 - 64*a^8*
c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2) + (2*b*log(x)*(3*a*c - 2*b^2))/a^5
```

3.28 $\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 318

$$\begin{aligned}\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx =& -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4}+\frac{b(5b^2-17ac)}{3a^3\left(b^2-4ac\right)x^3}-\frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2}\\ &+\frac{b(5b^4-27ab^2c+29a^2c^2)}{a^5\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^4\left(a+bx+cx^2\right)}\\ &+\frac{b(5b^6-42ab^4c+105a^2b^2c^2-70a^3c^3)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^6\left(b^2-4ac\right)^{3/2}}\\ &+\frac{(5b^4-12ab^2c+3a^2c^2)\log(x)}{a^6}\\ &-\frac{(5b^4-12ab^2c+3a^2c^2)\log\left(a+bx+cx^2\right)}{2a^6}\end{aligned}$$

```
[Out] 1/4*(12*a*c-5*b^2)/a^2/(-4*a*c+b^2)/x^4+1/3*b*(-17*a*c+5*b^2)/a^3/(-4*a*c+b
^2)/x^3+1/2*(-12*a^2*c^2+22*a*b^2*c-5*b^4)/a^4/(-4*a*c+b^2)/x^2+b*(29*a^2*c
^2-27*a*b^2*c+5*b^4)/a^5/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^
4/(c*x^2+b*x+a)+b*(-70*a^3*c^3+105*a^2*b^2*c^2-42*a*b^4*c+5*b^6)*arctanh((2
*c*x+b)/(-4*a*c+b^2)^(1/2))/a^6/(-4*a*c+b^2)^(3/2)+(3*a^2*c^2-12*a*b^2*c+5*
b^4)*ln(x)/a^6-1/2*(3*a^2*c^2-12*a*b^2*c+5*b^4)*ln(c*x^2+b*x+a)/a^6
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1599, 754, 814, 648, 632, 212, 642}

$$\begin{aligned}\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx &= \frac{b(5b^2-17ac)}{3a^3x^3\left(b^2-4ac\right)}-\frac{5b^2-12ac}{4a^2x^4\left(b^2-4ac\right)}\\ &\quad-\frac{\left(3a^2c^2-12ab^2c+5b^4\right)\log\left(a+bx+cx^2\right)}{2a^6}\\ &\quad+\frac{\log(x)\left(3a^2c^2-12ab^2c+5b^4\right)}{a^6}\\ &\quad+\frac{b(29a^2c^2-27ab^2c+5b^4)}{a^5x\left(b^2-4ac\right)}-\frac{12a^2c^2-22ab^2c+5b^4}{2a^4x^2\left(b^2-4ac\right)}\\ &\quad+\frac{b(-70a^3c^3+105a^2b^2c^2-42ab^4c+5b^6)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^6\left(b^2-4ac\right)^{3/2}}\\ &\quad+\frac{-2ac+b^2+bcx}{ax^4\left(b^2-4ac\right)\left(a+bx+cx^2\right)}\end{aligned}$$

```
[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x]
```

```
[Out] -1/4*(5*b^2 - 12*a*c)/(a^2*(b^2 - 4*a*c)*x^4) + (b*(5*b^2 - 17*a*c))/(3*a^3
*(b^2 - 4*a*c)*x^3) - (5*b^4 - 22*a*b^2*c + 12*a^2*c^2)/(2*a^4*(b^2 - 4*a*c
)*x^2) + (b*(5*b^4 - 27*a*b^2*c + 29*a^2*c^2))/(a^5*(b^2 - 4*a*c)*x) + (b^2
 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^4*(a + b*x + c*x^2)) + (b*(5*b^6 - 42*
a*b^4*c + 105*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*
c]])/(a^6*(b^2 - 4*a*c)^(3/2)) + ((5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[x])/
a^6 - ((5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[a + b*x + c*x^2])/(2*a^6)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^5 \left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^4\left(a+bx+cx^2\right)} - \frac{\int \frac{-5b^2+12ac-5bcx}{x^5\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^4\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{-5b^2+12ac}{ax^5} + \frac{5b^3-17abc}{a^2x^4} + \frac{-5b^4+22ab^2c-12a^2c^2}{a^3x^3} + \frac{5b^5-27ab^3c+29a^2bc^2}{a^4x^2} + \frac{\left(b^2-4ac\right)\left(-5b^4+12ab^2c-3a^2c^2\right)}{a^5x} + \frac{b(5b}{} \right.}{a\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b(5b^2-17ac)}{3a^3\left(b^2-4ac\right)x^3} - \frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2} + \frac{b(5b^4-27ab^2c+29a^2c^2)}{a^5\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^4\left(a+bx+cx^2\right)} + \frac{\left(5b^4-12ab^2c+3a^2c^2\right)\log(x)}{a^6} - \frac{\int \frac{b\left(5b^6-37ab^4c+78a^2b^2c^2-41a^3c^3\right)+c\left(5b^6-32ab^4c+51a^2b^2c^2-12a^3c^3\right)x}{a+bx+cx^2}\,dx}{a^6\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b(5b^2-17ac)}{3a^3\left(b^2-4ac\right)x^3} - \frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2} + \frac{b(5b^4-27ab^2c+29a^2c^2)}{a^5\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^4\left(a+bx+cx^2\right)} + \frac{\left(5b^4-12ab^2c+3a^2c^2\right)\log(x)}{a^6} - \frac{\left(5b^4-12ab^2c+3a^2c^2\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^6} - \frac{\left(b(5b^6-42ab^4c+105a^2b^2c^2-70a^3c^3)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a^6\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b(5b^2-17ac)}{3a^3\left(b^2-4ac\right)x^3} - \frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2} + \frac{b(5b^4-27ab^2c+29a^2c^2)}{a^5\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^4\left(a+bx+cx^2\right)} + \frac{\left(5b^4-12ab^2c+3a^2c^2\right)\log(x)}{a^6} - \frac{\left(5b^4-12ab^2c+3a^2c^2\right)\log\left(a+bx+cx^2\right)}{2a^6} + \frac{\left(b(5b^6-42ab^4c+105a^2b^2c^2-70a^3c^3)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{a^6\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b(5b^2-17ac)}{3a^3\left(b^2-4ac\right)x^3} - \frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2} + \frac{b(5b^4-27ab^2c+29a^2c^2)}{a^5\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^4\left(a+bx+cx^2\right)} + \frac{b(5b^6-42ab^4c+105a^2b^2c^2-70a^3c^3)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^6\left(b^2-4ac\right)^{3/2}} + \frac{\left(5b^4-12ab^2c+3a^2c^2\right)\log(x)}{a^6} - \frac{\left(5b^4-12ab^2c+3a^2c^2\right)\log\left(a+bx+cx^2\right)}{2a^6}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx$$

$$= \frac{-\frac{3a^4}{x^4}+\frac{8a^3b}{x^3}+\frac{6a^2(-3b^2+2ac)}{x^2}-\frac{24ab(-2b^2+3ac)}{x}-\frac{12a(-b^6+6ab^4c-9a^2b^2c^2+2a^3c^3-b^5cx+5ab^3c^2x-5a^2bc^3x)}{(b^2-4ac)(a+x(b+cx))}+\frac{12b(5b^6-42ab^4c}{}}{}$$

```
[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x]
```

```
[Out] ((-3*a^4)/x^4 + (8*a^3*b)/x^3 + (6*a^2*(-3*b^2 + 2*a*c))/x^2 - (24*a*b*(-2*
b^2 + 3*a*c))/x - (12*a*(-b^6 + 6*a*b^4*c - 9*a^2*b^2*c^2 + 2*a^3*c^3 - b^5
*c*x + 5*a*b^3*c^2*x - 5*a^2*b*c^3*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) +
(12*b*(5*b^6 - 42*a*b^4*c + 105*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTan[(b + 2*c*x
)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 12*(5*b^4 - 12*a*b^2*c + 3*a^
2*c^2)*Log[x] - 6*(5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[a + x*(b + c*x)])/(1
2*a^6)
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.13

method	result
default	$-\frac{1}{4a^2x^4}-\frac{-2ac+3b^2}{2x^2a^4}+\frac{(3a^2c^2-12a\,b^2c+5b^4)\ln(x)}{a^6}+\frac{2b}{3a^3x^3}-\frac{2b(3ac-2b^2)}{a^5x}-\frac{\frac{acb(5a^2c^2-5a\,b^2c+b^4)x}{4ac-b^2}-\frac{a(2c^3a^3-9a^2b^2c^2}{4ac-b^2}}{c\,x^2+bx+a}$
risch	Expression too large to display

```
[In] int(1/x/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^2/x^4-1/2*(-2*a*c+3*b^2)/x^2/a^4+(3*a^2*c^2-12*a*b^2*c+5*b^4)*ln(x)/
a^6+2/3/a^3*b/x^3-2*b*(3*a*c-2*b^2)/a^5/x-1/a^6*((a*c*b*(5*a^2*c^2-5*a*b^2*
c+b^4)/(4*a*c-b^2)*x-a*(2*a^3*c^3-9*a^2*b^2*c^2+6*a*b^4*c-b^6)/(4*a*c-b^2))
/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(12*a^3*c^4-51*a^2*b^2*c^3+32*a*b^4*c^2-5
*b^6*c)/c*ln(c*x^2+b*x+a)+2*(41*b*c^3*a^3-78*b^3*c^2*a^2+37*b^5*c*a-5*b^7-1
/2*(12*a^3*c^4-51*a^2*b^2*c^3+32*a*b^4*c^2-5*b^6*c)*b/c)/(4*a*c-b^2)^(1/2)*
arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 810 vs. 2(306) = 612.

Time = 0.73 (sec) , antiderivative size = 1640, normalized size of antiderivative = 5.16

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*a^5*b^4 - 24*a^6*b^2*c + 48*a^7*c^2 - 12*(5*a*b^7*c - 47*a^2*b^5*
c^2 + 137*a^3*b^3*c^3 - 116*a^4*b*c^4)*x^5 - 6*(10*a*b^8 - 99*a^2*b^6*c + 3
16*a^3*b^4*c^2 - 332*a^4*b^2*c^3 + 48*a^5*c^4)*x^4 - 2*(15*a^2*b^7 - 146*a^
3*b^5*c + 448*a^4*b^3*c^2 - 416*a^5*b*c^3)*x^3 + (10*a^3*b^6 - 89*a^4*b^4*c
 + 232*a^5*b^2*c^2 - 144*a^6*c^3)*x^2 - 6*((5*b^7*c - 42*a*b^5*c^2 + 105*a^
2*b^3*c^3 - 70*a^3*b*c^4)*x^6 + (5*b^8 - 42*a*b^6*c + 105*a^2*b^4*c^2 - 70*
a^3*b^2*c^3)*x^5 + (5*a*b^7 - 42*a^2*b^5*c + 105*a^3*b^3*c^2 - 70*a^4*b*c^3
)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
- 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 5*(a^4*b^5 - 8*a^5*b^3*c + 16*a^
6*b*c^2)*x + 6*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4
 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^
3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4
*b^2*c^3 + 48*a^5*c^4)*x^4)*log(c*x^2 + b*x + a) - 12*((5*b^8*c - 52*a*b^6*
c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b
^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 5
2*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + 48*a^5*c^4)*x^4)*log(x))/
((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*x^6 + (a^6*b^5 - 8*a^7*b^3*c + 16
*a^8*b*c^2)*x^5 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*x^4), -1/12*(3*a^5*b
^4 - 24*a^6*b^2*c + 48*a^7*c^2 - 12*(5*a*b^7*c - 47*a^2*b^5*c^2 + 137*a^3*b
^3*c^3 - 116*a^4*b*c^4)*x^5 - 6*(10*a*b^8 - 99*a^2*b^6*c + 316*a^3*b^4*c^2
- 332*a^4*b^2*c^3 + 48*a^5*c^4)*x^4 - 2*(15*a^2*b^7 - 146*a^3*b^5*c + 448*a
^4*b^3*c^2 - 416*a^5*b*c^3)*x^3 + (10*a^3*b^6 - 89*a^4*b^4*c + 232*a^5*b^2*
c^2 - 144*a^6*c^3)*x^2 - 12*((5*b^7*c - 42*a*b^5*c^2 + 105*a^2*b^3*c^3 - 70
*a^3*b*c^4)*x^6 + (5*b^8 - 42*a*b^6*c + 105*a^2*b^4*c^2 - 70*a^3*b^2*c^3)*x
^5 + (5*a*b^7 - 42*a^2*b^5*c + 105*a^3*b^3*c^2 - 70*a^4*b*c^3)*x^4)*sqrt(-b
^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 5*(a^4*
b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x + 6*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*
b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2
*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c +
179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + 48*a^5*c^4)*x^4)*log(c*x^2 + b*x + a) -
 12*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c
^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*
b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 +
48*a^5*c^4)*x^4)*log(x))/((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*x^6 + (a
^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*c^2)*x^5 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*
c^2)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx=-\frac{\left(5\,b^7-42\,ab^5c+105\,a^2b^3c^2-70\,a^3bc^3\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^6b^2-4\,a^7c\right)\sqrt{-b^2+4\,ac}}-\frac{\left(5\,b^4-12\,ab^2c+3\,a^2c^2\right)\log\left(cx^2+bx+a\right)}{2\,a^6}+\frac{\left(5\,b^4-12\,ab^2c+3\,a^2c^2\right)\log\left(|x|\right)}{a^6}-\frac{3\,a^5b^2-12\,a^6c-12\,(5\,ab^5c-27\,a^2b^3c^2+29\,a^3bc^3)x^5-6\,(10\,ab^6-59\,a^2b^4c+80\,a^3b^2c^2-12\,a^4c^3)x^4-\cdots}{12\,(cx^2+bx+a)(b^2-\cdots}$$

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] -(5*b^7 - 42*a*b^5*c + 105*a^2*b^3*c^2 - 70*a^3*b*c^3)*arctan((2*c*x + b)/s
qrt(-b^2 + 4*a*c))/((a^6*b^2 - 4*a^7*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(5*b^4 -
12*a*b^2*c + 3*a^2*c^2)*log(c*x^2 + b*x + a)/a^6 + (5*b^4 - 12*a*b^2*c + 3*
a^2*c^2)*log(abs(x))/a^6 - 1/12*(3*a^5*b^2 - 12*a^6*c - 12*(5*a*b^5*c - 27*
a^2*b^3*c^2 + 29*a^3*b*c^3)*x^5 - 6*(10*a*b^6 - 59*a^2*b^4*c + 80*a^3*b^2*c
^2 - 12*a^4*c^3)*x^4 - 2*(15*a^2*b^5 - 86*a^3*b^3*c + 104*a^4*b*c^2)*x^3 +
(10*a^3*b^4 - 49*a^4*b^2*c + 36*a^5*c^2)*x^2 - 5*(a^4*b^3 - 4*a^5*b*c)*x)/(
(c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^6*x^4)
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 1260, normalized size of antiderivative = 3.96

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx = \frac{\ln\left(x\right)\,\left(3\,a^2\,c^2-12\,a\,b^2\,c+5\,b^4\right)}{a^6} - \frac{\frac{1}{4\,a}-\frac{x^2\,\left(9\,a\,c-10\,b^2\right)}{12\,a^3}-\frac{5\,b\,x}{12\,a^2}+\frac{x^4\,\left(-12\,a^3\,c^3+80\,a^2\,b^2\,c^2-59\,a\,b^4\,c+10\,b^6\right)}{2\,a^5\,\left(4\,a\,c-b^2\right)}+\frac{b\,x^3\,\left(26\,a\,c-15\,b^2\right)}{6\,a^4}+\frac{b\,c\,x^5\,\left(29\,a^2\,c^2-27\,a\,b^2\,c+5\,b^4\right)}{a^5\,\left(4\,a\,c-b^2\right)}}{c\,x^6+b\,x^5+a\,x^4} + \frac{\ln\left(288\,a^6\,c^5-10\,b^{11}\,x-10\,a\,b^{10}+10\,a\,b^7\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+139\,a^2\,b^8\,c+10\,b^8\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-717\right.}{} - \frac{\ln\left(10\,a\,b^{10}+10\,b^{11}\,x-288\,a^6\,c^5+10\,a\,b^7\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-139\,a^2\,b^8\,c+10\,b^8\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+717\right.}{}$$

```
[In] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x)
```

```
[Out] (log(x)*(5*b^4 + 3*a^2*c^2 - 12*a*b^2*c))/a^6 - (1/(4*a) - (x^2*(9*a*c - 10
*b^2))/(12*a^3) - (5*b*x)/(12*a^2) + (x^4*(10*b^6 - 12*a^3*c^3 + 80*a^2*b^2
*c^2 - 59*a*b^4*c))/(2*a^5*(4*a*c - b^2)) + (b*x^3*(26*a*c - 15*b^2))/(6*a^
4) + (b*c*x^5*(5*b^4 + 29*a^2*c^2 - 27*a*b^2*c))/(a^5*(4*a*c - b^2)))/(a*x^
4 + b*x^5 + c*x^6) + (log(288*a^6*c^5 - 10*b^11*x - 10*a*b^10 + 10*a*b^7*(-
(4*a*c - b^2)^3)^(1/2) + 139*a^2*b^8*c + 10*b^8*x*(-(4*a*c - b^2)^3)^(1/2)
- 717*a^3*b^6*c^2 + 1643*a^4*b^4*c^3 - 1508*a^5*b^2*c^4 - 69*a^2*b^5*c*(-(4
*a*c - b^2)^3)^(1/2) - 53*a^4*b*c^3*(-(4*a*c - b^2)^3)^(1/2) - 779*a^2*b^7*
c^2*x + 1916*a^3*b^5*c^3*x - 1998*a^4*b^3*c^4*x + 36*a^4*c^4*x*(-(4*a*c - b
^2)^3)^(1/2) + 144*a*b^9*c*x + 129*a^3*b^3*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5
68*a^5*b*c^5*x - 84*a*b^6*c*x*(-(4*a*c - b^2)^3)^(1/2) + 225*a^2*b^4*c^2*x*
(-(4*a*c - b^2)^3)^(1/2) - 206*a^3*b^2*c^3*x*(-(4*a*c - b^2)^3)^(1/2))*(a^3
*(466*b^4*c^3 - 35*b*c^3*(-(4*a*c - b^2)^3)^(1/2)) - a^2*((387*b^6*c^2)/2 -
 (105*b^3*c^2*(-(4*a*c - b^2)^3)^(1/2))/2) - (5*b^10)/2 + 96*a^5*c^5 + (5*b
^7*(-(4*a*c - b^2)^3)^(1/2))/2 + a*(36*b^8*c - 21*b^5*c*(-(4*a*c - b^2)^3)^
(1/2)) - 456*a^4*b^2*c^4))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^
2*c^2) - (log(10*a*b^10 + 10*b^11*x - 288*a^6*c^5 + 10*a*b^7*(-(4*a*c - b^2
)^3)^(1/2) - 139*a^2*b^8*c + 10*b^8*x*(-(4*a*c - b^2)^3)^(1/2) + 717*a^3*b^
6*c^2 - 1643*a^4*b^4*c^3 + 1508*a^5*b^2*c^4 - 69*a^2*b^5*c*(-(4*a*c - b^2)^
3)^(1/2) - 53*a^4*b*c^3*(-(4*a*c - b^2)^3)^(1/2) + 779*a^2*b^7*c^2*x - 1916
*a^3*b^5*c^3*x + 1998*a^4*b^3*c^4*x + 36*a^4*c^4*x*(-(4*a*c - b^2)^3)^(1/2)
 - 144*a*b^9*c*x + 129*a^3*b^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 568*a^5*b*c^5
*x - 84*a*b^6*c*x*(-(4*a*c - b^2)^3)^(1/2) + 225*a^2*b^4*c^2*x*(-(4*a*c - b
^2)^3)^(1/2) - 206*a^3*b^2*c^3*x*(-(4*a*c - b^2)^3)^(1/2))*(a^2*((387*b^6*c
^2)/2 + (105*b^3*c^2*(-(4*a*c - b^2)^3)^(1/2))/2) - a^3*(466*b^4*c^3 + 35*b
*c^3*(-(4*a*c - b^2)^3)^(1/2)) + (5*b^10)/2 - 96*a^5*c^5 + (5*b^7*(-(4*a*c
- b^2)^3)^(1/2))/2 - a*(36*b^8*c + 21*b^5*c*(-(4*a*c - b^2)^3)^(1/2)) + 456
*a^4*b^2*c^4))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)
```

3.29 $\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 257

$$\begin{aligned}\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx &= \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{960c^3}-\frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x}\\&\quad-\frac{(7b^2-16ac)\,x\sqrt{ax^2+bx^3+cx^4}}{240c^2}+\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}\\&\quad+\frac{b(7b^2-12ac)(b^2-4ac)\,x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{9/2}\sqrt{ax^2+bx^3+cx^4}}\end{aligned}$$

```
[Out] 1/256*b*(-12*a*c+7*b^2)*(-4*a*c+b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2
+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(9/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/960*
b*(-116*a*c+35*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/1920*(256*a^2*c^2-460*a
*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^4/x-1/240*(-16*a*c+7*b^2)*x*(c*
x^4+b*x^3+a*x^2)^(1/2)/c^2+1/40*x^2*(8*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{1933, 1963, 12, 1928, 635, 212\}$

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$= -\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x} + \frac{bx(7b^2-12ac)(b^2-4ac)\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{9/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{240c^2} + \frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}$$

```
[In] Int[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(960*c^3) - ((105*b^4 -
460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(1920*c^4*x) - ((7*
b^2 - 16*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(240*c^2) + (x^2*(b + 8*c*x)*S
qrt[a*x^2 + b*x^3 + c*x^4])/(40*c) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*x*Sq
rt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])
/(256*c^(9/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
```

```
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1933

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Dist[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), Int[x^(m - (n - 2*q
))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}+\frac{\int \frac{x^3\left(-3ab-\frac{1}{2}\left(7b^2-16ac\right)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{40c}\\
&= -\frac{\left(7b^2-16ac\right)x\sqrt{ax^2+bx^3+cx^4}}{240c^2}+\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}\\
&\quad-\frac{\int \frac{x^2\left(-a\left(7b^2-16ac\right)-\frac{1}{4}b\left(35b^2-116ac\right)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{120c^2}\\
&= \frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{960c^3}-\frac{\left(7b^2-16ac\right)x\sqrt{ax^2+bx^3+cx^4}}{240c^2}\\
&\quad+\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}+\frac{\int \frac{x\left(-\frac{1}{4}ab\left(35b^2-116ac\right)-\frac{1}{8}\left(105b^4-460ab^2c+256a^2c^2\right)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{240c^3}
\end{aligned}$$

$$= \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x} - \frac{(7b^2-16ac)x\sqrt{ax^2+bx^3+cx^4}}{240c^2} + \frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c} - \frac{\int -\frac{15b(7b^2-12ac)(b^2-4ac)x}{16\sqrt{ax^2+bx^3+cx^4}}\,dx}{240c^4}$$

$$= \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x} - \frac{(7b^2-16ac)x\sqrt{ax^2+bx^3+cx^4}}{240c^2} + \frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c} + \frac{(b(7b^2-12ac)(b^2-4ac))\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{256c^4}$$

$$= \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x} - \frac{(7b^2-16ac)x\sqrt{ax^2+bx^3+cx^4}}{240c^2} + \frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c} + \frac{\left(b(7b^2-12ac)(b^2-4ac)x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{256c^4\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x} - \frac{(7b^2-16ac)x\sqrt{ax^2+bx^3+cx^4}}{240c^2} + \frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c} + \frac{\left(b(7b^2-12ac)(b^2-4ac)x\sqrt{a+bx+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{128c^4\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x} - \frac{(7b^2-16ac)x\sqrt{ax^2+bx^3+cx^4}}{240c^2} + \frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c} + \frac{b(7b^2-12ac)(b^2-4ac)x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{9/2}\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.72

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$$

$$= \frac{2\sqrt{c}x(a+x(b+cx))\left(-105b^4+70b^3cx+4b^2c(115a-14cx^2)+8bc^2x(-29a+6cx^2)+128c^2(-2a^2+acx\ldots\right.}{3840c^{9/2}\sqrt{x^2(a\ldots}}$$

```
[In] Integrate[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(-105*b^4 + 70*b^3*c*x + 4*b^2*c*(115*a - 14
*c*x^2) + 8*b*c^2*x*(-29*a + 6*c*x^2) + 128*c^2*(-2*a^2 + a*c*x^2 + 3*c^2*x
^4)) - 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*x*Sqrt[a + x*(b + c*x)]*Log[c
^4*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(3840*c^(9/2)*Sqrt[x^2*(
a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.52

method	result
pseudoelliptic	$-\frac{2\left(\left(-\frac{45}{32}a^2bc^2+\frac{75}{64}ab^3c-\frac{105}{512}b^5\right)\ln\left(2\sqrt{cx^2+bx+a}\,\sqrt{c}+2cx+b\right)+\sqrt{cx^2+bx+a}\,\left(\left(\frac{7}{32}b^2x^2+\frac{29}{32}abx+a^2\right)c^{\frac{5}{2}}-\frac{115\left(\frac{7bx}{46}+a\right)b\ldots}{64}\ldots\right.\right.}{15c^{\frac{9}{2}}}$
risch	$-\frac{\left(-384c^4x^4-48bc^3x^3-128ac^3x^2+56b^2c^2x^2+232abc^2x-70b^3cx+256a^2c^2-460ab^2c+105b^4\right)\sqrt{x^2(cx^2+bx+a)}}{1920c^4x}+\frac{b(48a\ldots}{\ldots}$
default	$\frac{\sqrt{cx^4+bx^3+ax^2}\left(768x^2(cx^2+bx+a)^{\frac{3}{2}}c^{\frac{9}{2}}-672c^{\frac{7}{2}}(cx^2+bx+a)^{\frac{3}{2}}bx-512c^{\frac{7}{2}}(cx^2+bx+a)^{\frac{3}{2}}a+720c^{\frac{7}{2}}\sqrt{cx^2+bx+a}\,abx+56\ldots\right.}{\ldots}$

```
[In] int(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/c^(9/2)*((-45/32*a^2*b*c^2+75/64*a*b^3*c-105/512*b^5)*ln(2*(c*x^2+b*x
+a)^(1/2)*c^(1/2)+2*c*x+b)+(c*x^2+b*x+a)^(1/2)*((7/32*b^2*x^2+29/32*a*b*x+a
^2)*c^(5/2)-115/64*(7/46*b*x+a)*b^2*c^(3/2)-1/2*(3/8*b*x+a)*x^2*c^(7/2)-3/2
*c^(9/2)*x^4+105/256*c^(1/2)*b^4))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.52

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$= \left[\frac{15\,(7\,b^5-40\,ab^3c+48\,a^2bc^2)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)+4\,(384\,c^5x^4+48\,b\ldots}{7680\ldots}\right.$$
$$-\frac{15\,(7\,b^5-40\,ab^3c+48\,a^2bc^2)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-2\,(384\,c^5x^4+48\,bc^4x^3-105\,b^4c+\ldots}{3840\,c^5x}$$

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 +
 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a
*c)*x)/x) + 4*(384*c^5*x^4 + 48*b*c^4*x^3 - 105*b^4*c + 460*a*b^2*c^2 - 256
*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^2 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x)*s
qrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x), -1/3840*(15*(7*b^5 - 40*a*b^3*c + 48*a
^2*b*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqr
t(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(384*c^5*x^4 + 48*b*c^4*x^3 - 105*b^
4*c + 460*a*b^2*c^2 - 256*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^2 + 2*(35*b^
3*c^2 - 116*a*b*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x)]
```

Sympy [F]

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx=\int x^2\sqrt{x^2\,(a+bx+cx^2)}\,dx$$

```
[In] integrate(x**2*(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx=\int \sqrt{cx^4+bx^3+ax^2}x^2\,dx$$

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.07

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$=\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(8\,x\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x-\frac{7\,b^2c^2\mathrm{sgn}(x)-16\,ac^3\mathrm{sgn}(x)}{c^4}\right)x+\frac{35\,b^3c\mathrm{sgn}(x)\ldots}{}\right.\right.$$
$$-\frac{(7\,b^5\mathrm{sgn}(x)-40\,ab^3c\mathrm{sgn}(x)+48\,a^2bc^2\mathrm{sgn}(x))\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{256\,c^{\frac{9}{2}}}$$
$$+\frac{\left(105\,b^5\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-600\,ab^3c\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+720\,a^2bc^2\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+210\,\sqrt{a}b^4\sqrt{\ldots}\right.}{3840\,c^{\frac{9}{2}}}$$

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*x*sgn(x) + b*sgn(x)/c)*x - (7*b^2*
c^2*sgn(x) - 16*a*c^3*sgn(x))/c^4)*x + (35*b^3*c*sgn(x) - 116*a*b*c^2*sgn(x
))/c^4)*x - (105*b^4*sgn(x) - 460*a*b^2*c*sgn(x) + 256*a^2*c^2*sgn(x))/c^4)
 - 1/256*(7*b^5*sgn(x) - 40*a*b^3*c*sgn(x) + 48*a^2*b*c^2*sgn(x))*log(abs(2
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2) + 1/3840*(105*b^
5*log(abs(b - 2*sqrt(a)*sqrt(c))) - 600*a*b^3*c*log(abs(b - 2*sqrt(a)*sqrt(
c))) + 720*a^2*b*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) + 210*sqrt(a)*b^4*sqrt
(c) - 920*a^(3/2)*b^2*c^(3/2) + 512*a^(5/2)*c^(5/2))*sgn(x)/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx=\int x^2\,\sqrt{c\,x^4+b\,x^3+a\,x^2}\,dx$$

```
[In] int(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
[Out] int(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.30 $\int x\sqrt{ax^2+bx^3+cx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 205

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx=-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96c^2}+\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192c^3x}+\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{(b^2-4ac)(5b^2-4ac)x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -1/128*(-4*a*c+b^2)*(-4*a*c+5*b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b
*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(7/2)/(c*x^4+b*x^3+a*x^2)^(1/2)-1/96*(-1
2*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2+1/192*b*(-52*a*c+15*b^2)*(c*x^4+
b*x^3+a*x^2)^(1/2)/c^3/x+1/24*x*(6*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1933, 1963, 12, 1928, 635, 212}

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx=-\frac{x(b^2-4ac)(5b^2-4ac)\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192c^3x}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96c^2}+\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}$$

```
[In] Int[x*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] -1/96*((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/c^2 + (b*(15*b^2 - 52*
a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(192*c^3*x) + (x*(b + 6*c*x)*Sqrt[a*x^2 +
 b*x^3 + c*x^4])/(24*c) - ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*x*Sqrt[a + b*x + c
*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2)*
Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1933

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Dist[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), Int[x^(m - (n - 2*q
))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}+\frac{\int \frac{x^2\left(-2ab-\frac{1}{2}\left(5b^2-12ac\right)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{24c}\\
&= -\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2}+\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}\\
&\quad-\frac{\int \frac{x\left(-\frac{1}{2}a\left(5b^2-12ac\right)-\frac{1}{4}b\left(15b^2-52ac\right)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{48c^2}\\
&= -\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2}+\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x}\\
&\quad+\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}+\frac{\int -\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)x}{8\sqrt{ax^2+bx^3+cx^4}}\,dx}{48c^3}\\
&= -\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2}+\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x}\\
&\quad+\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\left(\left(b^2-4ac\right)\left(5b^2-4ac\right)\right)\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{128c^3}\\
&= -\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2}\\
&\quad+\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x}+\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}\\
&\quad-\frac{\left(\left(b^2-4ac\right)\left(5b^2-4ac\right)x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{128c^3\sqrt{ax^2+bx^3+cx^4}}\\
&= -\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2}\\
&\quad+\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x}+\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}\\
&\quad-\frac{\left(\left(b^2-4ac\right)\left(5b^2-4ac\right)x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{64c^3\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}$$

$$= -\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192c^3x} + \frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c} - \frac{(b^2-4ac)(5b^2-4ac)x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.73

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx$$

$$= \frac{2\sqrt{c}x(a+x(b+cx))\left(15b^3-10b^2cx+24c^2x(a+2cx^2)+b(-52ac+8c^2x^2)\right)+3(5b^4-24ab^2c+16a^2c^2)\,x}{384c^{7/2}\sqrt{x^2(a+x(b+cx))}}$$

```
[In] Integrate[x*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(15*b^3 - 10*b^2*c*x + 24*c^2*x*(a + 2*c*x^2
) + b*(-52*a*c + 8*c^2*x^2)) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x*Sqrt[a
 + x*(b + c*x)]*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c^(7
/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.52

method	result
pseudoelliptic	$-\frac{\left(a^2c^2-\frac{3}{2}ab^2c+\frac{5}{16}b^4\right)\ln\left(2\sqrt{cx^2+bx+a}\sqrt{c}+2cx+b\right)+\frac{13\sqrt{cx^2+bx+a}\left(b\left(\frac{5bx}{26}+a\right)c^{\frac{3}{2}}-\frac{6\left(\frac{bx}{3}+a\right)xc^{\frac{5}{2}}}{13}-\frac{15\sqrt{c}\,b^3}{52}-\frac{12c^{\frac{7}{2}}x^3}{13}\right)}{6}}{8c^{\frac{7}{2}}}$
risch	$-\frac{\left(-48c^3x^3-8bc^2x^2-24ac^2x+10b^2cx+52abc-15b^3\right)\sqrt{x^2(cx^2+bx+a)}}{192c^3x}-\frac{\left(16a^2c^2-24ab^2c+5b^4\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right.}{128c^{\frac{7}{2}}x\sqrt{cx^2+bx+a}}$
default	$\frac{\sqrt{cx^4+bx^3+ax^2}\left(96x(cx^2+bx+a)^{\frac{3}{2}}c^{\frac{7}{2}}-48c^{\frac{7}{2}}\sqrt{cx^2+bx+a}\,ax-80c^{\frac{5}{2}}(cx^2+bx+a)^{\frac{3}{2}}b+60c^{\frac{5}{2}}\sqrt{cx^2+bx+a}\,b^2x-24c^{\frac{5}{2}}\sqrt{cx^2-}\right.}{}$

```
[In] int(x*(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/c^(7/2)*((a^2*c^2-3/2*a*b^2*c+5/16*b^4)*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/
2)+2*c*x+b)+13/6*(c*x^2+b*x+a)^(1/2)*(b*(5/26*b*x+a)*c^(3/2)-6/13*(1/3*b*x+
a)*x*c^(5/2)-15/52*c^(1/2)*b^3-12/13*c^(7/2)*x^3))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.59

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$= \left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)+4\,(48\,c^4x^3+8\,bc^3x}{768\,c^4x}\right.$$

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b
*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*
x)/x) + 4*(48*c^4*x^3 + 8*b*c^3*x^2 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2
- 12*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^4*x), 1/384*(3*(5*b^4 - 24*a
*b^2*c + 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c
*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(48*c^4*x^3 + 8*b*c^3*x^2
 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x)*sqrt(c*x^4 + b*x^3 +
 a*x^2))/(c^4*x)]
```

Sympy [F]

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx=\int x\sqrt{x^2\,(a+bx+cx^2)}\,dx$$

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx=\int\sqrt{cx^4+bx^3+ax^2}x\,dx$$

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x, x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.08

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$=\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,x\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x-\frac{5\,b^2c\mathrm{sgn}(x)-12\,ac^2\mathrm{sgn}(x)}{c^3}\right)x+\frac{15\,b^3\mathrm{sgn}(x)-52\,ab}{c^3}\right.$$
$$+\frac{\left(5\,b^4\mathrm{sgn}(x)-24\,ab^2c\mathrm{sgn}(x)+16\,a^2c^2\mathrm{sgn}(x)\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{7}{2}}}$$
$$-\frac{\left(15\,b^4\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-72\,ab^2c\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+48\,a^2c^2\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+30\,\sqrt{a}b^3\sqrt{c}-104\right.}{384\,c^{\frac{7}{2}}}$$

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*x*sgn(x) + b*sgn(x)/c)*x - (5*b^2*c*sg
n(x) - 12*a*c^2*sgn(x))/c^3)*x + (15*b^3*sgn(x) - 52*a*b*c*sgn(x))/c^3) + 1
/128*(5*b^4*sgn(x) - 24*a*b^2*c*sgn(x) + 16*a^2*c^2*sgn(x))*log(abs(2*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2) - 1/384*(15*b^4*log(ab
s(b - 2*sqrt(a)*sqrt(c))) - 72*a*b^2*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 48
*a^2*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^3*sqrt(c) - 104*a^(
3/2)*b*c^(3/2))*sgn(x)/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx=\int x\,\sqrt{c\,x^4+b\,x^3+a\,x^2}\,dx$$

```
[In] int(x*(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

```
[Out] int(x*(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.31 $\int \sqrt{ax^2+bx^3+cx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 163

$$\int \sqrt{ax^2+bx^3+cx^4}\,dx=-\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x}+\frac{(a+bx+cx^2)\sqrt{ax^2+bx^3+cx^4}}{3cx}+\frac{b(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}x\sqrt{a+bx+cx^2}}$$

```
[Out] -1/8*b*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2/x+1/3*(c*x^2+b*x+a)*(c*x^4+b
*x^3+a*x^2)^(1/2)/c/x+1/16*b*(-4*a*c+b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*
x^2+b*x+a)^(1/2))*(c*x^4+b*x^3+a*x^2)^(1/2)/c^(5/2)/x/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1917, 654, 626, 635, 212}

$$\int \sqrt{ax^2+bx^3+cx^4}\,dx=\frac{b(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}x\sqrt{a+bx+cx^2}}-\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x}+\frac{(a+bx+cx^2)\sqrt{ax^2+bx^3+cx^4}}{3cx}$$

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] -1/8*(b*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c^2*x) + ((a + b*x + c*x^
2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*c*x) + (b*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x
```

```
^3 + c*x^4]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(
5/2)*x*Sqrt[a + b*x + c*x^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1917

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{ax^2+bx^3+cx^4}\int x\sqrt{a+bx+cx^2}\,dx}{x\sqrt{a+bx+cx^2}} \\
&= \frac{(a+bx+cx^2)\sqrt{ax^2+bx^3+cx^4}}{3cx} - \frac{\left(b\sqrt{ax^2+bx^3+cx^4}\right)\int\sqrt{a+bx+cx^2}\,dx}{2cx\sqrt{a+bx+cx^2}} \\
&= -\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x} + \frac{(a+bx+cx^2)\sqrt{ax^2+bx^3+cx^4}}{3cx} \\
&\quad + \frac{\left(b(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}\right)\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{16c^2x\sqrt{a+bx+cx^2}}
\end{aligned}
$$

$$= -\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x} + \frac{(a+bx+cx^2)\sqrt{ax^2+bx^3+cx^4}}{3cx} + \frac{\left(b(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{8c^2x\sqrt{a+bx+cx^2}}$$

$$= -\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x} + \frac{(a+bx+cx^2)\sqrt{ax^2+bx^3+cx^4}}{3cx} + \frac{b(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}x\sqrt{a+bx+cx^2}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.74

$$\int \sqrt{ax^2+bx^3+cx^4}\,dx$$

$$= \frac{2\sqrt{c}x(a+x(b+cx))\left(-3b^2+2bcx+8c(a+cx^2)\right)-3(b^3-4abc)\,x\sqrt{a+x(b+cx)}\log\left(c^2\left(b+2cx-2\sqrt{\right.\right.}{48c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(-3*b^2 + 2*b*c*x + 8*c*(a + c*x^2)) - 3*(b^
3 - 4*a*b*c)*x*Sqrt[a + x*(b + c*x)]*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)])])/(48*c^(5/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.74

method	result
risch	$\frac{(8c^2x^2+2bcx+8ac-3b^2)\sqrt{x^2(cx^2+bx+a)}}{24c^2x}-\frac{b(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\sqrt{x^2(cx^2+bx+a)}}{16c^{\frac{5}{2}}x\sqrt{cx^2+bx+a}}$
pseudoelliptic	$\frac{16x^2\sqrt{cx^2+bx+a}\,c^{\frac{5}{2}}+4c^{\frac{3}{2}}\sqrt{cx^2+bx+a}\,bx+16a\,c^{\frac{3}{2}}\sqrt{cx^2+bx+a}-6\sqrt{c}\sqrt{cx^2+bx+a}\,b^2-12\ln\left(2\sqrt{cx^2+bx+a}\sqrt{c}+2cx+b\right)a}{48c^{\frac{5}{2}}}$
default	$\frac{\sqrt{cx^4+bx^3+ax^2}\left(16(cx^2+bx+a)^{\frac{3}{2}}c^{\frac{5}{2}}-12c^{\frac{5}{2}}\sqrt{cx^2+bx+a}\,bx-6c^{\frac{3}{2}}\sqrt{cx^2+bx+a}\,b^2-12\ln\left(\frac{2\sqrt{cx^2+bx+a}\sqrt{c}+2cx+b}{2\sqrt{c}}\right)ab\,c^2-\right.}{48x\sqrt{cx^2+bx+a}\,c^{\frac{7}{2}}}$

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(8*c^2*x^2+2*b*c*x+8*a*c-3*b^2)/c^2*(x^2*(c*x^2+b*x+a))^(1/2)/x-1/16*b
*(4*a*c-b^2)/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*(x^2*(c*x^
2+b*x+a))^(1/2)/x/(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.60

$$\int \sqrt{ax^2 + bx^3 + cx^4}\, dx$$

$$= \left[-\frac{3\,(b^3 - 4\,abc)\sqrt{c}x \log\left(-\frac{8\,c^2x^3 + 8\,bcx^2 - 4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c} + (b^2+4\,ac)x}{x}\right) - 4\,(8\,c^3x^2 + 2\,bc^2x - 3\,b^2c + 8\,ac}{96\,c^3x} \right.$$

$$-\frac{3\,(b^3 - 4\,abc)\sqrt{-c}x \arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right) - 2\,(8\,c^3x^2 + 2\,bc^2x - 3\,b^2c + 8\,ac^2)\sqrt{cx^4 + bx^3 + a}}{48\,c^3x}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3 - 4*a*b*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*
x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*(8*c^3*x
^2 + 2*b*c^2*x - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x), -
1/48*(3*(b^3 - 4*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(
2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(8*c^3*x^2 + 2*b*c^2*x
 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x)]
```

Sympy [F]

$$\int \sqrt{ax^2 + bx^3 + cx^4}\, dx = \int \sqrt{ax^2 + bx^3 + cx^4}\, dx$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**2 + b*x**3 + c*x**4), x)
```

Maxima [F]

$$\int \sqrt{ax^2 + bx^3 + cx^4}\, dx = \int \sqrt{cx^4 + bx^3 + ax^2}\, dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98

$$\int \sqrt{ax^2+bx^3+cx^4}\,dx$$
$$=\frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(4\,x\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x-\frac{3\,b^2\mathrm{sgn}(x)-8\,ac\mathrm{sgn}(x)}{c^2}\right)$$
$$-\frac{\left(b^3\mathrm{sgn}(x)-4\,abc\mathrm{sgn}(x)\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{5}{2}}}$$
$$+\frac{\left(3\,b^3\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-12\,abc\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+6\,\sqrt{a}b^2\sqrt{c}-16\,a^{\frac{3}{2}}c^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{48\,c^{\frac{5}{2}}}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*(4*x*sgn(x) + b*sgn(x)/c)*x - (3*b^2*sgn(x) -
 8*a*c*sgn(x))/c^2) - 1/16*(b^3*sgn(x) - 4*a*b*c*sgn(x))*log(abs(2*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2) + 1/48*(3*b^3*log(abs(b -
 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)
*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))*sgn(x)/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ax^2+bx^3+cx^4}\,dx=\int \sqrt{c\,x^4+b\,x^3+a\,x^2}\,dx$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.32 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 119

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx = \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{(b^2-4ac)\,x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -1/8*(-4*a*c+b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x
^2+b*x+a)^(1/2)/c^(3/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/4*(2*c*x+b)*(c*x^4+b*x^
3+a*x^2)^(1/2)/c/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1932, 1928, 635, 212}

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx = \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{x(b^2-4ac)\,\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]
```

```
[Out] ((b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c*x) - ((b^2 - 4*a*c)*x*Sqrt[a
 + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*
c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1932

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1))), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{(b^2-4ac)\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8c} \\
&= \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{\left((b^2-4ac)\,x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8c\sqrt{ax^2+bx^3+cx^4}} \\
&= \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{\left((b^2-4ac)\,x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4c\sqrt{ax^2+bx^3+cx^4}} \\
&= \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{(b^2-4ac)\,x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx = \frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{c}(b+2cx)+\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a}-\sqrt{a+x(b+cx)}}\right)}{\sqrt{a+x(b+cx)}}\right)}{4c^{3/2}x}$$

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[c]*(b + 2*c*x) + ((b^2 - 4*a*c)*ArcTanh[
(Sqrt[c]*x)/(Sqrt[a] - Sqrt[a + x*(b + c*x)])])/Sqrt[a + x*(b + c*x)]))/(4*
c^(3/2)*x)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{4\sqrt{c\,x^2+bx+a}\,c^{\frac{3}{2}}x+4\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)ac-\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)b^2+2b\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{8c^{\frac{3}{2}}}$
risch	$\frac{(2cx+b)\sqrt{x^2(c\,x^2+bx+a)}}{4cx}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)\sqrt{x^2(c\,x^2+bx+a)}}{8c^{\frac{3}{2}}x\sqrt{c\,x^2+bx+a}}$
default	$\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\,\left(4c^{\frac{5}{2}}\sqrt{c\,x^2+bx+a}\,x+2c^{\frac{3}{2}}\sqrt{c\,x^2+bx+a}\,b+4\ln\left(\frac{2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)a\,c^2-\ln\left(\frac{2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)\right.}{8c^{\frac{5}{2}}\sqrt{c\,x^2+bx+a}\,x}$

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c^(3/2)*(4*(c*x^2+b*x+a)^(1/2)*c^(3/2)*x+4*ln(2*(c*x^2+b*x+a)^(1/2)*c^(
1/2)+2*c*x+b)*a*c-ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*b^2+2*b*(c*x^2+
b*x+a)^(1/2)*c^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx$$
$$= \left[-\frac{(b^2-4\,ac)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)-4\,\sqrt{cx^4+bx^3+ax^2}(2\,c^2x+bc)}{16\,c^2x},\right.$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/16*((b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4
+ b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4 +
 b*x^3 + a*x^2)*(2*c^2*x + b*c))/(c^2*x), 1/8*((b^2 - 4*a*c)*sqrt(-c)*x*arc
tan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2
 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c))/(c^2*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx = \int \frac{\sqrt{x^2\,(a+bx+cx^2)}}{x}\,dx$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx = \int \frac{\sqrt{cx^4+bx^3+ax^2}}{x}\,dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x, x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx$$
$$=\frac{1}{8}\left(2\sqrt{cx^2+bx+a}\left(2x+\frac{b}{c}\right)+\frac{(b^2-4ac)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{3}{2}}}\right)\mathrm{sgn}(x)$$
$$-\frac{\left(b^2\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)-4ac\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+2\sqrt{a}b\sqrt{c}\right)\mathrm{sgn}(x)}{8c^{\frac{3}{2}}}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(c*x^2 + b*x + a)*(2*x + b/c) + (b^2 - 4*a*c)*log(abs(2*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(3/2))*sgn(x) - 1/8*(b^2*log(ab
s(b - 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 2*sqrt(
a)*b*sqrt(c))*sgn(x)/c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x}\,dx$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x, x)
```

3.33 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} dx = \frac{\sqrt{ax^2+bx^3+cx^4}}{x} - \frac{\sqrt{a}x\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} + \frac{bx\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*a^(1/2)*(c*x^2+b*x+a)
^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/2*b*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x
^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+(c*x
^4+b*x^3+a*x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1935, 1947, 857, 635, 212, 738}

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} dx = -\frac{\sqrt{a}x\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} + \frac{bx\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}} + \frac{\sqrt{ax^2+bx^3+cx^4}}{x}$$

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]
```

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/x - (Sqrt[a]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2
*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4] +
```

```
(b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])])/(2*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1935

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*(2
*n - q) + 1)), x] + Dist[(n - q)*(p/(m + p*(2*n - q) + 1)), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1947

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
```

```
&& EqQ[q, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{ax^2+bx^3+cx^4}}{x}+\frac{1}{2}\int \frac{2a+bx}{\sqrt{ax^2+bx^3+cx^4}}\,dx\\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{x}+\frac{\left(x\sqrt{a+bx+cx^2}\right)\int \frac{2a+bx}{x\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{ax^2+bx^3+cx^4}}\\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{x}+\frac{\left(ax\sqrt{a+bx+cx^2}\right)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{\sqrt{ax^2+bx^3+cx^4}}\\
&\quad+\frac{\left(bx\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{ax^2+bx^3+cx^4}}\\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{x}-\frac{\left(2ax\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}\\
&\quad+\frac{\left(bx\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}\\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{x}-\frac{\sqrt{a}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}\\
&\quad+\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx = \frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}+4\sqrt{a}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)-b\log\left(b+2cx-2\sqrt{c}\sqrt{a+\right.\right.}{2\sqrt{c}\sqrt{x^2(a+x(b+cx))}}$$

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)] + 4*Sqrt[a]*Sqrt[
c]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] - b*Log[b + 2*c*x -
 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(2*Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.55

method	result	size
pseudoelliptic	$\frac{-2\left(-\ln(2)+\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)\right)\sqrt{c}\,\sqrt{a}+\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}$	95
default	$-\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\left(2\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)\sqrt{c}-b\ln\left(\frac{2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)-2\sqrt{c\,x^2+bx+a}\,\sqrt{c}\right)}{2x\sqrt{c\,x^2+bx+a}\,\sqrt{c}}$	126

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^(1/2)*(-2*(-ln(2)+ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2
)))*c^(1/2)*a^(1/2)+ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*b+2*(c*x^2+b*
x+a)^(1/2)*c^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 638, normalized size of antiderivative = 3.69

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx$$
$$= \left[\frac{b\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)+2\,\sqrt{a}cx\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3}}{x^3}\right.}{4\,cx}\right.$$
$$-\frac{b\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-\sqrt{a}cx\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)-2}{2\,cx}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^
2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 2*sqrt(a)*c*x*log(-(8*a*b*x^
2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)
*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c*x), -1/2*(b*sqrt(-c)*x
*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c
*x^2 + a*c*x)) - sqrt(a)*c*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x
- 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 2*sqrt(c*x^4 +
b*x^3 + a*x^2)*c)/(c*x), 1/4*(4*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^4 + b*x^3
+ a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + b*sqrt(c)*x*lo
g(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(
c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c*x), 1/2*(2*s
qrt(-a)*c*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*
```

```
c*x^3 + a*b*x^2 + a^2*x)) - b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*
x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x
^3 + a*x^2)*c)/(c*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx = \int \frac{\sqrt{x^2\left(a+bx+cx^2\right)}}{x^2}\,dx$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**2,x)
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx = \int \frac{\sqrt{cx^4+bx^3+ax^2}}{x^2}\,dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^2}\,dx = \int \frac{\sqrt{c\,x^4 + b\,x^3 + a\,x^2}}{x^2}\,dx$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^2,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^2, x)
```

3.34 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx=-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}-\frac{bx\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{c}x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -1/2*b*x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(
1/2)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x
^2+b*x+a)^(1/2))*c^(1/2)*(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)-(c*x
^4+b*x^3+a*x^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1934, 1947, 857, 635, 212, 738}

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx=-\frac{bx\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{c}x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}$$

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3,x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2) - (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3
+ c*x^4]) + (Sqrt[c]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1947

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[(A + B*x^(n - q))/(x^(q/
```

```
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}+\frac{1}{2}\int\frac{b+2cx}{\sqrt{ax^2+bx^3+cx^4}}\,dx\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}+\frac{\left(x\sqrt{a+bx+cx^2}\right)\int\frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{ax^2+bx^3+cx^4}}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}+\frac{\left(bx\sqrt{a+bx+cx^2}\right)\int\frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{ax^2+bx^3+cx^4}}\\
&\quad+\frac{\left(cx\sqrt{a+bx+cx^2}\right)\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{\sqrt{ax^2+bx^3+cx^4}}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}-\frac{\left(bx\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int\frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}\\
&\quad+\frac{\left(2cx\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\\
&\quad+\frac{\sqrt{c}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.72

$$
\int\frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx
$$

$$
=\frac{\sqrt{a+x(b+cx)}\left(bx\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)-\sqrt{a}\left(\sqrt{a+x(b+cx)}+\sqrt{c}x\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(\right.\right.\right.}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}
$$

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3,x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(b*x*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqr
t[a]] - Sqrt[a]*(Sqrt[a + x*(b + c*x)] + Sqrt[c]*x*Log[b + 2*c*x - 2*Sqrt[c
]*Sqrt[a + x*(b + c*x)]])))/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.58

method	result
pseudoelliptic	$\frac{2\sqrt{c}\,\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)x\sqrt{a}+bx\ln(2)-bx\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)-2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{2x\sqrt{a}}$
risch	$-\frac{\sqrt{x^2(c\,x^2+bx+a)}}{x^2}+\frac{\left(\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)-\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2\sqrt{a}}\right)\sqrt{x^2(c\,x^2+bx+a)}}{x\sqrt{c\,x^2+bx+a}}$
default	$\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\,\left(2x^2\sqrt{c\,x^2+bx+a}\,c^{\frac{5}{2}}-c^{\frac{3}{2}}\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)bx-2\left(c\,x^2+bx+a\right)^{\frac{3}{2}}c^{\frac{3}{2}}+2c^{\frac{3}{2}}\sqrt{c\,x^2+bx+a}\,bx+2\right.}{2x^2\sqrt{c\,x^2+bx+a}\,a\,c^{\frac{3}{2}}}$

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*c^(1/2)*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*x*a^(1/2)+b*x*ln(2
)-b*x*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2))-2*a^(1/2)*(c*x^
2+b*x+a)^(1/2))/x/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 653, normalized size of antiderivative = 3.77

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx$$
$$= \left[\frac{2\,a\sqrt{c}x^2\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)+\sqrt{a}bx^2\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+b}}{x^3}\right.}{4\,ax^2}\right.$$
$$-\frac{4\,a\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-\sqrt{a}bx^2\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)}{4\,ax^2}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 +
a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + sqrt(a)*b*x^2*log(-(8*a*
b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x +
2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2), -1/4*(4*a*sq
rt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2
*x^3 + b*c*x^2 + a*c*x)) - sqrt(a)*b*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^
3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*s
qrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(
```

```
c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) +
a*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(
2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*a)
/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x +
 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*a*sqrt(-c)*x^2*arctan(1/2*s
qrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)
) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^3}\,dx = \int \frac{\sqrt{x^2\,(a + bx + cx^2)}}{x^3}\,dx$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**3,x)
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^3}\,dx = \int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^3}\,dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^3}\,dx$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^3,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^3, x)
```

3.35 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2} + \frac{(b^2-4ac)\,\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}}$$

```
[Out] 1/8*(-4*a*c+b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))
/a^(3/2)-1/2*(c*x^4+b*x^3+a*x^2)^(1/2)/x^3-1/4*b*(c*x^4+b*x^3+a*x^2)^(1/2)/
a/x^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1934, 1965, 12, 1918, 212}

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx = \frac{(b^2-4ac)\,\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]
```

```
[Out] -1/2*Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3 - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a
*x^2) + ((b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^
3 + c*x^4])])/(8*a^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}+\frac{1}{4}\int \frac{b+2cx}{x\sqrt{ax^2+bx^3+cx^4}}\,dx$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}-\frac{\int \frac{b^2-4ac}{2\sqrt{ax^2+bx^3+cx^4}}\,dx}{4a}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2} - \frac{(b^2-4ac)\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8a}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2} + \frac{(b^2-4ac)\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{4a}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2} + \frac{(b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx = -\frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{a}(2a+bx)\sqrt{a+x(b+cx)}+(b^2-4ac)\,x^2\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)\right)}{4a^{3/2}x^3\sqrt{a+x(b+cx)}}$$

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]
```

```
[Out] -1/4*(Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(b + c*x)
] + (b^2 - 4*a*c)*x^2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])
)/(a^(3/2)*x^3*Sqrt[a + x*(b + c*x)])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$-\frac{\sqrt{c\,x^2+bx+a}\,(bx+2a)}{4x^2a}+\frac{(4ac-b^2)\left(\ln(2)-\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)\right)}{8a^{\frac{3}{2}}}$
risch	$-\frac{(bx+2a)\sqrt{x^2(c\,x^2+bx+a)}}{4x^3a}-\frac{(4ac-b^2)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)\sqrt{x^2(c\,x^2+bx+a)}}{8a^{\frac{3}{2}}\,x\sqrt{c\,x^2+bx+a}}$
default	$-\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\left(4c\,a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)x^2+2c\sqrt{c\,x^2+bx+a}\,b\,x^3-4c\sqrt{c\,x^2+bx+a}\,a\,x^2-\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{}}{}\right.\right.}{8x^3\sqrt{c\,x^2+bx+a}\,a^2}$

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(c*x^2+b*x+a)^(1/2)*(b*x+2*a)/x^2/a+1/8*(4*a*c-b^2)*(ln(2)-ln((2*a+b*x
+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))/a^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.98

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^4} dx$$
$$= \left[-\frac{(b^2 - 4\,ac)\sqrt{a}x^3 \log\left(-\frac{8\,abx^2 + (b^2 + 4\,ac)x^3 + 8\,a^2x - 4\,\sqrt{cx^4 + bx^3 + ax^2}(bx + 2\,a)\sqrt{a}}{x^3}\right) + 4\,\sqrt{cx^4 + bx^3 + ax^2}(abx + 2\,a^2)}{16\,a^2x^3}, -\frac{(b^2 - 4\,ac)\sqrt{-a}x^3 \arctan\left(\frac{\sqrt{cx^4 + bx^3 + ax^2}(bx + 2\,a)\sqrt{-a}}{2\,(acx^3 + abx^2 + a^2x)}\right) + 2\,\sqrt{cx^4 + bx^3 + ax^2}(abx + 2\,a^2)}{8\,a^2x^3} \right]$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/16*((b^2 - 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a
^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x
^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3), -1/8*((b^2 - 4*a*c)*sqrt(-a
)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3
+ a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x
^3)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^4} dx = \int \frac{\sqrt{x^2 (a + bx + cx^2)}}{x^4} dx$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^4} dx = \int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^4} dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument Va
lue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^4}\,dx$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^4,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^4, x)
```

3.36 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 155

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5} \, dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3} + \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2} - \frac{b(b^2-4ac)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{5/2}}$$

```
[Out] -1/16*b*(-4*a*c+b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1
/2))/a^(5/2)-1/3*(c*x^4+b*x^3+a*x^2)^(1/2)/x^4-1/12*b*(c*x^4+b*x^3+a*x^2)^(
1/2)/a/x^3+1/24*(-8*a*c+3*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/x^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1934, 1965, 12, 1918, 212}

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5} \, dx = -\frac{b(b^2-4ac)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{5/2}} + \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3} - \frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5,x]
```

```
[Out] -1/3*Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4 - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(12*
a*x^3) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*a^2*x^2) - (b*(b
^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])
])/(16*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4} + \frac{1}{6}\int \frac{b+2cx}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3} - \frac{\int \frac{\frac{1}{2}(3b^2-8ac)+bcx}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{12a}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3} + \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2} + \frac{\int \frac{3b(b^2-4ac)}{4\sqrt{ax^2+bx^3+cx^4}}\,dx}{12a^2}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3} + \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2} + \frac{(b(b^2-4ac))\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{16a^2}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3} + \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2} - \frac{(b(b^2-4ac))\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^2}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3} + \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2} - \frac{b(b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx$$
$$= \frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{a}\sqrt{a+x(b+cx)}(-8a^2+3b^2x^2-2ax(b+4cx))+3b(b^2-4ac)\,x^3\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{}}{}\right.\right.}{24a^{5/2}x^4\sqrt{a+x(b+cx)}}$$

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(-8*a^2 + 3*b^2
*x^2 - 2*a*x*(b + 4*c*x)) + 3*b*(b^2 - 4*a*c)*x^3*ArcTanh[(Sqrt[c]*x - Sqrt
[a + x*(b + c*x)])/Sqrt[a]]))/(24*a^(5/2)*x^4*Sqrt[a + x*(b + c*x)])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{b\,x^3\left(ac-\frac{b^2}{4}\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)+\left(-\frac{x(4cx+b)a^{\frac{3}{2}}}{3}+\frac{\sqrt{a}\,b^2x^2}{2}-\frac{4a^{\frac{5}{2}}}{3}\right)\sqrt{c\,x^2+bx+a}-\ln(2)x^3b\left(ac-\frac{b^2}{4}\right)}{4a^{\frac{5}{2}}x^3}$
risch	$-\frac{(8ac\,x^2-3b^2x^2+2abx+8a^2)\sqrt{x^2(c\,x^2+bx+a)}}{24x^4a^2}+\frac{(4ac-b^2)\,b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)\sqrt{x^2(c\,x^2+bx+a)}}{16a^{\frac{5}{2}}\,x\sqrt{c\,x^2+bx+a}}$
default	$\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\left(12c\,a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)b\,x^3+6c\sqrt{c\,x^2+bx+a}\,b^2x^4-12c\sqrt{c\,x^2+bx+a}\,ab\,x^3-3\sqrt{a}\,\ln\left(\frac{2a+bx\cdots}{}\right.\right.}{48x^4\sqrt{c\,x^2+bx\cdots}}$

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x^3*(a*c-1/4*b^2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/
2))+(-1/3*x*(4*c*x+b)*a^(3/2)+1/2*a^(1/2)*b^2*x^2-4/3*a^(5/2))*(c*x^2+b*x+a
)^(1/2)-ln(2)*x^3*b*(a*c-1/4*b^2))/a^(5/2)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx$$
$$=\left[-\frac{3\,(b^3-4\,abc)\sqrt{a}x^4\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)+4\,\sqrt{cx^4+bx^3+ax^2}(2\,a^2bx\cdots}{96\,a^3x^4}\right.$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^4*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 +
 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt
(c*x^4 + b*x^3 + a*x^2)*(2*a^2*b*x + 8*a^3 - (3*a*b^2 - 8*a^2*c)*x^2))/(a^3
*x^4), 1/48*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*sqrt(c*x^4 +
b*x^3 + a*x^2)*(2*a^2*b*x + 8*a^3 - (3*a*b^2 - 8*a^2*c)*x^2))/(a^3*x^4)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx=\int \frac{\sqrt{x^2\left(a+bx+cx^2\right)}}{x^5}\,dx$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**5,x)
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx=\int \frac{\sqrt{cx^4+bx^3+ax^2}}{x^5}\,dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument Va
lue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^5}\,dx$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^5,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^5, x)
```

3.37 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 205

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6} \, dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3} - \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192a^3x^2} + \frac{(b^2-4ac)(5b^2-4ac)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{7/2}}$$

```
[Out] 1/128*(-4*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*
x^3+a*x^2)^(1/2))/a^(7/2)-1/4*(c*x^4+b*x^3+a*x^2)^(1/2)/x^5-1/24*b*(c*x^4+b
*x^3+a*x^2)^(1/2)/a/x^4+1/96*(-12*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/
x^3-1/192*b*(-52*a*c+15*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/x^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used

= {1934, 1965, 12, 1918, 212}

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx = \frac{(b^2-4ac)\,(5b^2-4ac)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{7/2}} - \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192a^3x^2} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4} - \frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]
```

```
[Out] -1/4*Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5 - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*
a*x^4) + ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(96*a^2*x^3) - (b*(
15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(192*a^3*x^2) + ((b^2 - 4*a*c
)*(5*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c
*x^4])])/(128*a^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}+\frac{1}{8}\int \frac{b+2cx}{x^3\sqrt{ax^2+bx^3+cx^4}}\,dx \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4}-\frac{\int \frac{\frac{1}{2}(5b^2-12ac)+2bcx}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx}{24a} \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4} \\
&\quad +\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}+\frac{\int \frac{\frac{1}{4}b(15b^2-52ac)+\frac{1}{2}c(5b^2-12ac)x}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{48a^2} \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4}+\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3} \\
&\quad -\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192a^3x^2}-\frac{\int \frac{3(b^2-4ac)(5b^2-4ac)}{8\sqrt{ax^2+bx^3+cx^4}}\,dx}{48a^3} \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4}+\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3} \\
&\quad -\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192a^3x^2}-\frac{((b^2-4ac)(5b^2-4ac))\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{128a^3} \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4} \\
&\quad +\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192a^3x^2} \\
&\quad +\frac{((b^2-4ac)(5b^2-4ac))\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{64a^3} \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4}+\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3} \\
&\quad -\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192a^3x^2}+\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx = -\frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{a}\sqrt{a+x(b+cx)}(48a^3+15b^3x^3+8a^2x(b+3cx)-2abx^2(5b+26cx))+3(5b^4-\right.}{192a^{7/2}x^5\sqrt{a+x(b+cx)}}$$

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]
```

```
[Out] -1/192*(Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(48*a^3
+ 15*b^3*x^3 + 8*a^2*x*(b + 3*c*x) - 2*a*b*x^2*(5*b + 26*c*x)) + 3*(5*b^4 -
 24*a*b^2*c + 16*a^2*c^2)*x^4*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/S
qrt[a]]))/(a^(7/2)*x^5*Sqrt[a + x*(b + c*x)])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{x^4\left(ac-\frac{5b^2}{4}\right)\left(ac-\frac{b^2}{4}\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)+\left(\frac{5\left(\frac{26cx}{5}+b\right)x^2b\,a^{\frac{3}{2}}}{12}+\left(-c\,x^2-\frac{1}{3}bx\right)a^{\frac{5}{2}}-\frac{5\sqrt{a}\,b^3x^3}{8}-2a^{\frac{7}{2}}\right)\sqrt{c\,x^2+bx+a}}{8a^{\frac{7}{2}}x^4}$
risch	$-\frac{(-52abc\,x^3+15b^3x^3+24a^2c\,x^2-10a\,b^2x^2+8a^2bx+48a^3)\sqrt{x^2(c\,x^2+bx+a)}}{192x^5a^3}+\frac{(16a^2c^2-24a\,b^2c+5b^4)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x}}{x}\right.}{128a^{\frac{7}{2}}x\sqrt{c\,x^2+bx}}$
default	$\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\left(48c^2a^{\frac{5}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)x^4+24c^2\sqrt{c\,x^2+bx+a}\,ab\,x^5-72c\,a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)b^2\right.}{}$

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^(7/2)*(x^4*(a*c-5/4*b^2)*(a*c-1/4*b^2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b
*x+a)^(1/2))/x/a^(1/2))+(5/12*(26/5*c*x+b)*x^2*b*a^(3/2)+(-c*x^2-1/3*b*x)*a
^(5/2)-5/8*a^(1/2)*b^3*x^3-2*a^(7/2))*(c*x^2+b*x+a)^(1/2)-ln(2)*x^4*(a*c-5/
4*b^2)*(a*c-1/4*b^2))/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx$$
$$= \left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{a}x^5\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)-4\,(8\,a^3bx+48\,a^4}{768\,a^4x^5}\right.$$
$$-\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{-a}x^5\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\,(8\,a^3bx+48\,a^4+(15\,ab^3-52\,a}{384\,a^4x^5}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^5*log(-(8*a*b*x^2 + (
b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt
(a))/x^3) - 4*(8*a^3*b*x + 48*a^4 + (15*a*b^3 - 52*a^2*b*c)*x^3 - 2*(5*a^2*
b^2 - 12*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^5), -1/384*(3*(5*b
^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^5*arctan(1/2*sqrt(c*x^4 + b*x^3 +
a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*(8*a^3*b*x + 4
8*a^4 + (15*a*b^3 - 52*a^2*b*c)*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^2)*sqrt(c*
x^4 + b*x^3 + a*x^2))/(a^4*x^5)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx = \int \frac{\sqrt{x^2\,(a+bx+cx^2)}}{x^6}\,dx$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**6,x)
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx = \int \frac{\sqrt{cx^4+bx^3+ax^2}}{x^6}\,dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^6, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument Va
lue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^6}\,dx$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^6,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^6, x)
```

3.38 $\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 422

$$
\begin{aligned}
\int x\left(ax^2+bx^3\right.&\left.+cx^4\right)^{3/2}\,dx=\frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5}\\
&-\frac{b\left(3465b^6-30660ab^4c+81648a^2b^2c^2-58816a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{573440c^6x}\\
&-\frac{b\left(231b^4-1560ab^2c+2416a^2c^2\right)x\sqrt{ax^2+bx^3+cx^4}}{71680c^4}\\
&+\frac{\left(99b^4-568ab^2c+560a^2c^2\right)x^2\sqrt{ax^2+bx^3+cx^4}}{35840c^3}\\
&-\frac{x^3\left(b(11b^2+68ac)+10c(11b^2-28ac)x\right)\sqrt{ax^2+bx^3+cx^4}}{4480c^2}\\
&+\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}\\
&+\frac{3\left(b^2-4ac\right)^2\left(33b^4-72ab^2c+16a^2c^2\right)x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{13/2}\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}
$$

```
[Out] 1/112*x*(14*c*x+3*b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c+3/32768*(-4*a*c+b^2)^2*(16
*a^2*c^2-72*a*b^2*c+33*b^4)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))*(c*x^2+b*x+a)^(1/2)/c^(13/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/286720*(-672
0*a^3*c^3+18896*a^2*b^2*c^2-8988*a*b^4*c+1155*b^6)*(c*x^4+b*x^3+a*x^2)^(1/2
)/c^5-1/573440*b*(-58816*a^3*c^3+81648*a^2*b^2*c^2-30660*a*b^4*c+3465*b^6)*
(c*x^4+b*x^3+a*x^2)^(1/2)/c^6/x-1/71680*b*(2416*a^2*c^2-1560*a*b^2*c+231*b^
4)*x*(c*x^4+b*x^3+a*x^2)^(1/2)/c^4+1/35840*(560*a^2*c^2-568*a*b^2*c+99*b^4)
*x^2*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/4480*x^3*(b*(68*a*c+11*b^2)+10*c*(-28*
a*c+11*b^2)*x)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1933, 1959, 1963, 12, 1928, 635, 212}

$$\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx=\frac{3x\left(b^2-4ac\right)^2\left(16a^2c^2-72ab^2c+33b^4\right)\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{13/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax^2+bx^3+cx^4}}{71680c^4}+\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{35840c^3}-\frac{b\left(-58816a^3c^3+81648a^2b^2c^2-30660ab^4c+3465b^6\right)\sqrt{ax^2+bx^3+cx^4}}{573440c^6x}+\frac{\left(-6720a^3c^3+18896a^2b^2c^2-8988ab^4c+1155b^6\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5}-\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{4480c^2}+\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}$$

```
[In] Int[x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] ((1155*b^6 - 8988*a*b^4*c + 18896*a^2*b^2*c^2 - 6720*a^3*c^3)*Sqrt[a*x^2 +
b*x^3 + c*x^4])/(286720*c^5) - (b*(3465*b^6 - 30660*a*b^4*c + 81648*a^2*b^2
*c^2 - 58816*a^3*c^3)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(573440*c^6*x) - (b*(231
*b^4 - 1560*a*b^2*c + 2416*a^2*c^2)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(71680*c
^4) + ((99*b^4 - 568*a*b^2*c + 560*a^2*c^2)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]
)/(35840*c^3) - (x^3*(b*(11*b^2 + 68*a*c) + 10*c*(11*b^2 - 28*a*c)*x)*Sqrt[
a*x^2 + b*x^3 + c*x^4])/(4480*c^2) + (x*(3*b + 14*c*x)*(a*x^2 + b*x^3 + c*x
^4)^(3/2))/(112*c) + (3*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*
x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(32768*c^(13/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1933

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Dist[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), Int[x^(m - (n - 2*q
))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1959

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
```

```
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c} \\
&\quad + \frac{3\int x^2\left(-4ab-\frac{1}{2}(11b^2-28ac)\,x\right)\sqrt{ax^2+bx^3+cx^4}\,dx}{112c} \\
&= -\frac{x^3(b(11b^2+68ac)+10c(11b^2-28ac)\,x)\sqrt{ax^2+bx^3+cx^4}}{4480c^2} \\
&\quad + \frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c} + \frac{\int \frac{x^4\left(2ab(11b^2-52ac)+\frac{1}{4}(99b^4-568ab^2c+560a^2c^2)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2240c^2} \\
&= \frac{(99b^4-568ab^2c+560a^2c^2)\,x^2\sqrt{ax^2+bx^3+cx^4}}{35840c^3} \\
&\quad - \frac{x^3(b(11b^2+68ac)+10c(11b^2-28ac)\,x)\sqrt{ax^2+bx^3+cx^4}}{4480c^2} \\
&\quad + \frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c} \\
&\quad - \frac{\int \frac{x^3\left(\frac{3}{4}a(99b^4-568ab^2c+560a^2c^2)+\frac{3}{8}b(231b^4-1560ab^2c+2416a^2c^2)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8960c^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{b(231b^4-1560ab^2c+2416a^2c^2)\,x\sqrt{ax^2+bx^3+cx^4}}{71680c^4} \\
&\quad + \frac{(99b^4-568ab^2c+560a^2c^2)\,x^2\sqrt{ax^2+bx^3+cx^4}}{35840c^3} \\
&\quad - \frac{x^3(b(11b^2+68ac)+10c(11b^2-28ac)\,x)\sqrt{ax^2+bx^3+cx^4}}{4480c^2} \\
&\quad + \frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c} \\
&\quad + \frac{\int \frac{x^2\left(\frac{3}{4}ab(231b^4-1560ab^2c+2416a^2c^2)+\frac{3}{16}(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{26880c^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{286720c^5} \\
&\quad - \frac{b(231b^4-1560ab^2c+2416a^2c^2)\,x\sqrt{ax^2+bx^3+cx^4}}{71680c^4} \\
&\quad + \frac{(99b^4-568ab^2c+560a^2c^2)\,x^2\sqrt{ax^2+bx^3+cx^4}}{35840c^3} \\
&\quad - \frac{x^3(b(11b^2+68ac)+10c(11b^2-28ac)\,x)\sqrt{ax^2+bx^3+cx^4}}{4480c^2} \\
&\quad + \frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c} \\
&\quad - \frac{\int \frac{x\left(\frac{3}{16}a(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3)+\frac{3}{32}b(3465b^6-30660ab^4c+81648a^2b^2c^2-58816a^3c^3)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{53760c^5}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{286720c^5} \\
&\quad - \frac{b(3465b^6-30660ab^4c+81648a^2b^2c^2-58816a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{573440c^6x} \\
&\quad - \frac{b(231b^4-1560ab^2c+2416a^2c^2)\,x\sqrt{ax^2+bx^3+cx^4}}{71680c^4} \\
&\quad + \frac{(99b^4-568ab^2c+560a^2c^2)\,x^2\sqrt{ax^2+bx^3+cx^4}}{35840c^3} \\
&\quad - \frac{x^3(b(11b^2+68ac)+10c(11b^2-28ac)\,x)\sqrt{ax^2+bx^3+cx^4}}{4480c^2} \\
&\quad + \frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c} + \frac{\int \frac{315(b^2-4ac)^2(33b^4-72ab^2c+16a^2c^2)x}{64\sqrt{ax^2+bx^3+cx^4}}\,dx}{53760c^6}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(1155b^6 - 8988ab^4c + 18896a^2b^2c^2 - 6720a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{286720c^5} \\
&\quad - \frac{b(3465b^6 - 30660ab^4c + 81648a^2b^2c^2 - 58816a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{573440c^6x} \\
&\quad - \frac{b(231b^4 - 1560ab^2c + 2416a^2c^2)\,x\sqrt{ax^2+bx^3+cx^4}}{71680c^4} \\
&\quad + \frac{(99b^4 - 568ab^2c + 560a^2c^2)\,x^2\sqrt{ax^2+bx^3+cx^4}}{35840c^3} \\
&\quad - \frac{x^3(b(11b^2+68ac) + 10c(11b^2-28ac)\,x)\sqrt{ax^2+bx^3+cx^4}}{4480c^2} \\
&\quad + \frac{x(3b+14cx)\,(ax^2+bx^3+cx^4)^{3/2}}{112c} \\
&\quad + \frac{\left(3(b^2-4ac)^2\,(33b^4 - 72ab^2c + 16a^2c^2)\right)\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{32768c^6}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(1155b^6 - 8988ab^4c + 18896a^2b^2c^2 - 6720a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{286720c^5} \\
&\quad - \frac{b(3465b^6 - 30660ab^4c + 81648a^2b^2c^2 - 58816a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{573440c^6x} \\
&\quad - \frac{b(231b^4 - 1560ab^2c + 2416a^2c^2)\,x\sqrt{ax^2+bx^3+cx^4}}{71680c^4} \\
&\quad + \frac{(99b^4 - 568ab^2c + 560a^2c^2)\,x^2\sqrt{ax^2+bx^3+cx^4}}{35840c^3} \\
&\quad - \frac{x^3(b(11b^2+68ac) + 10c(11b^2-28ac)\,x)\sqrt{ax^2+bx^3+cx^4}}{4480c^2} \\
&\quad + \frac{x(3b+14cx)\,(ax^2+bx^3+cx^4)^{3/2}}{112c} \\
&\quad + \frac{\left(3(b^2-4ac)^2\,(33b^4 - 72ab^2c + 16a^2c^2)\,x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{32768c^6\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}
$$

$$= \frac{(1155b^6 - 8988ab^4c + 18896a^2b^2c^2 - 6720a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{286720c^5} - \frac{b(3465b^6 - 30660ab^4c + 81648a^2b^2c^2 - 58816a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{573440c^6x} - \frac{b(231b^4 - 1560ab^2c + 2416a^2c^2)\,x\sqrt{ax^2+bx^3+cx^4}}{71680c^4} + \frac{(99b^4 - 568ab^2c + 560a^2c^2)\,x^2\sqrt{ax^2+bx^3+cx^4}}{35840c^3} - \frac{x^3(b(11b^2+68ac) + 10c(11b^2-28ac)\,x)\sqrt{ax^2+bx^3+cx^4}}{4480c^2} + \frac{x(3b+14cx)\,(ax^2+bx^3+cx^4)^{3/2}}{112c} + \frac{\left(3(b^2-4ac)^2\,(33b^4 - 72ab^2c + 16a^2c^2)\,x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{16384c^6\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{(1155b^6 - 8988ab^4c + 18896a^2b^2c^2 - 6720a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{286720c^5} - \frac{b(3465b^6 - 30660ab^4c + 81648a^2b^2c^2 - 58816a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{573440c^6x} - \frac{b(231b^4 - 1560ab^2c + 2416a^2c^2)\,x\sqrt{ax^2+bx^3+cx^4}}{71680c^4} + \frac{(99b^4 - 568ab^2c + 560a^2c^2)\,x^2\sqrt{ax^2+bx^3+cx^4}}{35840c^3} - \frac{x^3(b(11b^2+68ac) + 10c(11b^2-28ac)\,x)\sqrt{ax^2+bx^3+cx^4}}{4480c^2} + \frac{x(3b+14cx)\,(ax^2+bx^3+cx^4)^{3/2}}{112c} + \frac{3(b^2-4ac)^2\,(33b^4 - 72ab^2c + 16a^2c^2)\,x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{13/2}\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.72

$$\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx = \frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}(-3465b^7 + 2310b^6cx + 84b^5c(365a-22cx^2) + 24b^4c^2\right.}{}$$

```
[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^7 + 2310
*b^6*c*x + 84*b^5*c*(365*a - 22*c*x^2) + 24*b^4*c^2*x*(-749*a + 66*c*x^2) +
```

```
32*b^2*c^3*x*(1181*a^2 - 284*a*c*x^2 + 40*c^2*x^4) - 16*b^3*c^2*(5103*a^2
- 780*a*c*x^2 + 88*c^2*x^4) + 4480*c^4*x*(-3*a^3 + 2*a^2*c*x^2 + 24*a*c^2*x
^4 + 16*c^3*x^6) + 64*b*c^3*(919*a^3 - 302*a^2*c*x^2 + 104*a*c^2*x^4 + 1360
*c^3*x^6)) - 105*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*Log[b +
2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(1146880*c^(13/2)*Sqrt[x^2*(a +
x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.78

method	result
risch	$\frac{(71680c^7x^7+87040b\,c^6x^6+107520a\,c^6x^5+1280b^2c^5x^5+6656ab\,c^5x^4-1408b^3c^4x^4+8960a^2c^5x^3-9088a\,b^2c^4x^3+1584b^4c^3x^3-19328a^2b}{}$
default	$\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(-80640c^{\frac{9}{2}}(c\,x^2+bx+a)^{\frac{3}{2}}a\,b^2x-127680c^{\frac{9}{2}}\sqrt{c\,x^2+bx+a}\,a^2b^2x+85680c^{\frac{7}{2}}\sqrt{c\,x^2+bx+a}\,a\,b^4x+143360x^3(c\,x^2+bx+\right.}{}$

```
[In] int(x*(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/573440*(71680*c^7*x^7+87040*b*c^6*x^6+107520*a*c^6*x^5+1280*b^2*c^5*x^5+6
656*a*b*c^5*x^4-1408*b^3*c^4*x^4+8960*a^2*c^5*x^3-9088*a*b^2*c^4*x^3+1584*b
^4*c^3*x^3-19328*a^2*b*c^4*x^2+12480*a*b^3*c^3*x^2-1848*b^5*c^2*x^2-13440*a
^3*c^4*x+37792*a^2*b^2*c^3*x-17976*a*b^4*c^2*x+2310*b^6*c*x+58816*a^3*b*c^3
-81648*a^2*b^3*c^2+30660*a*b^5*c-3465*b^7)/c^6*(x^2*(c*x^2+b*x+a))^(1/2)/x+
3/32768*(256*a^4*c^4-1280*a^3*b^2*c^3+1120*a^2*b^4*c^2-336*a*b^6*c+33*b^8)/
c^(13/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*(x^2*(c*x^2+b*x+a))^(1
/2)/x/(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.57

$$\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx=\left[\frac{105\,(33\,b^8-336\,ab^6c+1120\,a^2b^4c^2-1280\,a^3b^2c^3+256\,a^4c^4)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx}}{}\right.}{}\right.$$

$$-\frac{105\,(33\,b^8-336\,ab^6c+1120\,a^2b^4c^2-1280\,a^3b^2c^3+256\,a^4c^4)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-2\,(71}{}$$

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2293760*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3
 + 256*a^4*c^4)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(71680*c^8*x^7 + 8
7040*b*c^7*x^6 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a
^3*b*c^4 + 1280*(b^2*c^6 + 84*a*c^7)*x^5 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^
4 + 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^3 - 15
60*a*b^3*c^4 + 2416*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 188
96*a^2*b^2*c^4 - 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^7*x), -1/
1146880*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 +
256*a^4*c^4)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*
sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(71680*c^8*x^7 + 87040*b*c^7*x^6
- 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 + 1280
*(b^2*c^6 + 84*a*c^7)*x^5 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^4 + 16*(99*b^4*
c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^3 - 1560*a*b^3*c^4 +
2416*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4
- 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^7*x)]
```

Sympy [F]

$$\int x(ax^2 + bx^3 + cx^4)^{3/2} \, dx = \int x\left(x^2(a + bx + cx^2)\right)^{\frac{3}{2}} \, dx$$

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2)**(3/2),x)
[Out] Integral(x*(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int x(ax^2 + bx^3 + cx^4)^{3/2} \, dx = \int (cx^4 + bx^3 + ax^2)^{\frac{3}{2}} x \, dx$$

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)*x, x)
```

Giac [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.21

$$\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx=\frac{1}{573440}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\left(4\left(14\,cx\mathrm{sgn}(x)+17\,b\mathrm{sgn}(x)\right)x+\frac{b^2c^6\mathrm{sgn}(x)+84\,ac^7}{c^7}\right.\right.\right.\right.\right.\right.$$
$$-\frac{3\left(33\,b^8\mathrm{sgn}(x)-336\,ab^6c\mathrm{sgn}(x)+1120\,a^2b^4c^2\mathrm{sgn}(x)-1280\,a^3b^2c^3\mathrm{sgn}(x)+256\,a^4c^4\mathrm{sgn}(x)\right)\log\left(\left|2\left(\sqrt{c}x-\right.\right.\right.}{32768\,c^{\frac{13}{2}}}$$
$$+\frac{\left(3465\,b^8\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-35280\,ab^6c\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+117600\,a^2b^4c^2\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-134400\right.}{}$$

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/573440*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*(14*c*x*sgn(x) + 17*b*sgn
(x))*x + (b^2*c^6*sgn(x) + 84*a*c^7*sgn(x))/c^7)*x - (11*b^3*c^5*sgn(x) - 5
2*a*b*c^6*sgn(x))/c^7)*x + (99*b^4*c^4*sgn(x) - 568*a*b^2*c^5*sgn(x) + 560*
a^2*c^6*sgn(x))/c^7)*x - (231*b^5*c^3*sgn(x) - 1560*a*b^3*c^4*sgn(x) + 2416
*a^2*b*c^5*sgn(x))/c^7)*x + (1155*b^6*c^2*sgn(x) - 8988*a*b^4*c^3*sgn(x) +
18896*a^2*b^2*c^4*sgn(x) - 6720*a^3*c^5*sgn(x))/c^7)*x - (3465*b^7*c*sgn(x)
 - 30660*a*b^5*c^2*sgn(x) + 81648*a^2*b^3*c^3*sgn(x) - 58816*a^3*b*c^4*sgn(
x))/c^7) - 3/32768*(33*b^8*sgn(x) - 336*a*b^6*c*sgn(x) + 1120*a^2*b^4*c^2*s
gn(x) - 1280*a^3*b^2*c^3*sgn(x) + 256*a^4*c^4*sgn(x))*log(abs(2*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(13/2) + 1/1146880*(3465*b^8*log(a
bs(b - 2*sqrt(a)*sqrt(c))) - 35280*a*b^6*c*log(abs(b - 2*sqrt(a)*sqrt(c)))
+ 117600*a^2*b^4*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) - 134400*a^3*b^2*c^3*l
og(abs(b - 2*sqrt(a)*sqrt(c))) + 26880*a^4*c^4*log(abs(b - 2*sqrt(a)*sqrt(c
))) + 6930*sqrt(a)*b^7*sqrt(c) - 61320*a^(3/2)*b^5*c^(3/2) + 163296*a^(5/2)
*b^3*c^(5/2) - 117632*a^(7/2)*b*c^(7/2))*sgn(x)/c^(13/2)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx=\int x\,\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}\,dx$$

```
[In] int(x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] int(x*(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.39 $\int (ax^2+bx^3+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 364

$$\int (ax^2+bx^3+cx^4)^{3/2}\,dx = -\frac{b(105b^4-728ab^2c+1168a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{17920c^4} + \frac{(315b^6-2520ab^4c+5488a^2b^2c^2-2048a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{35840c^5x} + \frac{(7b^2-32ac)(3b^2-4ac)x\sqrt{ax^2+bx^3+cx^4}}{4480c^3} - \frac{b(9b^2-44ac)x^2\sqrt{ax^2+bx^3+cx^4}}{2240c^2} + \frac{x^3(b^2+24ac+10bcx)\sqrt{ax^2+bx^3+cx^4}}{280c} + \frac{1}{7}x(ax^2+bx^3+cx^4)^{3/2} - \frac{3b(b^2-4ac)^2(3b^2-4ac)x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2048c^{11/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] 1/7*x*(c*x^4+b*x^3+a*x^2)^(3/2)-3/2048*b*(-4*a*c+b^2)^2*(-4*a*c+3*b^2)*x*ar
ctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(11/
2)/(c*x^4+b*x^3+a*x^2)^(1/2)-1/17920*b*(1168*a^2*c^2-728*a*b^2*c+105*b^4)*(
c*x^4+b*x^3+a*x^2)^(1/2)/c^4+1/35840*(-2048*a^3*c^3+5488*a^2*b^2*c^2-2520*a
*b^4*c+315*b^6)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^5/x+1/4480*(-32*a*c+7*b^2)*(-4*
a*c+3*b^2)*x*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/2240*b*(-44*a*c+9*b^2)*x^2*(c*
x^4+b*x^3+a*x^2)^(1/2)/c^2+1/280*x^3*(10*b*c*x+24*a*c+b^2)*(c*x^4+b*x^3+a*x
^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1920, 1959, 1963, 12, 1928, 635, 212}

$$\int \left(ax^2+bx^3+cx^4\right)^{3/2} \, dx = -\frac{b(1168a^2c^2-728ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{17920c^4} + \frac{(-2048a^3c^3+5488a^2b^2c^2-2520ab^4c+315b^6)\sqrt{ax^2+bx^3+cx^4}}{35840c^5x} - \frac{3bx(b^2-4ac)^2(3b^2-4ac)\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2048c^{11/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{x(7b^2-32ac)(3b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}{4480c^3} - \frac{bx^2(9b^2-44ac)\sqrt{ax^2+bx^3+cx^4}}{2240c^2} + \frac{x^3(24ac+b^2+10bcx)\sqrt{ax^2+bx^3+cx^4}}{280c} + \frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] -1/17920*(b*(105*b^4 - 728*a*b^2*c + 1168*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x
^4])/c^4 + ((315*b^6 - 2520*a*b^4*c + 5488*a^2*b^2*c^2 - 2048*a^3*c^3)*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(35840*c^5*x) + ((7*b^2 - 32*a*c)*(3*b^2 - 4*a*c)*
x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4480*c^3) - (b*(9*b^2 - 44*a*c)*x^2*Sqrt[a*
x^2 + b*x^3 + c*x^4])/(2240*c^2) + (x^3*(b^2 + 24*a*c + 10*b*c*x)*Sqrt[a*x^
2 + b*x^3 + c*x^4])/(280*c) + (x*(a*x^2 + b*x^3 + c*x^4)^(3/2))/7 - (3*b*(b
^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2048*c^(11/2)*Sqrt[a*x^2 + b*x^3 + c*x^
4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1920

```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Simp[x*((a*x^q + b*x^n + c*x^(2*n - q))^p/(p*(2*n - q) + 1)), x] + Dis
t[(n - q)*(p/(p*(2*n - q) + 1)), Int[x^q*(2*a + b*x^(n - q))*(a*x^q + b*x^n
 + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*
n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0]
&& NeQ[p*(2*n - q) + 1, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1959

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}+\frac{3}{14}\int x^2(2a+bx)\sqrt{ax^2+bx^3+cx^4}\,dx$$

$$=\frac{x^3(b^2+24ac+10bcx)\sqrt{ax^2+bx^3+cx^4}}{280c}+\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}+\frac{\int\frac{x^4\left(-4a(b^2-6ac)-\frac{1}{2}b(9b^2-44ac)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{280c}$$

$$=-\frac{b(9b^2-44ac)x^2\sqrt{ax^2+bx^3+cx^4}}{2240c^2}+\frac{x^3(b^2+24ac+10bcx)\sqrt{ax^2+bx^3+cx^4}}{280c}+\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}-\frac{\int\frac{x^3\left(-\frac{3}{2}ab(9b^2-44ac)-\frac{3}{4}(7b^2-32ac)(3b^2-4ac)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{1120c^2}$$

$$=\frac{(7b^2-32ac)(3b^2-4ac)x\sqrt{ax^2+bx^3+cx^4}}{4480c^3}-\frac{b(9b^2-44ac)x^2\sqrt{ax^2+bx^3+cx^4}}{2240c^2}+\frac{x^3(b^2+24ac+10bcx)\sqrt{ax^2+bx^3+cx^4}}{280c}+\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}+\frac{\int\frac{x^2\left(-\frac{3}{2}a(7b^2-32ac)(3b^2-4ac)-\frac{3}{8}b(105b^4-728ab^2c+1168a^2c^2)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{3360c^3}$$

$$=-\frac{b(105b^4-728ab^2c+1168a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}+\frac{(7b^2-32ac)(3b^2-4ac)x\sqrt{ax^2+bx^3+cx^4}}{4480c^3}-\frac{b(9b^2-44ac)x^2\sqrt{ax^2+bx^3+cx^4}}{2240c^2}+\frac{x^3(b^2+24ac+10bcx)\sqrt{ax^2+bx^3+cx^4}}{280c}+\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}-\frac{\int\frac{x\left(-\frac{3}{8}ab(105b^4-728ab^2c+1168a^2c^2)-\frac{3}{16}(315b^6-2520ab^4c+5488a^2b^2c^2-2048a^3c^3)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{6720c^4}$$

$$=-\frac{b(105b^4-728ab^2c+1168a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}+\frac{(315b^6-2520ab^4c+5488a^2b^2c^2-2048a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{35840c^5x}+\frac{(7b^2-32ac)(3b^2-4ac)x\sqrt{ax^2+bx^3+cx^4}}{4480c^3}-\frac{b(9b^2-44ac)x^2\sqrt{ax^2+bx^3+cx^4}}{2240c^2}+\frac{x^3(b^2+24ac+10bcx)\sqrt{ax^2+bx^3+cx^4}}{280c}+\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}+\frac{\int-\frac{315b(b^2-4ac)^2(3b^2-4ac)x}{32\sqrt{ax^2+bx^3+cx^4}}\,dx}{6720c^5}$$

$$= -\frac{b(105b^4 - 728ab^2c + 1168a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4}$$
$$+ \frac{(315b^6 - 2520ab^4c + 5488a^2b^2c^2 - 2048a^3c^3)\sqrt{ax^2 + bx^3 + cx^4}}{35840c^5x}$$
$$+ \frac{(7b^2 - 32ac)(3b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}}{4480c^3}$$
$$- \frac{b(9b^2 - 44ac)x^2\sqrt{ax^2 + bx^3 + cx^4}}{2240c^2} + \frac{x^3(b^2 + 24ac + 10bcx)\sqrt{ax^2 + bx^3 + cx^4}}{280c}$$
$$+ \frac{1}{7}x\left(ax^2 + bx^3 + cx^4\right)^{3/2} - \frac{\left(3b(b^2 - 4ac)^2(3b^2 - 4ac)\right)\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2048c^5}$$

$$= -\frac{b(105b^4 - 728ab^2c + 1168a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4}$$
$$+ \frac{(315b^6 - 2520ab^4c + 5488a^2b^2c^2 - 2048a^3c^3)\sqrt{ax^2 + bx^3 + cx^4}}{35840c^5x}$$
$$+ \frac{(7b^2 - 32ac)(3b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}}{4480c^3}$$
$$- \frac{b(9b^2 - 44ac)x^2\sqrt{ax^2 + bx^3 + cx^4}}{2240c^2}$$
$$+ \frac{x^3(b^2 + 24ac + 10bcx)\sqrt{ax^2 + bx^3 + cx^4}}{280c} + \frac{1}{7}x\left(ax^2 + bx^3 + cx^4\right)^{3/2}$$
$$- \frac{\left(3b(b^2 - 4ac)^2(3b^2 - 4ac)x\sqrt{a + bx + cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2048c^5\sqrt{ax^2 + bx^3 + cx^4}}$$

$$= -\frac{b(105b^4 - 728ab^2c + 1168a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4}$$
$$+ \frac{(315b^6 - 2520ab^4c + 5488a^2b^2c^2 - 2048a^3c^3)\sqrt{ax^2 + bx^3 + cx^4}}{35840c^5x}$$
$$+ \frac{(7b^2 - 32ac)(3b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}}{4480c^3}$$
$$- \frac{b(9b^2 - 44ac)x^2\sqrt{ax^2 + bx^3 + cx^4}}{2240c^2}$$
$$+ \frac{x^3(b^2 + 24ac + 10bcx)\sqrt{ax^2 + bx^3 + cx^4}}{280c} + \frac{1}{7}x\left(ax^2 + bx^3 + cx^4\right)^{3/2}$$
$$- \frac{\left(3b(b^2 - 4ac)^2(3b^2 - 4ac)x\sqrt{a + bx + cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{1024c^5\sqrt{ax^2 + bx^3 + cx^4}}$$

$$
\begin{aligned}
&= -\frac{b(105b^4 - 728ab^2c + 1168a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} \\
&\quad + \frac{(315b^6 - 2520ab^4c + 5488a^2b^2c^2 - 2048a^3c^3)\sqrt{ax^2 + bx^3 + cx^4}}{35840c^5x} \\
&\quad + \frac{(7b^2 - 32ac)(3b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}}{4480c^3} \\
&\quad - \frac{b(9b^2 - 44ac)x^2\sqrt{ax^2 + bx^3 + cx^4}}{2240c^2} \\
&\quad + \frac{x^3(b^2 + 24ac + 10bcx)\sqrt{ax^2 + bx^3 + cx^4}}{280c} + \frac{1}{7}x\left(ax^2 + bx^3 + cx^4\right)^{3/2} \\
&\quad - \frac{3b(b^2 - 4ac)^2(3b^2 - 4ac)x\sqrt{a + bx + cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2048c^{11/2}\sqrt{ax^2 + bx^3 + cx^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.66

$$
\int \left(ax^2 + bx^3 + cx^4\right)^{3/2} dx = \frac{x\sqrt{a + x(b + cx)}\left(2\sqrt{c}\sqrt{a + x(b + cx)}\left(315b^6 - 210b^5cx + 16b^3c^2x(91a - 9cx^2) + 168b^4c(-15\right.\right.}{}
$$

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(315*b^6 - 210*b^
5*c*x + 16*b^3*c^2*x*(91*a - 9*c*x^2) + 168*b^4*c*(-15*a + c*x^2) + 1024*c^
3*(a + c*x^2)^2*(-2*a + 5*c*x^2) + 16*b^2*c^2*(343*a^2 - 62*a*c*x^2 + 8*c^2
*x^4) + 32*b*c^3*x*(-73*a^2 + 22*a*c*x^2 + 200*c^2*x^4)) + 105*b*(b^2 - 4*a
*c)^2*(3*b^2 - 4*a*c)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(7
1680*c^(11/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{(-5120c^6x^6 - 6400b\,c^5x^5 - 8192a\,c^5x^4 - 128b^2c^4x^4 - 704ab\,c^4x^3 + 144b^3c^3x^3 - 1024a^2c^4x^2 + 992a\,b^2c^3x^2 - 168b^4c^2x^2 + 2336a^2b\,c^3x - 1}{35840c^5x}$
default	$\frac{(c\,x^4 + b\,x^3 + a\,x^2)^{\frac{3}{2}}\left(10240x^2(c\,x^2 + bx + a)^{\frac{5}{2}}c^{\frac{11}{2}} - 7680c^{\frac{9}{2}}(c\,x^2 + bx + a)^{\frac{5}{2}}bx - 4096c^{\frac{9}{2}}(c\,x^2 + bx + a)^{\frac{5}{2}}a + 4480c^{\frac{9}{2}}(c\,x^2 + bx + a)^{\frac{3}{2}}abx + 67\right.}{}$

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35840*(-5120*c^6*x^6-6400*b*c^5*x^5-8192*a*c^5*x^4-128*b^2*c^4*x^4-704*a
*b*c^4*x^3+144*b^3*c^3*x^3-1024*a^2*c^4*x^2+992*a*b^2*c^3*x^2-168*b^4*c^2*x
^2+2336*a^2*b*c^3*x-1456*a*b^3*c^2*x+210*b^5*c*x+2048*a^3*c^3-5488*a^2*b^2*
c^2+2520*a*b^4*c-315*b^6)/c^5*(x^2*(c*x^2+b*x+a))^(1/2)/x+3/2048*b*(64*a^3*
c^3-80*a^2*b^2*c^2+28*a*b^4*c-3*b^6)/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*(x^2*(c*x^2+b*x+a))^(1/2)/x/(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.53

$$\int \left(ax^2+bx^3+cx^4\right)^{3/2}\,dx = \left[-\frac{105\left(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3\right)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b\right.}{x}\right.$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/143360*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(c
)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*(5120*c^7*x^6 + 6400*b*c^6*x^5 + 315*b^6
*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 128*(b^2*c^5 + 64*a
*c^6)*x^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^3 + 8*(21*b^4*c^3 - 124*a*b^2*c^4
 + 128*a^2*c^5)*x^2 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x)*s
qrt(c*x^4 + b*x^3 + a*x^2))/(c^6*x), 1/71680*(105*(3*b^7 - 28*a*b^5*c + 80*
a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^
2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(5120*c^7*x^6 + 64
00*b*c^6*x^5 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4
 + 128*(b^2*c^5 + 64*a*c^6)*x^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^3 + 8*(21*b
^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*x^2 - 2*(105*b^5*c^2 - 728*a*b^3*c^3
+ 1168*a^2*b*c^4)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^6*x)]
```

Sympy [F]

$$\int \left(ax^2+bx^3+cx^4\right)^{3/2} dx = \int \left(ax^2+bx^3+cx^4\right)^{\frac{3}{2}} dx$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2),x)
[Out] Integral((a*x**2 + b*x**3 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \left(ax^2+bx^3+cx^4\right)^{3/2} dx = \int \left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}} dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.15

$$\int \left(ax^2+bx^3 +cx^4\right)^{3/2} dx = \frac{1}{35840}\sqrt{cx^2+bx+a}\Bigg(2\Bigg(4\Bigg(2\Bigg(8\Bigg(10\left(4\,cx\mathrm{sgn}(x)+5\,b\mathrm{sgn}(x)\right)x+\frac{b^2c^5\mathrm{sgn}(x)+64\,ac^6\mathrm{sgn}(x)}{c^6} + \frac{3\left(3\,b^7\mathrm{sgn}(x)-28\,ab^5c\mathrm{sgn}(x)+80\,a^2b^3c^2\mathrm{sgn}(x)-64\,a^3bc^3\mathrm{sgn}(x)\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2048\,c^{\frac{11}{2}}} - \frac{\left(315\,b^7\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-2940\,ab^5c\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+8400\,a^2b^3c^2\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-6720\,a^3bc^3\,\mathrm{l}}{71680\,c^{\frac{11}{2}}}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] 1/35840*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*c*x*sgn(x) + 5*b*sgn(x))*x
 + (b^2*c^5*sgn(x) + 64*a*c^6*sgn(x))/c^6)*x - (9*b^3*c^4*sgn(x) - 44*a*b*c
^5*sgn(x))/c^6)*x + (21*b^4*c^3*sgn(x) - 124*a*b^2*c^4*sgn(x) + 128*a^2*c^5
*sgn(x))/c^6)*x - (105*b^5*c^2*sgn(x) - 728*a*b^3*c^3*sgn(x) + 1168*a^2*b*c
^4*sgn(x))/c^6)*x + (315*b^6*c*sgn(x) - 2520*a*b^4*c^2*sgn(x) + 5488*a^2*b^
2*c^3*sgn(x) - 2048*a^3*c^4*sgn(x))/c^6) + 3/2048*(3*b^7*sgn(x) - 28*a*b^5*
c*sgn(x) + 80*a^2*b^3*c^2*sgn(x) - 64*a^3*b*c^3*sgn(x))*log(abs(2*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(11/2) - 1/71680*(315*b^7*log(ab
```

```
s(b - 2*sqrt(a)*sqrt(c))) - 2940*a*b^5*c*log(abs(b - 2*sqrt(a)*sqrt(c))) +
8400*a^2*b^3*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) - 6720*a^3*b*c^3*log(abs(b
 - 2*sqrt(a)*sqrt(c))) + 630*sqrt(a)*b^6*sqrt(c) - 5040*a^(3/2)*b^4*c^(3/2)
 + 10976*a^(5/2)*b^2*c^(5/2) - 4096*a^(7/2)*c^(7/2))*sgn(x)/c^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int (ax^2 + bx^3 + cx^4)^{3/2} \, dx = \int (c\,x^4 + b\,x^3 + a\,x^2)^{3/2} \, dx$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2),x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.40 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 288

$$\begin{aligned}\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx &= \frac{(35b^4-216ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} \\ &- \frac{b(105b^4-760ab^2c+1296a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{7680c^4x} \\ &- \frac{x(b(7b^2+12ac)+6c(7b^2-20ac)x)\sqrt{ax^2+bx^3+cx^4}}{960c^2} \\ &+ \frac{(3b+10cx)(ax^2+bx^3+cx^4)^{3/2}}{60cx} \\ &+ \frac{(b^2-4ac)^2(7b^2-4ac)x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}\sqrt{ax^2+bx^3+cx^4}}\end{aligned}$$

```
[Out] 1/60*(10*c*x+3*b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c/x+1/1024*(-4*a*c+b^2)^2*(-4*a
*c+7*b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a
)^(1/2)/c^(9/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/3840*(240*a^2*c^2-216*a*b^2*c+3
5*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/7680*b*(1296*a^2*c^2-760*a*b^2*c+105
*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^4/x-1/960*x*(b*(12*a*c+7*b^2)+6*c*(-20*a*
c+7*b^2)*x)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1933, 1948, 1963, 12, 1928, 635, 212}

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x} \, dx = -\frac{b(1296a^2c^2-760ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{7680c^4x} + \frac{(240a^2c^2-216ab^2c+35b^4)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} + \frac{x(7b^2-4ac)(b^2-4ac)^2\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}\sqrt{ax^2+bx^3+cx^4}} - \frac{x(6cx(7b^2-20ac)+b(12ac+7b^2))\sqrt{ax^2+bx^3+cx^4}}{960c^2} + \frac{(3b+10cx)(ax^2+bx^3+cx^4)^{3/2}}{60cx}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]
```

```
[Out] ((35*b^4 - 216*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3840*c^
3) - (b*(105*b^4 - 760*a*b^2*c + 1296*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
/(7680*c^4*x) - (x*(b*(7*b^2 + 12*a*c) + 6*c*(7*b^2 - 20*a*c)*x)*Sqrt[a*x^2
 + b*x^3 + c*x^4])/(960*c^2) + ((3*b + 10*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2
))/(60*c*x) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2)*Sqrt[a*x^
2 + b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1933

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Dist[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), Int[x^(m - (n - 2*q
))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1948

```
Int[((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_)*((A_) + (
B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x*(b*B*(n - q)*p + A*c*(p*q + (n - q)*(
2*p + 1) + 1) + B*c*(p*(2*n - q) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/(c*(p*(2*n - q) + 1)*(p*q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n
 - q)*(p/(c*(p*(2*n - q) + 1)*(p*q + (n - q)*(2*p + 1) + 1))), Int[x^q*(2*a
*A*c*(p*q + (n - q)*(2*p + 1) + 1) - a*b*B*(p*q + 1) + (2*a*B*c*(p*(2*n - q
) + 1) + A*b*c*(p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(p*q + (n - q)*p + 1))
*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b,
 c, A, B, n, q}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] &&
 NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p*(2*n - q) + 1, 0] && NeQ[p*q + (
n - q)*(2*p + 1) + 1, 0]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
```

```
) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx} \\
&\quad + \frac{\int \left(-2ab+\frac{1}{2}\left(-7b^2+20ac\right)x\right)\sqrt{ax^2+bx^3+cx^4}\,dx}{20c} \\
&= -\frac{x\left(b\left(7b^2+12ac\right)+6c\left(7b^2-20ac\right)x\right)\sqrt{ax^2+bx^3+cx^4}}{960c^2} \\
&\quad + \frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx} \\
&\quad + \frac{\int \frac{x^2\left(-16a^2bc-ab\left(-7b^2+20ac\right)+\left(-8ab^2c-\frac{5}{4}b^2\left(-7b^2+20ac\right)+3ac\left(-7b^2+20ac\right)\right)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{480c^2} \\
&= \frac{\left(35b^4-216ab^2c+240a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} \\
&\quad - \frac{x\left(b\left(7b^2+12ac\right)+6c\left(7b^2-20ac\right)x\right)\sqrt{ax^2+bx^3+cx^4}}{960c^2} \\
&\quad + \frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx} \\
&\quad - \frac{\int \frac{x\left(\frac{1}{4}a\left(35b^4-216ab^2c+240a^2c^2\right)+\frac{1}{8}b\left(105b^4-760ab^2c+1296a^2c^2\right)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{960c^3} \\
&= \frac{\left(35b^4-216ab^2c+240a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} \\
&\quad - \frac{b\left(105b^4-760ab^2c+1296a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{7680c^4x} \\
&\quad - \frac{x\left(b\left(7b^2+12ac\right)+6c\left(7b^2-20ac\right)x\right)\sqrt{ax^2+bx^3+cx^4}}{960c^2} \\
&\quad + \frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx} + \frac{\int \frac{15\left(b^2-4ac\right)^2\left(7b^2-4ac\right)x}{16\sqrt{ax^2+bx^3+cx^4}}\,dx}{960c^4} \\
&= \frac{\left(35b^4-216ab^2c+240a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} \\
&\quad - \frac{b\left(105b^4-760ab^2c+1296a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{7680c^4x} \\
&\quad - \frac{x\left(b\left(7b^2+12ac\right)+6c\left(7b^2-20ac\right)x\right)\sqrt{ax^2+bx^3+cx^4}}{960c^2} \\
&\quad + \frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx} + \frac{\left(\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\right)\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{1024c^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(35b^4-216ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} \\
&\quad - \frac{b(105b^4-760ab^2c+1296a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{7680c^4x} \\
&\quad - \frac{x(b(7b^2+12ac)+6c(7b^2-20ac)x)\sqrt{ax^2+bx^3+cx^4}}{960c^2} \\
&\quad + \frac{(3b+10cx)(ax^2+bx^3+cx^4)^{3/2}}{60cx} \\
&\quad + \frac{\left((b^2-4ac)^2(7b^2-4ac)x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{1024c^4\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(35b^4-216ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} \\
&\quad - \frac{b(105b^4-760ab^2c+1296a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{7680c^4x} \\
&\quad - \frac{x(b(7b^2+12ac)+6c(7b^2-20ac)x)\sqrt{ax^2+bx^3+cx^4}}{960c^2} \\
&\quad + \frac{(3b+10cx)(ax^2+bx^3+cx^4)^{3/2}}{60cx} \\
&\quad + \frac{\left((b^2-4ac)^2(7b^2-4ac)x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{512c^4\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(35b^4-216ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} \\
&\quad - \frac{b(105b^4-760ab^2c+1296a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{7680c^4x} \\
&\quad - \frac{x(b(7b^2+12ac)+6c(7b^2-20ac)x)\sqrt{ax^2+bx^3+cx^4}}{960c^2} \\
&\quad + \frac{(3b+10cx)(ax^2+bx^3+cx^4)^{3/2}}{60cx} \\
&\quad + \frac{(b^2-4ac)^2(7b^2-4ac)x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.73

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx = \frac{x\sqrt{a+x(b+cx)}\Big(2\sqrt{c}\sqrt{a+x(b+cx)}(-105b^5+70b^4cx+8b^3c(95a-7cx^2)\ldots}{\ldots}$$

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^5 + 70*b^
4*c*x + 8*b^3*c*(95*a - 7*c*x^2) + 48*b^2*c^2*x*(-9*a + c*x^2) + 160*c^3*x*
(3*a^2 + 14*a*c*x^2 + 8*c^2*x^4) + 16*b*c^2*(-81*a^2 + 18*a*c*x^2 + 104*c^2
*x^4)) - 15*(b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[
a + x*(b + c*x)]]))/(15360*c^(9/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{(-1280c^5x^5-1664b\,c^4x^4-2240a\,c^4x^3-48b^2c^3x^3-288ab\,c^3x^2+56c^2x^2b^3-480a^2c^3x+432a\,b^2c^2x-70xc\,b^4+1296a^2b\,c^2-760a\,b^3c+\ldots}{7680c^4x}$
default	$\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\Big(2560x(c\,x^2+bx+a)^{\frac{5}{2}}c^{\frac{9}{2}}-640c^{\frac{9}{2}}(c\,x^2+bx+a)^{\frac{3}{2}}ax-960c^{\frac{9}{2}}\sqrt{c\,x^2+bx+a}\,a^2x-1792c^{\frac{7}{2}}(c\,x^2+bx+a)^{\frac{5}{2}}b+1120c^{\frac{7}{2}}\ldots}{\ldots}$

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7680*(-1280*c^5*x^5-1664*b*c^4*x^4-2240*a*c^4*x^3-48*b^2*c^3*x^3-288*a*b
*c^3*x^2+56*b^3*c^2*x^2-480*a^2*c^3*x+432*a*b^2*c^2*x-70*b^4*c*x+1296*a^2*b
*c^2-760*a*b^3*c+105*b^5)/c^4*(x^2*(c*x^2+b*x+a))^(1/2)/x-1/1024*(64*a^3*c^
3-144*a^2*b^2*c^2+60*a*b^4*c-7*b^6)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))*(x^2*(c*x^2+b*x+a))^(1/2)/x/(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.65

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x} \, dx = \left[-\frac{15\,(7\,b^6 - 60\,ab^4c + 144\,a^2b^2c^2 - 64\,a^3c^3)\sqrt{c}x \log\left(-\frac{8\,c^2x^3 + 8\,bcx^2 - 4\,\sqrt{cx^4 + bx^3 + ax^2}}{x}\right.}{} \right.$$

$$-\frac{15\,(7\,b^6 - 60\,ab^4c + 144\,a^2b^2c^2 - 64\,a^3c^3)\sqrt{-c}x \arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right) - 2\,(1280\,c^6x^5 + 1664\,bc^5x}{}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*x
*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sq
rt(c) + (b^2 + 4*a*c)*x)/x) - 4*(1280*c^6*x^5 + 1664*b*c^5*x^4 - 105*b^5*c
+ 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 16*(3*b^2*c^4 + 140*a*c^5)*x^3 - 8*(7*b^
3*c^3 - 36*a*b*c^4)*x^2 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x)*s
qrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x), -1/15360*(15*(7*b^6 - 60*a*b^4*c + 144
*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2
)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(1280*c^6*x^5 + 166
4*b*c^5*x^4 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 16*(3*b^2*c^4 +
140*a*c^5)*x^3 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^2 + 2*(35*b^4*c^2 - 216*a*b^2
*c^3 + 240*a^2*c^4)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x)]
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x} \, dx = \int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x} \, dx$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x} \, dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x} \, dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x, x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.23

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx = \frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\,(10\,cx\text{sgn}(x)+13\,b\text{sgn}(x))x+\frac{3\,b^2c^4\text{sgn}(x)}{\cdots}\right.\right.\right.\right.$$
$$-\frac{(7\,b^6\text{sgn}(x)-60\,ab^4c\text{sgn}(x)+144\,a^2b^2c^2\text{sgn}(x)-64\,a^3c^3\text{sgn}(x))\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{1024\,c^{\frac{9}{2}}}$$
$$+\frac{\left(105\,b^6\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)-900\,ab^4c\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+2160\,a^2b^2c^2\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)-960\,a^3c^3\log\cdots\right.}{15360\,c^{\frac{9}{2}}}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*x*sgn(x) + 13*b*sgn(x))*x +
(3*b^2*c^4*sgn(x) + 140*a*c^5*sgn(x))/c^5)*x - (7*b^3*c^3*sgn(x) - 36*a*b*c
^4*sgn(x))/c^5)*x + (35*b^4*c^2*sgn(x) - 216*a*b^2*c^3*sgn(x) + 240*a^2*c^4
*sgn(x))/c^5)*x - (105*b^5*c*sgn(x) - 760*a*b^3*c^2*sgn(x) + 1296*a^2*b*c^3
*sgn(x))/c^5) - 1/1024*(7*b^6*sgn(x) - 60*a*b^4*c*sgn(x) + 144*a^2*b^2*c^2*
sgn(x) - 64*a^3*c^3*sgn(x))*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*s
qrt(c) + b))/c^(9/2) + 1/15360*(105*b^6*log(abs(b - 2*sqrt(a)*sqrt(c))) - 9
00*a*b^4*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 2160*a^2*b^2*c^2*log(abs(b - 2
*sqrt(a)*sqrt(c))) - 960*a^3*c^3*log(abs(b - 2*sqrt(a)*sqrt(c))) + 210*sqrt
(a)*b^5*sqrt(c) - 1520*a^(3/2)*b^3*c^(3/2) + 2592*a^(5/2)*b*c^(5/2))*sgn(x)
/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx = \int \frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}{x}\,dx$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x, x)
```

3.41 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 198

$$\begin{aligned}\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx &= \frac{3b(b^2-4ac)\,(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x}\\ &-\frac{b(b+2cx)\,(ax^2+bx^3+cx^4)^{3/2}}{16c^2x^3}+\frac{(ax^2+bx^3+cx^4)^{5/2}}{5cx^5}\\ &-\frac{3b(b^2-4ac)^2\,x\sqrt{a+bx+cx^2}\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}\sqrt{ax^2+bx^3+cx^4}}\end{aligned}$$

```
[Out] -1/16*b*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c^2/x^3+1/5*(c*x^4+b*x^3+a*x^2)
^(5/2)/c/x^5-3/256*b*(-4*a*c+b^2)^2*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+
b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(7/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+3/128*b
*(-4*a*c+b^2)*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3/x
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1931, 1932, 1928, 635, 212}

$$\begin{aligned}\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx &= -\frac{3bx(b^2-4ac)^2\,\sqrt{a+bx+cx^2}\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}\sqrt{ax^2+bx^3+cx^4}}\\ &+\frac{3b(b^2-4ac)\,(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x}\\ &-\frac{b(b+2cx)\,(ax^2+bx^3+cx^4)^{3/2}}{16c^2x^3}+\frac{(ax^2+bx^3+cx^4)^{5/2}}{5cx^5}\end{aligned}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(128*c^3*x) - (
b*(b + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(16*c^2*x^3) + (a*x^2 + b*x^3
+ c*x^4)^(5/2)/(5*c*x^5) - (3*b*(b^2 - 4*a*c)^2*x*Sqrt[a + b*x + c*x^2]*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(7/2)*Sqrt[a*x^
2 + b*x^3 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1931

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n)*((a*x^(n - 1) + b*x^n + c*x^(n + 1))^(p + 1)
/(2*c*(p + 1))), x] - Dist[b/(2*c), Int[x^(m - 1)*(a*x^(n - 1) + b*x^n + c*
x^(n + 1))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n -
q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p,
 q] && EqQ[m + p*(n - 1) - 1, 0]
```

Rule 1932

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1))), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rubi steps

$$\text{integral} = \frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5} - \frac{b\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^3}\,dx}{2c}$$

$$= -\frac{b(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{16c^2x^3} + \frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5} + \frac{(3b(b^2-4ac))\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx}{32c^2}$$

$$= \frac{3b(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} - \frac{b(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{16c^2x^3} + \frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5} - \frac{\left(3b(b^2-4ac)^2\right)\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{256c^3}$$

$$= \frac{3b(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} - \frac{b(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{16c^2x^3} + \frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5} - \frac{\left(3b(b^2-4ac)^2\,x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{256c^3\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{3b(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} - \frac{b(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{16c^2x^3} + \frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5} - \frac{\left(3b(b^2-4ac)^2\,x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{128c^3\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{3b(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} - \frac{b(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{16c^2x^3} + \frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5} - \frac{3b(b^2-4ac)^2\,x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.81

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^2}\,dx = \frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}\left(15b^4-10b^3cx+128c^2(a+cx^2)^2+4b^2\right.\right.}{1280c^{7}}$$

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^4 - 10*b^3*
c*x + 128*c^2*(a + c*x^2)^2 + 4*b^2*c*(-25*a + 2*c*x^2) + 8*b*c^2*x*(7*a +
22*c*x^2)) + 15*b*(b^2 - 4*a*c)^2*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b +
 c*x)]]))/(1280*c^(7/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{-\frac{15b\left(ac-\frac{b^2}{4}\right)^2\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)}{16}+\left(\left(\frac{1}{16}b^2x^2+\frac{7}{16}abx+a^2\right)c^{\frac{5}{2}}-\frac{25b^2\left(\frac{bx}{10}+a\right)c^{\frac{3}{2}}}{32}+\left(\frac{11}{8}b\,x^3+2a\,x^2\right)c^{\frac{7}{2}}+c^{\frac{9}{2}}x^4+\frac{15\sqrt{c}}{128}\right.}{5c^{\frac{7}{2}}}$
risch	$\frac{(128c^4x^4+176b\,c^3x^3+256a\,c^3x^2+8b^2c^2x^2+56ab\,c^2x-10b^3cx+128a^2c^2-100a\,b^2c+15b^4)\sqrt{x^2(c\,x^2+bx+a)}}{640c^3x}-\frac{3b(16a^2c^2-}{}$
default	$\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(256(c\,x^2+bx+a)^{\frac{5}{2}}c^{\frac{7}{2}}-160c^{\frac{7}{2}}(c\,x^2+bx+a)^{\frac{3}{2}}bx-80c^{\frac{5}{2}}(c\,x^2+bx+a)^{\frac{3}{2}}b^2-240c^{\frac{7}{2}}\sqrt{c\,x^2+bx+a}\,abx+60c\right.}{}$

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/c^(7/2)*(-15/16*b*(a*c-1/4*b^2)^2*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*
x+b)+((1/16*b^2*x^2+7/16*a*b*x+a^2)*c^(5/2)-25/32*b^2*(1/10*b*x+a)*c^(3/2)+
(11/8*b*x^3+2*a*x^2)*c^(7/2)+c^(9/2)*x^4+15/128*c^(1/2)*b^4)*(c*x^2+b*x+a)^
(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.94

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx = \left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4}{x}\right.}{}\right.$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*
b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)
*x)/x) + 4*(128*c^5*x^4 + 176*b*c^4*x^3 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^
2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x)*sqrt(c*x
^4 + b*x^3 + a*x^2))/(c^4*x), 1/1280*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*s
qrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*
x^3 + b*c*x^2 + a*c*x)) + 2*(128*c^5*x^4 + 176*b*c^4*x^3 + 15*b^4*c - 100*a
*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2 - 28*a*b
*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^4*x)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx=\int \frac{(x^2(a+bx+cx^2))^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**2,x)
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx=\int \frac{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.39

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx=\frac{1}{640}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\,cx\text{sgn}(x)+11\,b\text{sgn}(x)\right)x+\frac{b^2c^3\text{sgn}(x)+32\,ac^4\text{s}}{c^4}\right.\right.\right.$$
$$+\frac{3\left(b^5\text{sgn}(x)-8\,ab^3c\text{sgn}(x)+16\,a^2bc^2\text{sgn}(x)\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{256\,c^{\frac{7}{2}}}$$
$$-\frac{\left(15\,b^5\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-120\,ab^3c\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+240\,a^2bc^2\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+30\,\sqrt{a}b^4\sqrt{c}-20\right.}{1280\,c^{\frac{7}{2}}}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")
[Out] 1/640*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*c*x*sgn(x) + 11*b*sgn(x))*x + (b^2*
c^3*sgn(x) + 32*a*c^4*sgn(x))/c^4)*x - (5*b^3*c^2*sgn(x) - 28*a*b*c^3*sgn(x
))/c^4)*x + (15*b^4*c*sgn(x) - 100*a*b^2*c^2*sgn(x) + 128*a^2*c^3*sgn(x))/c
^4) + 3/256*(b^5*sgn(x) - 8*a*b^3*c*sgn(x) + 16*a^2*b*c^2*sgn(x))*log(abs(2
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2) - 1/1280*(15*b^5
*log(abs(b - 2*sqrt(a)*sqrt(c))) - 120*a*b^3*c*log(abs(b - 2*sqrt(a)*sqrt(c
))) + 240*a^2*b*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^4*sqrt(c
) - 200*a^(3/2)*b^2*c^(3/2) + 256*a^(5/2)*c^(5/2))*sgn(x)/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^2}\,dx=\int \frac{{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}}{x^2}\,dx$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2, x)
```

3.42 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 165

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx = -\frac{3(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{8cx^3} + \frac{3(b^2-4ac)^2x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] 1/8*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c/x^3+3/128*(-4*a*c+b^2)^2*x*arctan
h(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(5/2)/(c
*x^4+b*x^3+a*x^2)^(1/2)-3/64*(-4*a*c+b^2)*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/
2)/c^2/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1932, 1928, 635, 212}

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx = \frac{3x(b^2-4ac)^2\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}\sqrt{ax^2+bx^3+cx^4}} - \frac{3(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{8cx^3}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*c^2*x) + ((b
 + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(8*c*x^3) + (3*(b^2 - 4*a*c)^2*x*S
```

```
qrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(128*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1932

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1))), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rubi steps

$$\text{integral} = \frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3} - \frac{\left(3(b^2-4ac)\right)\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx}{16c}$$

$$= -\frac{3(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3} + \frac{\left(3(b^2-4ac)^2\right)\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{128c^2}$$

$$
\begin{aligned}
&= -\frac{3(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3} \\
&\quad + \frac{\left(3(b^2-4ac)^2 x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{128c^2\sqrt{ax^2+bx^3+cx^4}} \\
&= -\frac{3(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3} \\
&\quad + \frac{\left(3(b^2-4ac)^2 x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{64c^2\sqrt{ax^2+bx^3+cx^4}} \\
&= -\frac{3(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3} \\
&\quad + \frac{3(b^2-4ac)^2 x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.81

$$
\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^3}\,dx = \frac{\left(x^2(a+x(b+cx))\right)^{3/2}\left(\frac{\sqrt{c}(b+2cx)\left(-3b^2+8bcx+4c(5a+2cx^2)\right)}{a+x(b+cx)} + \frac{3(b^2-4ac)^2\text{arctanh}\left(\frac{\cdots}{-\sqrt{a}+\cdots}\right.}{(a+x(b+cx))^{3/2}}\right.}{64c^{5/2}x^3}
$$

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] ((x^2*(a + x*(b + c*x)))^(3/2)*((Sqrt[c]*(b + 2*c*x)*(-3*b^2 + 8*b*c*x + 4*
c*(5*a + 2*c*x^2)))/(a + x*(b + c*x)) + (3*(b^2 - 4*a*c)^2*ArcTanh[(Sqrt[c]
*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(a + x*(b + c*x))^(3/2)))/(64*c^(5
/2)*x^3)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$\frac{\frac{3\left(ac-\frac{b^2}{4}\right)^2 \ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)}{8}+\frac{5\sqrt{c\,x^2+bx+a}\,\left(b\left(\frac{bx}{10}+a\right)c^{\frac{3}{2}}+\left(\frac{6}{5}b\,x^2+2ax\right)c^{\frac{5}{2}}-\frac{3\sqrt{c}\,b^3}{20}+\frac{4c^{\frac{7}{2}}x^3}{5}\right)}{16}}{c^{\frac{5}{2}}}$
risch	$\frac{(16c^3x^3+24b\,c^2x^2+40a\,c^2x+2b^2cx+20abc-3b^3)\sqrt{x^2(c\,x^2+bx+a)}}{64c^2x}+\frac{3(16a^2c^2-8a\,b^2c+b^4)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)\sqrt{x}}{128c^{\frac{5}{2}}x\sqrt{c\,x^2+bx+a}}$
default	$\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(32x(c\,x^2+bx+a)^{\frac{3}{2}}c^{\frac{7}{2}}+16c^{\frac{5}{2}}(c\,x^2+bx+a)^{\frac{3}{2}}b+48c^{\frac{7}{2}}\sqrt{c\,x^2+bx+a}\,ax-12c^{\frac{5}{2}}\sqrt{c\,x^2+bx+a}\,b^2x+24c^{\frac{5}{2}}\sqrt{c}\right.}{}$

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8/c^(5/2)*((a*c-1/4*b^2)^2*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)+5/6*
(c*x^2+b*x+a)^(1/2)*(b*(1/10*b*x+a)*c^(3/2)+(6/5*b*x^2+2*a*x)*c^(5/2)-3/20*
c^(1/2)*b^3+4/5*c^(7/2)*x^3))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.94

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx = \left[\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac}{x}\right.}{}\right.$$

$$-\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-2\,(16\,c^4x^3+24\,bc^3x^2-3\,b^3c+20\,abc^2+}{128\,c^3x}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*
x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/
x) + 4*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*
a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x), -1/128*(3*(b^4 - 8*a*b^2*c
+ 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b
^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/
(c^3*x)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx = \int \frac{(x^2(a+bx+cx^2))^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**3,x)
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx = \int \frac{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^3, x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.36

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx = \frac{1}{64}\sqrt{cx^2+bx+a}\left(2\left(4\,(2\,cx\mathrm{sgn}(x)+3\,b\mathrm{sgn}(x))x+\frac{b^2c^2\mathrm{sgn}(x)+20\,ac^3\mathrm{sgn}(x)}{c^3}\right.\right.$$
$$-\frac{3\,(b^4\mathrm{sgn}(x)-8\,ab^2c\mathrm{sgn}(x)+16\,a^2c^2\mathrm{sgn}(x))\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{5}{2}}}$$
$$+\frac{\left(3\,b^4\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-24\,ab^2c\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+48\,a^2c^2\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+6\,\sqrt{a}b^3\sqrt{c}-40\,a^{\frac{3}{2}}bc^{\frac{3}{2}}\right.}{128\,c^{\frac{5}{2}}}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")
[Out] 1/64*sqrt(c*x^2 + b*x + a)*(2*(4*(2*c*x*sgn(x) + 3*b*sgn(x))*x + (b^2*c^2*s
gn(x) + 20*a*c^3*sgn(x))/c^3)*x - (3*b^3*c*sgn(x) - 20*a*b*c^2*sgn(x))/c^3)
 - 3/128*(b^4*sgn(x) - 8*a*b^2*c*sgn(x) + 16*a^2*c^2*sgn(x))*log(abs(2*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2) + 1/128*(3*b^4*log(ab
s(b - 2*sqrt(a)*sqrt(c))) - 24*a*b^2*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 48
*a^2*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^3*sqrt(c) - 40*a^(3/
2)*b*c^(3/2))*sgn(x)/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx=\int \frac{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}{x^3}\,dx$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3, x)
```

3.43 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 227

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} dx = \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} - \frac{a^{3/2}x\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} - \frac{b(b^2-12ac)\,x\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] 1/3*(c*x^4+b*x^3+a*x^2)^(3/2)/x^3-a^(3/2)*x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(
c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)-1/16*b*(-
12*a*c+b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x
+a)^(1/2)/c^(3/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/8*(2*b*c*x+8*a*c+b^2)*(c*x^4+
b*x^3+a*x^2)^(1/2)/c/x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used

= {1935, 1959, 1947, 857, 635, 212, 738}

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx = -\frac{a^{3/2}x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} - \frac{bx(b^2-12ac)\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{(8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*c*x) + (a*x^2 + b*
x^3 + c*x^4)^(3/2)/(3*x^3) - (a^(3/2)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a
+ b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4] - (b
*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/(16*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1935

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*(2
*n - q) + 1)), x] + Dist[(n - q)*(p/(m + p*(2*n - q) + 1)), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1947

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 1959

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}+\frac{1}{2}\int\frac{(2a+bx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx\\ &= \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx}+\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}+\frac{\int\frac{8a^2c-\frac{1}{2}b(b^2-12ac)x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8c}\\ &= \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx}+\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}\\ &\quad+\frac{\left(x\sqrt{a+bx+cx^2}\right)\int\frac{8a^2c-\frac{1}{2}b(b^2-12ac)x}{x\sqrt{a+bx+cx^2}}\,dx}{8c\sqrt{ax^2+bx^3+cx^4}}\end{aligned}$$

$$= \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} + \frac{\left(a^2x\sqrt{a+bx+cx^2}\right)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(b(b^2-12ac)\,x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{16c\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} - \frac{\left(2a^2x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(b(b^2-12ac)\,x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{8c\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} - \frac{a^{3/2}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} - \frac{b(b^2-12ac)\,x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.72

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx = \frac{x\sqrt{a+x(b+cx)}\left(-3b(b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+2\sqrt{c}\left(\sqrt{a+x(b\right.\right.}{48c^{3/2}\sqrt{x^2(a+}$$

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(-3*b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(Sqrt[a + x*(b + c*x)]*(3*b^2 + 14*b*
c*x + 8*c*(4*a + c*x^2)) + 48*a^(3/2)*c*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b
+ c*x)])/Sqrt[a]])))/(48*c^(3/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$\frac{16x^2\sqrt{c\,x^2+bx+a}\,c^{\frac{5}{2}}+48\ln(2)a^{\frac{3}{2}}c^{\frac{3}{2}}-48\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)a^{\frac{3}{2}}c^{\frac{3}{2}}+28c^{\frac{3}{2}}\sqrt{c\,x^2+bx+a}\,bx+36\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}\right.}{48c^{\frac{3}{2}}}$
default	$-\frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(48c^{\frac{5}{2}}a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)-16\left(c\,x^2+bx+a\right)^{\frac{3}{2}}c^{\frac{5}{2}}-12c^{\frac{5}{2}}\sqrt{c\,x^2+bx+a}\,bx-48c^{\frac{5}{2}}\sqrt{c\,x^2+bx+}\right.}{48x^3\left(c\,x^2+bx+a\right)^{\frac{3}{2}}c^{\frac{5}{2}}}$

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/c^(3/2)*(16*x^2*(c*x^2+b*x+a)^(1/2)*c^(5/2)+48*ln(2)*a^(3/2)*c^(3/2)-4
8*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2))*a^(3/2)*c^(3/2)+28*
c^(3/2)*(c*x^2+b*x+a)^(1/2)*b*x+36*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b
)*a*b*c-3*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*b^3+64*a*c^(3/2)*(c*x^2
+b*x+a)^(1/2)+6*c^(1/2)*(c*x^2+b*x+a)^(1/2)*b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 791, normalized size of antiderivative = 3.48

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx = \left[\frac{48\,a^{\frac{3}{2}}c^2x\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)-3\,(b^3-12\,abc)}{}\right.$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^(3/2)*c^2*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*s
qrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 3*(b^3 - 12*a*b*c)*s
qrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x
 + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c +
 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/48*(24*a^(3/2)*c^2*x*log
(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*
(b*x + 2*a)*sqrt(a))/x^3) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c
*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2
*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))
/(c^2*x), 1/96*(96*sqrt(-a)*a*c^2*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*
(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 3*(b^3 - 12*a*b*c)*sqrt
(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x +
b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32
*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/48*(48*sqrt(-a)*a*c^2*x*arc
tan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2
```

```
 + a^2*x)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 +
a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(8*c^3*x^2 + 1
4*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x)]
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4}\,dx = \int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**4,x)
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4}\,dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx = \int \frac{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}{x^4}\,dx$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4, x)
```

3.44 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 219

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx = \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} - \frac{3\sqrt{a}bx\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}} + \frac{3(b^2+4ac)\,x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -(c*x^4+b*x^3+a*x^2)^(3/2)/x^4-3/2*b*x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2
+b*x+a)^(1/2))*a^(1/2)*(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+3/8*(4
*a*c+b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a
)^(1/2)/c^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+3/4*(2*c*x+3*b)*(c*x^4+b*x^3+a*x^
2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

= {1934, 1959, 1947, 857, 635, 212, 738}

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx = \frac{3x(4ac+b^2)\sqrt{a+bx+cx^2}\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\sqrt{ax^2+bx^3+cx^4}} - \frac{3\sqrt{a}bx\sqrt{a+bx+cx^2}\,\mathrm{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} + \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (3*(3*b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x) - (a*x^2 + b*x^3 + c*x^
4)^(3/2)/x^4 - (3*Sqrt[a]*b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*
Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (3*(b^2
+ 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(8*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1947

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 1959

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}+\frac{3}{2}\int\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx$$

$$=\frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}+\frac{3\int\frac{4abc+c(b^2+4ac)x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8c}$$

$$=\frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}+\frac{\left(3x\sqrt{a+bx+cx^2}\right)\int\frac{4abc+c(b^2+4ac)x}{x\sqrt{a+bx+cx^2}}\,dx}{8c\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} + \frac{\left(3abx\sqrt{a+bx+cx^2}\right)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(3(b^2+4ac)\,x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} - \frac{\left(3abx\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(3(b^2+4ac)\,x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} - \frac{3\sqrt{a}bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}} + \frac{3(b^2+4ac)\,x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.71

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx = \frac{\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}(-4a+x(5b+2cx)) + 24\sqrt{a}b\sqrt{c}x\text{arctanh}\right.}{8\sqrt{c}\sqrt{x^2(a+x}}$$

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-4*a + x*(5*b + 2*
c*x)) + 24*Sqrt[a]*b*Sqrt[c]*x*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/
Sqrt[a]] - 3*(b^2 + 4*a*c)*x*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)
]]))/(8*Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{4c^{\frac{3}{2}}x^2\sqrt{c\,x^2+bx+a}-12\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)bx\sqrt{a}\,\sqrt{c}+12\ln(2)bx\sqrt{a}\,\sqrt{c}+10b\sqrt{c\,x^2+bx+a}\,x\sqrt{c}+12\ln\left(2\sqrt{c\,x^2+bx\right.}{8x\sqrt{c}}$
risch	$-\frac{a\sqrt{x^2(c\,x^2+bx+a)}}{x^2}+\frac{\left(\frac{3b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8\sqrt{c}}+\frac{3a\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2}+\frac{c\sqrt{c\,x^2+bx+a}\,x}{2}+\frac{5\sqrt{c\,x^2+bx+a}\,b}{4}-\right.}{x\sqrt{c\,x^2+bx+a}}$
default	$\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(8c^{\frac{5}{2}}(c\,x^2+bx+a)^{\frac{3}{2}}x^2+12c^{\frac{5}{2}}\sqrt{c\,x^2+bx+a}\,a\,x^2-12c^{\frac{3}{2}}a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)bx-8(c\,x^2+bx+\right.}{8x^4(c}$

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(4*c^(3/2)*x^2*(c*x^2+b*x+a)^(1/2)-12*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+
a)^(1/2))/x/a^(1/2))*b*x*a^(1/2)*c^(1/2)+12*ln(2)*b*x*a^(1/2)*c^(1/2)+10*b*
(c*x^2+b*x+a)^(1/2)*x*c^(1/2)+12*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*
a*c*x+3*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*b^2*x-8*a*(c*x^2+b*x+a)^(
1/2)*c^(1/2))/x/c^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 757, normalized size of antiderivative = 3.46

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx = \left[\frac{12\,\sqrt{a}bcx^2\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)+3\,(b^2+4\,ac}{}\right.$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(12*sqrt(a)*b*c*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4
*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 3*(b^2 + 4*a*c)*sq
rt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*
x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2
*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/8*(6*sqrt(a)*b*c*x^2*log(-(8*a*b*x^2 +
(b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqr
t(a))/x^3) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a
*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*
x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/16*(24*sqrt(-a)*b*c*
x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 +
a*b*x^2 + a^2*x)) + 3*(b^2 + 4*a*c)*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2
```

```
 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x)
+ 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/8
*(12*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sq
rt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1
/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*
c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2
)]
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^5} \, dx = \int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^5} \, dx$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**5,x)
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**5, x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^5} \, dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^5} \, dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^5} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument Va
lue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^5}\, dx = \int \frac{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}{x^5}\, dx$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5, x)
```

3.45 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 219

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx = -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5} - \frac{3(b^2+4ac)\,x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} + \frac{3b\sqrt{c}x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -1/2*(c*x^4+b*x^3+a*x^2)^(3/2)/x^5-3/8*(4*a*c+b^2)*x*arctanh(1/2*(b*x+2*a)/
a^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/a^(1/2)/(c*x^4+b*x^3+a*x^2
)^(1/2)+3/2*b*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)*
(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)-3/4*(-2*c*x+b)*(c*x^4+b*x^3+a
*x^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used

= {1934, 1955, 1947, 857, 635, 212, 738}

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx = -\frac{3x(4ac+b^2)\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} + \frac{3b\sqrt{c}x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}} - \frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (-3*(b - 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x^2) - (a*x^2 + b*x^3 + c*x
^4)^(3/2)/(2*x^5) - (3*(b^2 + 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a +
 b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c
*x^4]) + (3*b*Sqrt[c]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1947

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 1955

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n -
 q)*(p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}+\frac{3}{4}\int\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx$$

$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}-\frac{3}{8}\int\frac{-b^2-4ac-4bcx}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$

$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}-\frac{\left(3x\sqrt{a+bx+cx^2}\right)\int\frac{-b^2-4ac-4bcx}{x\sqrt{a+bx+cx^2}}\,dx}{8\sqrt{ax^2+bx^3+cx^4}}$$

$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5} + \frac{\left(3bcx\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(3(-b^2-4ac)\,x\sqrt{a+bx+cx^2}\right)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{8\sqrt{ax^2+bx^3+cx^4}}$$

$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5} + \frac{\left(3bcx\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(3(-b^2-4ac)\,x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{4\sqrt{ax^2+bx^3+cx^4}}$$

$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5} - \frac{3(b^2+4ac)\,x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} + \frac{3b\sqrt{c}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.73

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx = \frac{\sqrt{x^2(a+x(b+cx))}\left(3(b^2+4ac)\,x^2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right) - \sqrt{a}\left((2a+x(5\right.\right.}{4\sqrt{a}x^3\sqrt{a+}}$$

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(3*(b^2 + 4*a*c)*x^2*ArcTanh[(Sqrt[c]*x - Sqrt
[a + x*(b + c*x)])/Sqrt[a]] - Sqrt[a]*((2*a + x*(5*b - 4*c*x))*Sqrt[a + x*(
b + c*x)] + 6*b*Sqrt[c]*x^2*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]
])))/(4*Sqrt[a]*x^3*Sqrt[a + x*(b + c*x)])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$-\frac{3\left(x^{2}\left(ac+\frac{b^{2}}{4}\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)-\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)b\,x^{2}\sqrt{c}\,\sqrt{a}+\frac{\left(a^{\frac{3}{2}}+\left(-2c\,x^{2}+\frac{5}{2}bx\right)\sqrt{a}\right)\sqrt{c\,x^{2}+b}}{3}\right.}{2\sqrt{a}\,x^{2}}$
risch	$-\frac{(5bx+2a)\sqrt{x^{2}(c\,x^{2}+bx+a)}}{4x^{3}}+\frac{\left(c\sqrt{c\,x^{2}+bx+a}+\frac{3b\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2}-\frac{3\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)c}{2}-\frac{3\ln(}{}\right.}{x\sqrt{c\,x^{2}+bx+a}}$
default	$-\frac{(c\,x^{4}+b\,x^{3}+a\,x^{2})^{\frac{3}{2}}\left(12a^{\frac{5}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)c^{\frac{5}{2}}x^{2}-2c^{\frac{5}{2}}(c\,x^{2}+bx+a)^{\frac{3}{2}}b\,x^{3}+3a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)\right.}{}$

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*(x^2*(a*c+1/4*b^2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2
))-ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*b*x^2*c^(1/2)*a^(1/2)+1/3*(a^(
3/2)+(-2*c*x^2+5/2*b*x)*a^(1/2))*(c*x^2+b*x+a)^(1/2)-ln(2)*(a*c+1/4*b^2)*x^
2)/a^(1/2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 757, normalized size of antiderivative = 3.46

$$\int\frac{(ax^{2}+bx^{3}+cx^{4})^{3/2}}{x^{6}}\,dx=\left[\frac{12\,ab\sqrt{c}x^{3}\log\left(-\frac{8\,c^{2}x^{3}+8\,bcx^{2}+4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}(2\,cx+b)\sqrt{c}+(b^{2}+4\,ac)x}{x}\right)+3\,(b^{2}+4\,ac)\sqrt{}}{}\right.$$

$$-\frac{24\,ab\sqrt{-c}x^{3}\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}(2\,cx+b)\sqrt{-c}}{2\,(c^{2}x^{3}+bcx^{2}+acx)}\right)-3\,(b^{2}+4\,ac)\sqrt{a}x^{3}\log\left(-\frac{8\,abx^{2}+(b^{2}+4\,ac)x^{3}+8\,a^{2}x-4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}}{x^{3}}\right.}{16\,ax^{3}}$$

$$-\frac{12\,ab\sqrt{-c}x^{3}\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}(2\,cx+b)\sqrt{-c}}{2\,(c^{2}x^{3}+bcx^{2}+acx)}\right)-3\,(b^{2}+4\,ac)\sqrt{-a}x^{3}\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}(bx+2\,a)\sqrt{-a}}{2\,(acx^{3}+abx^{2}+a^{2}x)}\right)-2\,\sqrt{c}}{8\,ax^{3}}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/16*(12*a*b*sqrt(c)*x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 3*(b^2 + 4*a*c)*sqrt
(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c
*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), -1/16*(24*a*b*sqrt(-c)*x^3*arctan(1/2*sqr
```

```
t(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x))
- 3*(b^2 + 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x
 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 +
 b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), 1/8*(6*a*b*sqrt(c)*
x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 3*(b^2 + 4*a*c)*sqrt(-a)*x^3*arctan(1/2*sq
rt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))
 + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), -1
/8*(12*a*b*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*
sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 3*(b^2 + 4*a*c)*sqrt(-a)*x^3*arctan
(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 +
a^2*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x
^3)]
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^6}\,dx = \int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**6,x)
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**6, x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^6}\,dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^6, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^6}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument Va
lue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx=\int \frac{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}{x^6}\,dx$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6, x)
```

3.46 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 257

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx = \frac{(b^2-8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6} - \frac{b(ax^2+bx^3+cx^4)^{3/2}}{4ax^5} + \frac{b(b^2-12ac)\,x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{c^{3/2}x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -1/3*(c*x^4+b*x^3+a*x^2)^(3/2)/x^6-1/4*b*(c*x^4+b*x^3+a*x^2)^(3/2)/a/x^5+1/
16*b*(-12*a*c+b^2)*x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*
x^2+b*x+a)^(1/2)/a^(3/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+c^(3/2)*x*arctanh(1/2*(2
*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2
)^(1/2)+1/8*(2*b*c*x-8*a*c+b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a/x^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{1934, 1965, 1955, 1947, 857, 635, 212, 738\}$

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx = \frac{bx(b^2-12ac)\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{c^{3/2}x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} + \frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6} - \frac{b(ax^2+bx^3+cx^4)^{3/2}}{4ax^5}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] ((b^2 - 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*a*x^2) - (a*x^2 +
b*x^3 + c*x^4)^(3/2)/(3*x^6) - (b*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(4*a*x^5)
+ (b*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + b*x + c*x^2])])/(16*a^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (c^(3/2
)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1947

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 1955

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n -
 q)*(p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6}+\frac{1}{2}\int\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx\\
&= -\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6}-\frac{b(ax^2+bx^3+cx^4)^{3/2}}{4ax^5}-\frac{\int\frac{\left(\frac{1}{2}(b^2-8ac)-bcx\right)\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx}{4a}\\
&= \frac{(b^2-8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6}\\
&\quad-\frac{b(ax^2+bx^3+cx^4)^{3/2}}{4ax^5}+\frac{\int\frac{-\frac{1}{2}b(b^2-12ac)+8ac^2x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8a}\\
&= \frac{(b^2-8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6}\\
&\quad-\frac{b(ax^2+bx^3+cx^4)^{3/2}}{4ax^5}+\frac{\left(x\sqrt{a+bx+cx^2}\right)\int\frac{-\frac{1}{2}b(b^2-12ac)+8ac^2x}{x\sqrt{a+bx+cx^2}}\,dx}{8a\sqrt{ax^2+bx^3+cx^4}}\\
&= \frac{(b^2-8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6}-\frac{b(ax^2+bx^3+cx^4)^{3/2}}{4ax^5}\\
&\quad+\frac{\left(c^2x\sqrt{a+bx+cx^2}\right)\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(b(b^2-12ac)\,x\sqrt{a+bx+cx^2}\right)\int\frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{16a\sqrt{ax^2+bx^3+cx^4}}\\
&= \frac{(b^2-8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6}\\
&\quad-\frac{b(ax^2+bx^3+cx^4)^{3/2}}{4ax^5}+\frac{\left(2c^2x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}\\
&\quad+\frac{\left(b(b^2-12ac)\,x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int\frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{8a\sqrt{ax^2+bx^3+cx^4}}\\
&= \frac{(b^2-8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6}\\
&\quad-\frac{b(ax^2+bx^3+cx^4)^{3/2}}{4ax^5}+\frac{b(b^2-12ac)\,x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}\sqrt{ax^2+bx^3+cx^4}}\\
&\quad+\frac{c^{3/2}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.67

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx =$$

$$-\frac{\sqrt{x^2(a+x(b+cx))}\Big(3b(b^2-12ac)\,x^3\operatorname{arctanh}\Big(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\Big)+\sqrt{a}\Big(\sqrt{a+x(b+cx)}(8a^2+3b^2x^2+2ax}{24a^{3/2}x^4\sqrt{a+x(b+cx)}}$$

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -1/24*(Sqrt[x^2*(a + x*(b + c*x))]*(3*b*(b^2 - 12*a*c)*x^3*ArcTanh[(Sqrt[c]
*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + Sqrt[a]*(Sqrt[a + x*(b + c*x)]*(8*a^
2 + 3*b^2*x^2 + 2*a*x*(7*b + 16*c*x)) + 24*a*c^(3/2)*x^3*Log[b + 2*c*x - 2*
Sqrt[c]*Sqrt[a + x*(b + c*x)]])))/(a^(3/2)*x^4*Sqrt[a + x*(b + c*x)])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.58

method	result
pseudoelliptic	$-\frac{3\left(b\,x^3\left(ac-\frac{b^2}{12}\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)-\frac{4c^{\frac{3}{2}}a^{\frac{3}{2}}\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)x^3}{3}+\left(\frac{7x\left(\frac{16cx}{7}+b\right)a^{\frac{3}{2}}}{9}+\frac{\sqrt{a}\,b^2x^2}{6}+\frac{4a^{\frac{5}{2}}}{9}\right.\right.}{4a^{\frac{3}{2}}x^3}$
risch	$-\frac{(32ac\,x^2+3b^2x^2+14abx+8a^2)\sqrt{x^2(c\,x^2+bx+a)}}{24x^4a}+\frac{\left(16a\,c^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)-\frac{b\left(12ac-b^2\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c}}{x}\right.}{\sqrt{a}}\right.}{16ax\sqrt{c\,x^2+bx+a}}$
default	$\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(32c^{\frac{7}{2}}(c\,x^2+bx+a)^{\frac{3}{2}}a\,x^4-36c^{\frac{5}{2}}a^{\frac{5}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)b\,x^3+48c^{\frac{7}{2}}\sqrt{c\,x^2+bx+a}\,a^2x^4-2c^{\frac{5}{2}}(c\right.}{}$

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*(b*x^3*(a*c-1/12*b^2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(
1/2))-4/3*c^(3/2)*a^(3/2)*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*x^3+(7/
9*x*(16/7*c*x+b)*a^(3/2)+1/6*a^(1/2)*b^2*x^2+4/9*a^(5/2))*(c*x^2+b*x+a)^(1/
2)-ln(2)*(a*c-1/12*b^2)*x^3*b)/a^(3/2)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 815, normalized size of antiderivative = 3.17

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx = \left[\frac{48\,a^2c^{\frac{3}{2}}x^4\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)-3\,(b^3-12\,abc)\cdots}{\cdots}\right.$$

$$-\frac{96\,a^2\sqrt{-c}cx^4\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)+3\,(b^3-12\,abc)\sqrt{a}x^4\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+\cdots}}{x^3}\cdots\right.}{96\,a^2x^4}$$

$$-\frac{48\,a^2\sqrt{-c}cx^4\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)+3\,(b^3-12\,abc)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\cdots}{48\,a^2x^4}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^2*c^(3/2)*x^4*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 3*(b^3 - 12*a*b*c)*s
qrt(a)*x^4*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b
*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(14
*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), -1/96*(96*a^2*sqrt
(-c)*c*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2
*x^3 + b*c*x^2 + a*c*x)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^4*log(-(8*a*b*x^2 +
 (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sq
rt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2
+ 32*a^2*c)*x^2))/(a^2*x^4), 1/48*(24*a^2*c^(3/2)*x^4*log(-(8*c^2*x^3 + 8*b
*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*
x)/x) - 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x
^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*sqrt(c*x^4 + b*x^
3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), -1/4
8*(48*a^2*sqrt(-c)*c*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^4*ar
ctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^
2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2
+ 32*a^2*c)*x^2))/(a^2*x^4)]
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^7}\,dx = \int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^7}\,dx$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**7,x)
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**7, x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^7}\,dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^7}\,dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^7, x)
```

Giac [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^7}\,dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^7}\,dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^7}\,dx = \int \frac{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}{x^7}\,dx$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7, x)
```

3.47 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 197

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = -\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3} + \frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2} - \frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7} - \frac{3(b^2-4ac)^2\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{5/2}}$$

```
[Out] -1/4*(c*x^4+b*x^3+a*x^2)^(3/2)/x^7-3/128*(-4*a*c+b^2)^2*arctanh(1/2*x*(b*x+
2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(5/2)-1/32*(-12*a*c+b^2)*(c*x^4+b
*x^3+a*x^2)^(1/2)/a/x^3+1/64*b*(-20*a*c+3*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^
2/x^2-1/8*(6*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/x^4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1934, 1955, 1965, 12, 1918, 212}

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = -\frac{3(b^2-4ac)^2\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{5/2}} + \frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2} - \frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3} - \frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] -1/32*((b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^3) + (b*(3*b^2 - 20
*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*a^2*x^2) - ((b + 6*c*x)*Sqrt[a*x^2 +
 b*x^3 + c*x^4])/(8*x^4) - (a*x^2 + b*x^3 + c*x^4)^(3/2)/(4*x^7) - (3*(b^2
- 4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]
)/(128*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1955

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n -
 q)*(p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
```

```
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7} + \frac{3}{8}\int \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx$$

$$= -\frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7} + \frac{1}{16}\int \frac{b^2-12ac-4bcx}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx$$

$$= -\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3} - \frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7} - \frac{\int \frac{\frac{1}{2}b(3b^2-20ac)+c(b^2-12ac)x}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{32a}$$

$$= -\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3} + \frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2} - \frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7} + \frac{\int \frac{3(b^2-4ac)^2}{4\sqrt{ax^2+bx^3+cx^4}}\,dx}{32a^2}$$

$$= -\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3} + \frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2} - \frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7} + \frac{\left(3(b^2-4ac)^2\right)\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{128a^2}$$

$$= -\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}+\frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2} - \frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7} - \frac{\left(3(b^2-4ac)^2\right)\text{Subst}\left(\int\frac{1}{4a-x^2}\,dx,x,\frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{64a^2}$$

$$= -\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}+\frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2} - \frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7} - \frac{3(b^2-4ac)^2\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.72

$$\int\frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = \frac{\sqrt{x^2(a+x(b+cx))}\left(-\sqrt{a}(2a+bx)\sqrt{a+x(b+cx)}(8a^2-3b^2x^2+4ax(2b+5\right.}{64a^{5/2}x^5\sqrt{a+x(b+cx)}}$$

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-(Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(b + c*x)]*(
8*a^2 - 3*b^2*x^2 + 4*a*x*(2*b + 5*c*x))) + 3*(b^2 - 4*a*c)^2*x^4*ArcTanh[(
Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]]))/(64*a^(5/2)*x^5*Sqrt[a + x*(b
 + c*x)])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{3\left(x^4\left(ac-\frac{b^2}{4}\right)^2\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)+\left(\frac{b\,x^2(10cx+b)a^{\frac{3}{2}}}{12}+x\left(\frac{5cx}{3}+b\right)a^{\frac{5}{2}}-\frac{\sqrt{a}\,b^3x^3}{8}+\frac{2a^{\frac{7}{2}}}{3}\right)\sqrt{c\,x^2+bx+a}-\ln(2)x^4}{8a^{\frac{5}{2}}x^4}$
risch	$-\frac{(20abc\,x^3-3b^3x^3+40a^2c\,x^2+2a\,b^2x^2+24a^2bx+16a^3)\sqrt{x^2(c\,x^2+bx+a)}}{64x^5a^2}-\frac{3(16a^2c^2-8a\,b^2c+b^4)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+}}{x}\right.}{128a^{\frac{5}{2}}x\sqrt{c\,x^2+bx+}}$
default	$-\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(48c^2a^{\frac{7}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)x^4+24c^2(c\,x^2+bx+a)^{\frac{3}{2}}ab\,x^5-24c\,a^{\frac{5}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+b}}{x}\right.\right.}{}$

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*(x^4*(a*c-1/4*b^2)^2*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1
/2))+(1/12*b*x^2*(10*c*x+b)*a^(3/2)+x*(5/3*c*x+b)*a^(5/2)-1/8*a^(1/2)*b^3*x
^3+2/3*a^(7/2))*(c*x^2+b*x+a)^(1/2)-ln(2)*x^4*(a*c-1/4*b^2)^2)/a^(5/2)/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.69

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = \left[\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{a}x^5\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)}{x^3}\right.}{}\right.$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^5*log(-(8*a*b*x^2 + (b^2
 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a)
)/x^3) - 4*(24*a^3*b*x + 16*a^4 - (3*a*b^3 - 20*a^2*b*c)*x^3 + 2*(a^2*b^2 +
 20*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^3*x^5), 1/128*(3*(b^4 - 8*a
*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^5*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b
*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(24*a^3*b*x + 16*a^4 -
(3*a*b^3 - 20*a^2*b*c)*x^3 + 2*(a^2*b^2 + 20*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3
 + a*x^2))/(a^3*x^5)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = \int \frac{\left(x^2(a+bx+cx^2)\right)^{\frac{3}{2}}}{x^8}\,dx$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**8,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**8, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = \int \frac{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}{x^8}\,dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^8, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^8} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument Va
lue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^8} \, dx = \int \frac{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}{x^8} \, dx$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8, x)
```

3.48 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 249

$$\begin{aligned}\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx &= -\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4} \\ &+ \frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3} - \frac{(15b^4-100ab^2c+128a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{640a^3x^2} \\ &- \frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8} \\ &+ \frac{3b(b^2-4ac)^2\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{256a^{7/2}}\end{aligned}$$

```
[Out] -1/5*(c*x^4+b*x^3+a*x^2)^(3/2)/x^8+3/256*b*(-4*a*c+b^2)^2*arctanh(1/2*x*(b*
x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(7/2)-1/80*(-8*a*c+b^2)*(c*x^4+
b*x^3+a*x^2)^(1/2)/a/x^4+1/320*b*(-28*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/
a^2/x^3-1/640*(128*a^2*c^2-100*a*b^2*c+15*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^
3/x^2-3/40*(4*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/x^5
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

= {1934, 1955, 1965, 12, 1918, 212}

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx = \frac{3b(b^2-4ac)^2\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{256a^{7/2}} + \frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3} - \frac{(128a^2c^2-100ab^2c+15b^4)\sqrt{ax^2+bx^3+cx^4}}{640a^3x^2} - \frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8} - \frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -1/80*((b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^4) + (b*(5*b^2 - 28*
a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(320*a^2*x^3) - ((15*b^4 - 100*a*b^2*c +
128*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(640*a^3*x^2) - (3*(b + 4*c*x)*Sq
rt[a*x^2 + b*x^3 + c*x^4])/(40*x^5) - (a*x^2 + b*x^3 + c*x^4)^(3/2)/(5*x^8)
 + (3*b*(b^2 - 4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x
^3 + c*x^4])])/(256*a^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1955

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n -
 q)*(p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8}+\frac{3}{10}\int\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx\\
&= -\frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8}+\frac{3}{160}\int\frac{2(b^2-8ac)-4bcx}{x^3\sqrt{ax^2+bx^3+cx^4}}\,dx\\
&= -\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}-\frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5}\\
&\quad-\frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8}-\frac{\int\frac{b(5b^2-28ac)+4c(b^2-8ac)x}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx}{160a}\\
&= -\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}+\frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}\\
&\quad-\frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8}\\
&\quad+\frac{\int\frac{\frac{1}{2}(15b^4-100ab^2c+128a^2c^2)+bc(5b^2-28ac)x}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{320a^2}
\end{aligned}
$$

$$= -\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4} + \frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3} - \frac{(15b^4-100ab^2c+128a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{640a^3x^2} - \frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8} - \frac{\int \frac{15b(b^2-4ac)^2}{4\sqrt{ax^2+bx^3+cx^4}}dx}{320a^3}$$

$$= -\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4} + \frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3} - \frac{(15b^4-100ab^2c+128a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{640a^3x^2} - \frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8} - \frac{\left(3b(b^2-4ac)^2\right)\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}dx}{256a^3}$$

$$= -\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4} + \frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3} - \frac{(15b^4-100ab^2c+128a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{640a^3x^2} - \frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8} + \frac{\left(3b(b^2-4ac)^2\right)\text{Subst}\left(\int \frac{1}{4a-x^2}dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^3}$$

$$= -\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4} + \frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3} - \frac{(15b^4-100ab^2c+128a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{640a^3x^2} - \frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8} + \frac{3b(b^2-4ac)^2\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{256a^{7/2}}$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.71

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}dx = -\frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{a}\sqrt{a+x(b+cx)}(128a^4+15b^4x^4-10ab^2x^3(b+10cx)+16a^3x(11b+16cx)+8a\right.}{640a^{7/2}x^6\sqrt{a+x(b+cx)}}$$

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x]
[Out] -1/640*(Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(128*a^4
 + 15*b^4*x^4 - 10*a*b^2*x^3*(b + 10*c*x) + 16*a^3*x*(11*b + 16*c*x) + 8*a^
2*x^2*(b^2 + 7*b*c*x + 16*c^2*x^2)) + 15*b*(b^2 - 4*a*c)^2*x^5*ArcTanh[(Sqr
t[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]]))/(a^(7/2)*x^6*Sqrt[a + x*(b + c*x
)])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{\frac{3x^5b\left(ac-\frac{b^2}{4}\right)^2\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x\sqrt{a}}\right)}{16}+\frac{3\left(-\frac{x^2\left(16c^2x^2+7bcx+b^2\right)a^{\frac{5}{2}}}{15}+\frac{b^2x^3(10cx+b)a^{\frac{3}{2}}}{12}-\frac{22x\left(\frac{16cx}{11}+b\right)a^{\frac{7}{2}}}{15}-\frac{b^4x^4\sqrt{a}}{8}-\frac{16a}{15}\right.}{16}}{a^{\frac{7}{2}}x^5}$
risch	$-\frac{\left(128a^2c^2x^4-100a\,b^2cx^4+15b^4x^4+56a^2bcx^3-10a\,b^3x^3+256a^3cx^2+8a^2b^2x^2+176a^3bx+128a^4\right)\sqrt{x^2\left(cx^2+bx+a\right)}}{640x^6a^3}+\frac{3(16}{}$
default	$\frac{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(240c^2a^{\frac{7}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)bx^5+120c^2\left(cx^2+bx+a\right)^{\frac{3}{2}}ab^2x^6-120ca^{\frac{5}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+}}{x}\right.\right.}{}$

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] 3/16/a^(7/2)*(x^5*b*(a*c-1/4*b^2)^2*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/
2))/x/a^(1/2))+(-1/15*x^2*(16*c^2*x^2+7*b*c*x+b^2)*a^(5/2)+1/12*b^2*x^3*(10
*c*x+b)*a^(3/2)-22/15*x*(16/11*c*x+b)*a^(7/2)-1/8*b^4*x^4*a^(1/2)-16/15*a^(
9/2))*(c*x^2+b*x+a)^(1/2)-ln(2)*x^5*b*(a*c-1/4*b^2)^2)/x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.58

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx = \left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{a}x^6\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}(bx+2}{x^3}\right.}{}\right.$$

$$-\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\,(176\,a^4bx+128\,a^5+(15\,ab^4-100\,a}{1280\,a^4x^6}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^6*log(-(8*a*b*x^2 +
(b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqr
t(a))/x^3) - 4*(176*a^4*b*x + 128*a^5 + (15*a*b^4 - 100*a^2*b^2*c + 128*a^3
*c^2)*x^4 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^2)*sq
rt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^6), -1/1280*(15*(b^5 - 8*a*b^3*c + 16*a^2
*b*c^2)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqr
t(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*(176*a^4*b*x + 128*a^5 + (15*a*b^4 -
 100*a^2*b^2*c + 128*a^3*c^2)*x^4 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^3 + 8*(a^3
*b^2 + 32*a^4*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^6)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx = \int \frac{(x^2(a+bx+cx^2))^{\frac{3}{2}}}{x^9}\,dx$$

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**9,x)
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**9, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx = \int \frac{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}{x^9}\,dx$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^9, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument Va
lue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx = \int \frac{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}{x^9}\,dx$$

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9, x)
```

3.49 $\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 143

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}+\frac{(3b^2-4ac)\,x\sqrt{a+bx+cx^2}\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] 1/8*(-4*a*c+3*b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*
x^2+b*x+a)^(1/2)/c^(5/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/2*(c*x^4+b*x^3+a*x^2)^
(1/2)/c-3/4*b*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2/x
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1942, 1963, 12, 1928, 635, 212}

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\frac{x(3b^2-4ac)\sqrt{a+bx+cx^2}\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}+\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}$$

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*c) - (3*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^
2*x) + ((3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1942

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - 2*n + q + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^(
p + 1)/(c*(m + p*q + 2*(n - q)*p + 1))), x] - Dist[1/(c*(m + p*q + 2*(n - q
)*p + 1)), Int[x^(m - 2*(n - q))*(a*(m + p*q - 2*(n - q) + 1) + b*(m + p*q
+ (n - q)*(p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
&& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && Rational
Q[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
```

```
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{\int \frac{x\left(a+\frac{3bx}{2}\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2c}\\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x} + \frac{\int \frac{(3b^2-4ac)x}{4\sqrt{ax^2+bx^3+cx^4}}\,dx}{2c^2}\\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x} + \frac{(3b^2-4ac)\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8c^2}\\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x} + \frac{\left((3b^2-4ac)\,x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8c^2\sqrt{ax^2+bx^3+cx^4}}\\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}\\
&\quad + \frac{\left((3b^2-4ac)\,x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4c^2\sqrt{ax^2+bx^3+cx^4}}\\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}\\
&\quad + \frac{(3b^2-4ac)\,x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{x\left(2\sqrt{c}(-3b+2cx)(a+x(b+cx)) + (-3b^2+4ac)\sqrt{a+x(b+cx)}\log\left(c^2\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right.\right.\right.}{8c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*(2*Sqrt[c]*(-3*b + 2*c*x)*(a + x*(b + c*x)) + (-3*b^2 + 4*a*c)*Sqrt[a +
x*(b + c*x)]*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(8*c^
(5/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{4\sqrt{c\,x^{2}+bx+a}\,c^{\frac{3}{2}}x-6b\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}-4\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)ac+3\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)b^{2}}{8c^{\frac{5}{2}}}$
risch	$-\frac{(-2cx+3b)(c\,x^{2}+bx+a)x}{4c^{2}\sqrt{x^{2}(c\,x^{2}+bx+a)}}-\frac{(4ac-3b^{2})\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)x\sqrt{c\,x^{2}+bx+a}}{8c^{\frac{5}{2}}\sqrt{x^{2}(c\,x^{2}+bx+a)}}$
default	$\frac{x\sqrt{c\,x^{2}+bx+a}\left(4c^{\frac{5}{2}}\sqrt{c\,x^{2}+bx+a}\,x-6c^{\frac{3}{2}}\sqrt{c\,x^{2}+bx+a}\,b-4\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)a\,c^{2}+3\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right.\right.}{8\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,c^{\frac{7}{2}}}$

```
[In] int(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(4*(c*x^2+b*x+a)^(1/2)*c^(3/2)*x-6*b*(c*x^2+b*x+a)^(1/2)*c^(1/2)-4*ln(2
*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*a*c+3*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2
)+2*c*x+b)*b^2)/c^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.58

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$
$$= \left[-\frac{(3\,b^2-4\,ac)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)-4\,\sqrt{cx^4+bx^3+ax^2}(2\,c^2x-3\,b}{16\,c^3x}\right.$$
$$\left.-\frac{(3\,b^2-4\,ac)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-2\,\sqrt{cx^4+bx^3+ax^2}(2\,c^2x-3\,bc)}{8\,c^3x}\right]$$

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^
4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4
 + b*x^3 + a*x^2)*(2*c^2*x - 3*b*c))/(c^3*x), -1/8*((3*b^2 - 4*a*c)*sqrt(-c
)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 +
b*c*x^2 + a*c*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x - 3*b*c))/(c^3*x
)]
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x^3}{\sqrt{x^2(a+bx+cx^2)}}\,dx$$

```
[In] integrate(x**3/(c*x**4+b*x**3+a*x**2)**(1/2),x)
[Out] Integral(x**3/sqrt(x**2*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x^3}{\sqrt{cx^4+bx^3+ax^2}}\,dx$$

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx &= \frac{1}{4}\sqrt{cx^2+bx+a}\left(\frac{2\,x}{c\mathrm{sgn}\left(x\right)} - \frac{3\,b}{c^2\mathrm{sgn}\left(x\right)}\right) \\ &+ \frac{\left(3\,b^2\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right) - 4\,ac\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right) + 6\,\sqrt{a}b\sqrt{c}\right)\mathrm{sgn}(x)}{8\,c^{\frac{5}{2}}} \\ &- \frac{\left(3\,b^2-4\,ac\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}\end{aligned}$$

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*x/(c*sgn(x)) - 3*b/(c^2*sgn(x))) + 1/8*(3*b^2*
log(abs(b - 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 6
*sqrt(a)*b*sqrt(c))*sgn(x)/c^(5/2) - 1/8*(3*b^2 - 4*a*c)*log(abs(2*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/(c^(5/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\int \frac{x^3}{\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

```
[In] int(x^3/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
[Out] int(x^3/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.50 $\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -1/2*b*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(
1/2)/c^(3/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+(c*x^4+b*x^3+a*x^2)^(1/2)/c/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1931, 1928, 635, 212}

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[In] Int[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(c*x) - (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3 +
 c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1931

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n)*((a*x^(n - 1) + b*x^n + c*x^(n + 1))^(p + 1)
/(2*c*(p + 1))), x] - Dist[b/(2*c), Int[x^(m - 1)*(a*x^(n - 1) + b*x^n + c*
x^(n + 1))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n -
q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p,
 q] && EqQ[m + p*(n - 1) - 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{b\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2c} \\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{\left(bx\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2c\sqrt{ax^2+bx^3+cx^4}} \\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{\left(bx\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c\sqrt{ax^2+bx^3+cx^4}} \\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{x\left(2\sqrt{c}(a+x(b+cx))-b\sqrt{a+x(b+cx)}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{2c^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

```
[In] Integrate[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*(2*Sqrt[c]*(a + x*(b + c*x)) - b*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(2*c^(3/2)*Sqrt[x^2*(a + x*(b + c*x
))])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$-\frac{\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)b-2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}}{2c^{\frac{3}{2}}}$	50
default	$\frac{x\sqrt{c\,x^{2}+bx+a}\,\left(2\sqrt{c\,x^{2}+bx+a}\,c^{\frac{3}{2}}-b\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)c\right)}{2\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,c^{\frac{5}{2}}}$	88
risch	$\frac{(c\,x^{2}+bx+a)x}{c\sqrt{x^{2}(c\,x^{2}+bx+a)}}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)x\sqrt{c\,x^{2}+bx+a}}{2c^{\frac{3}{2}}\,\sqrt{x^{2}(c\,x^{2}+bx+a)}}$	93

```
[In] int(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c^(3/2)*(ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*b-2*(c*x^2+b*x+a)^(
1/2)*c^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.83

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \left[\frac{b\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)+4\,\sqrt{cx^4+bx^3+ax^2}c}{4\,c^2x}, \frac{b\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx\cdots}}{2\,(c\cdots}\right.}{\cdots}\right.$$

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^
2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2
)*c)/(c^2*x), 1/2*(b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c
*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^
2)*c)/(c^2*x)]
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x^2}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

```
[In] integrate(x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**2*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x^2}{\sqrt{cx^4+bx^3+ax^2}}\,dx$$

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx = -\frac{\left(b\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+2\sqrt{a}\sqrt{c}\right)\operatorname{sgn}(x)}{2c^{\frac{3}{2}}} + \frac{b\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2c^{\frac{3}{2}}\operatorname{sgn}(x)} + \frac{\sqrt{cx^2+bx+a}}{c\operatorname{sgn}(x)}$$

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*log(abs(b - 2*sqrt(a)*sqrt(c))) + 2*sqrt(a)*sqrt(c))*sgn(x)/c^(3/2)
 + 1/2*b*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/(c^(3/
2)*sgn(x)) + sqrt(c*x^2 + b*x + a)/(c*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{ax^2 + bx^3 + cx^4}}\, dx = \int \frac{x^2}{\sqrt{c\,x^4 + b\,x^3 + a\,x^2}}\, dx$$

```
[In] int(x^2/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
[Out] int(x^2/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.51 $\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\frac{x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^
(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1928, 635, 212}

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\frac{x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

```
[In] Int[x/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rubi steps

$$\text{integral} = \frac{\left(x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{\left(2x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = -\frac{x\sqrt{a+bx+cx^2}\log\left(b+2cx-2\sqrt{c}\sqrt{a+bx+cx^2}\right)}{\sqrt{c}\sqrt{x^2(a+x(b+cx))}}$$

```
[In] Integrate[x/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
[Out] -((x*Sqrt[a + b*x + c*x^2]*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]]
)/(Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))]))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

method	result	size
pseudoelliptic	$\frac{\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)}{\sqrt{c}}$	29
default	$\frac{x\sqrt{c\,x^{2}+bx+a}\,\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)}{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,\sqrt{c}}$	65

```
[In] int(x/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^(1/2)*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.82

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \left[\frac{\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)}{2\,\sqrt{c}},\ -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)}{c}\right]$$

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b
)*sqrt(c) + (b^2 + 4*a*c)*x)/x)/sqrt(c), -sqrt(-c)*arctan(1/2*sqrt(c*x^4 +
b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x))/c]
```

Sympy [F]

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x}{\sqrt{x^2\,(a+bx+cx^2)}}\,dx$$

```
[In] integrate(x/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{ax^2 + bx^3 + cx^4}}\, dx = \int \frac{x}{\sqrt{cx^4 + bx^3 + ax^2}}\, dx$$

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \frac{x}{\sqrt{ax^2 + bx^3 + cx^4}}\, dx = \frac{\log\left(\left|b - 2\sqrt{a}\sqrt{c}\right|\right)\operatorname{sgn}(x)}{\sqrt{c}} - \frac{\log\left(\left|2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} + b\right|\right)}{\sqrt{c}\operatorname{sgn}(x)}$$

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] log(abs(b - 2*sqrt(a)*sqrt(c)))*sgn(x)/sqrt(c) - log(abs(2*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*sqrt(c) + b))/(sqrt(c)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{ax^2 + bx^3 + cx^4}}\, dx = \int \frac{x}{\sqrt{c\,x^4 + b\,x^3 + a\,x^2}}\, dx$$

```
[In] int(x/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
[Out] int(x/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.52 $\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1918, 212}

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

```
[In] Int[1/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
```

```
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.51

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{2x\sqrt{a+x(b+cx)}\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
[Out] (2*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt
[a]])/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$\frac{\ln(2)-\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)}{\sqrt{a}}$	42
default	$-\frac{x\sqrt{c\,x^2+bx+a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\sqrt{c\,x^4+b\,x^3+a\,x^2}\,\sqrt{a}}$	66

```
[In] int(1/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] (ln(2)-ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.89

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \left[\frac{\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)}{a}\right]$$

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(a))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**3 + c*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}\,dx$$

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}} + \frac{2\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] -2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(-(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

```
[In] int(1/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
[Out] int(1/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.53 $\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}+\frac{b\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}$$

```
[Out] 1/2*b*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(3/2)-(c
*x^4+b*x^3+a*x^2)^(1/2)/a/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1941, 1918, 212}

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{b\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}$$

```
[In] Int[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/(a*x^2)) + (b*ArcTanh[(x*(2*a + b*x))/(2*Sqrt
[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1941

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*((a*x^q + b*x^n + c*x^(2*n - q))^(p +
 1)/(2*a*(n - q)*(p + 1))), x] - Dist[b/(2*a), Int[x^(m + n - q)*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[
p, -1] && LtQ[p, 0] && RationalQ[m, q] && EqQ[m + p*q + 1, -2*(n - q)*(p +
1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2} - \frac{b\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2a} \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2} + \frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{a} \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2} + \frac{b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{-\sqrt{a}(a+x(b+cx)) - bx\sqrt{a+x(b+cx)}\,\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

```
[In] Integrate[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
[Out] (-(Sqrt[a]*(a + x*(b + c*x))) - b*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*
x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(a^(3/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

method	result	size
pseudoelliptic	$\frac{bx\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)-bx\ln\left(2\right)-2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{2a^{\frac{3}{2}}x}$	68
default	$-\frac{\sqrt{c\,x^{2}+bx+a}\,\left(2a^{\frac{3}{2}}\sqrt{c\,x^{2}+bx+a}-b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)ax\right)}{2\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,a^{\frac{5}{2}}}$	88
risch	$-\frac{c\,x^{2}+bx+a}{a\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)x\sqrt{c\,x^{2}+bx+a}}{2a^{\frac{3}{2}}\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}$	97

```
[In] int(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^(3/2)*(b*x*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2))-b*x*
ln(2)-2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.52

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \left[\frac{\sqrt{a}bx^{2}\log\left(-\frac{8\,abx^{2}+\left(b^{2}+4\,ac\right)x^{3}+8\,a^{2}x+4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\left(bx+2\,a\right)\sqrt{a}}{x^{3}}\right)-4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}a}{4\,a^{2}x^{2}},\ -\frac{\sqrt{-a}bx^{2}\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}\left(bx+2\,a\right)\sqrt{-a}}{2\,\left(acx^{3}+abx^{2}+a^{2}x\right)}\right)+2\,\sqrt{cx^{4}+bx^{3}+ax^{2}}a}{2\,a^{2}x^{2}}\right]$$

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(
c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a
*x^2)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*
x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x
^3 + a*x^2)*a)/(a^2*x^2)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{x\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(x**2*(a + b*x + c*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^3+ax^2}x}\,dx$$

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument Va
lue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{x\,\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

```
[In] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(1/2)),x)
[Out] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(1/2)), x)
```

3.54 $\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 119

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2} - \frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}}$$

```
[Out] -1/8*(-4*a*c+3*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/
2))/a^(5/2)-1/2*(c*x^4+b*x^3+a*x^2)^(1/2)/a/x^3+3/4*b*(c*x^4+b*x^3+a*x^2)^(
1/2)/a^2/x^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1943, 1965, 12, 1918, 212}

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = -\frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}} + \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

```
[In] Int[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] -1/2*Sqrt[a*x^2 + b*x^3 + c*x^4]/(a*x^3) + (3*b*Sqrt[a*x^2 + b*x^3 + c*x^4]
)/(4*a^2*x^2) - ((3*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*
x^2 + b*x^3 + c*x^4])])/(8*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1943

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - q + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
/(a*(m + p*q + 1))), x] - Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*(b*(m
 + p*q + (n - q)*(p + 1) + 1) + c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ
[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ
[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && LtQ[m + p*q + 1, 0]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}+\frac{\int \frac{-\frac{3b}{2}-cx}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{2a}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}-\frac{\int \frac{-\frac{3b^2}{4}+ac}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2a^2}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3} + \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2} + \frac{(3b^2-4ac)\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8a^2}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3} + \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2} - \frac{(3b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{4a^2}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3} + \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2} - \frac{(3b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx$$

$$= \frac{-\sqrt{a}(2a-3bx)(a+x(b+cx)) + (3b^2-4ac)\,x^2\sqrt{a+x(b+cx)}\,\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{4a^{5/2}x\sqrt{x^2(a+x(b+cx))}}$$

```
[In] Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] (-(Sqrt[a]*(2*a - 3*b*x)*(a + x*(b + c*x))) + (3*b^2 - 4*a*c)*x^2*Sqrt[a +
x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(4*a^(5/
2)*x*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{(c\,x^2+bx+a)(-3bx+2a)}{4a^2x\sqrt{x^2(c\,x^2+bx+a)}} + \frac{(4ac-3b^2)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)x\sqrt{c\,x^2+bx+a}}{8a^{\frac{5}{2}}\sqrt{x^2(c\,x^2+bx+a)}}$
pseudoelliptic	$\frac{4\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)ac\,x^2-3\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)b^2x^2-4\ln(2)ac\,x^2+3\ln(2)b^2x^2-4a^{\frac{3}{2}}\sqrt{c\,x^2+bx+a}+6b}{8a^{\frac{5}{2}}x^2}$
default	$-\frac{\sqrt{c\,x^2+bx+a}\,\left(-6a^{\frac{3}{2}}\sqrt{c\,x^2+bx+a}\,bx-4c\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)a^2x^2+3\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)a\,b^2x^2+4a^{\frac{5}{2}}\right.}{8x\sqrt{c\,x^4+b\,x^3+a\,x^2}\,a^{\frac{7}{2}}}$

```
[In] int(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(c*x^2+b*x+a)*(-3*b*x+2*a)/a^2/x/(x^2*(c*x^2+b*x+a))^(1/2)+1/8*(4*a*c-
3*b^2)/a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x/(x^2*(c*x^2+
b*x+a))^(1/2)*(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.95

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx$$
$$= \left[-\frac{(3\,b^2-4\,ac)\sqrt{a}x^3\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)-4\,\sqrt{cx^4+bx^3+ax^2}(3\,abx-2}{16\,a^3x^3}\right.$$

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b^2 - 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8
*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c
*x^4 + b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3), 1/8*((3*b^2 - 4*a*c)*sq
rt(-a)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c
*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))
/(a^3*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{x^2\sqrt{x^2\,(a+bx+cx^2)}}\,dx$$

```
[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x + c*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^3+ax^2}x^2}\,dx$$

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument Va
lue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{x^2\,\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

```
[In] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2)), x)
```

3.55 $\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 262

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2(b^2-4ac)} - \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4c^3(b^2-4ac)x} - \frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)} + \frac{3(5b^2-4ac)\,x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] 2*x^4*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)+3/8*(-4*a*c+5*b^2)*x
*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(
7/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/2*(-12*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2
)/c^2/(-4*a*c+b^2)-1/4*b*(-52*a*c+15*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3/(-4
*a*c+b^2)/x-2*b*x*(c*x^4+b*x^3+a*x^2)^(1/2)/c/(-4*a*c+b^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1937, 1963, 12, 1928, 635, 212}

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{3x(5b^2-4ac)\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}\sqrt{ax^2+bx^3+cx^4}} - \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4c^3x(b^2-4ac)} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2(b^2-4ac)} + \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)}$$

```
[In] Int[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^4*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) + ((5*b^2 -
 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*c^2*(b^2 - 4*a*c)) - (b*(15*b^2 -
52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^3*(b^2 - 4*a*c)*x) - (2*b*x*Sqrt[
a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + (3*(5*b^2 - 4*a*c)*x*Sqrt[a + b
*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7
/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1937

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - 2*n + q + 1))*(2*a + b*x^(n - q))*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/((n - q)*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1
/((n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n
- q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n +
 c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[
p, -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int\frac{x^3(6a+3bx)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{b^2-4ac}\\
&= \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)}+\frac{2\int\frac{x^2\left(6ab+\frac{3}{2}(5b^2-12ac)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{3c\,(b^2-4ac)}\\
&= \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}+\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2\,(b^2-4ac)}\\
&\quad-\frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)}-\frac{\int\frac{x\left(\frac{3}{2}a(5b^2-12ac)+\frac{3}{4}b(15b^2-52ac)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{3c^2\,(b^2-4ac)}\\
&= \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}+\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2\,(b^2-4ac)}\\
&\quad-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4c^3\,(b^2-4ac)\,x}\\
&\quad-\frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)}+\frac{\int\frac{9(b^2-4ac)(5b^2-4ac)x}{8\sqrt{ax^2+bx^3+cx^4}}\,dx}{3c^3\,(b^2-4ac)}\\
&= \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}+\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2\,(b^2-4ac)}\\
&\quad-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4c^3\,(b^2-4ac)\,x}-\frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)}\\
&\quad+\frac{(3(5b^2-4ac))\int\frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8c^3}
\end{aligned}$$

$$= \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2(b^2-4ac)} - \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4c^3(b^2-4ac)x} - \frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)} + \frac{\left(3(5b^2-4ac)x\sqrt{a+bx+cx^2}\right)\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8c^3\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2(b^2-4ac)} - \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4c^3(b^2-4ac)x} - \frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)} + \frac{\left(3(5b^2-4ac)x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4c^3\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2(b^2-4ac)} - \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4c^3(b^2-4ac)x} - \frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)} + \frac{3(5b^2-4ac)x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.71

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{x\Big(2\sqrt{c}(4a^2c(-13b+6cx)+b^2x(15b^2+5bcx-2c^2x^2)+a(15b^3-62b^2cx-20}{8c^{7/2}(}$$

```
[In] Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*Sqrt[c]*(4*a^2*c*(-13*b + 6*c*x) + b^2*x*(15*b^2 + 5*b*c*x - 2*c^2*x^
2) + a*(15*b^3 - 62*b^2*c*x - 20*b*c^2*x^2 + 8*c^3*x^3)) + 3*(5*b^4 - 24*a*
b^2*c + 16*a^2*c^2)*Sqrt[a + x*(b + c*x)]*Log[c^3*(b + 2*c*x - 2*Sqrt[c]*Sq
rt[a + x*(b + c*x)])]))/(8*c^(7/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x)
)])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$-\frac{48\left(\left(-\frac{5}{24}b^3x^2+\frac{31}{12}b^2ax+\frac{13}{6}a^2b\right)c^{\frac{3}{2}}+\left(\frac{1}{12}b^2x^3+\frac{5}{6}ab\,x^2-a^2x\right)c^{\frac{5}{2}}-\frac{a\,c^{\frac{7}{2}}x^3}{3}-\frac{5b^3\sqrt{c}\,(bx+a)}{8}+\frac{\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)\sqrt{c\,x^2}}{16}\right.}{\sqrt{c\,x^2+bx+a}\,c^{\frac{7}{2}}\,(32ac-8b^2)}$
default	$\frac{x^3\,(c\,x^2+bx+a)\left(16c^{\frac{9}{2}}a\,x^3-4c^{\frac{7}{2}}b^2x^3-40c^{\frac{7}{2}}ab\,x^2+48c^{\frac{7}{2}}a^2x+10c^{\frac{5}{2}}b^3x^2-124c^{\frac{5}{2}}a\,b^2x-104c^{\frac{5}{2}}a^2b+30c^{\frac{3}{2}}b^4x+30c^{\frac{3}{2}}a\,b^3-48\ln\right.}{8c^{\frac{9}{2}}}$
risch	$-\frac{(-2cx+7b)(c\,x^2+bx+a)x}{4c^3\sqrt{x^2(c\,x^2+bx+a)}}-\frac{\left(\frac{8c\,a^2(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{14b^2a(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+(-4abc-7b^3)\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2c}{c\left(4ac-b^2\right)}\right.\right.}{}$

```
[In] int(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -48/(c*x^2+b*x+a)^(1/2)/c^(7/2)*((-5/24*b^3*x^2+31/12*b^2*a*x+13/6*a^2*b)*c
^(3/2)+(1/12*b^2*x^3+5/6*a*b*x^2-a^2*x)*c^(5/2)-1/3*a*c^(7/2)*x^3-5/8*b^3*c
^(1/2)*(b*x+a)+1/16*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*(c*x^2+b*x+a)
^(1/2)*(16*a^2*c^2-24*a*b^2*c+5*b^4))/(32*a*c-8*b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 616, normalized size of antiderivative = 2.35

$$\int \frac{x^7}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \left[-\frac{3\left((5\,b^4c-24\,ab^2c^2+16\,a^2c^3)x^3+(5\,b^5-24\,ab^3c+16\,a^2bc^2)x^2+(5\,ab^4-2\right.}{}\right.$$

$$-\frac{3\left((5\,b^4c-24\,ab^2c^2+16\,a^2c^3)x^3+(5\,b^5-24\,ab^3c+16\,a^2bc^2)x^2+(5\,ab^4-24\,a^2b^2c+16\,a^3c^2)x\right)\sqrt{-c}\,\mathrm{arctan}}{8\left((b^2c^5-4\right.}$$

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^3 + (5*b^5 - 24*a*b^3*c
+ 16*a^2*b*c^2)*x^2 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x)*sqrt(c)*log(
-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c)
 + (b^2 + 4*a*c)*x)/x) + 4*(15*a*b^3*c - 52*a^2*b*c^2 - 2*(b^2*c^3 - 4*a*c^
4)*x^3 + 5*(b^3*c^2 - 4*a*b*c^3)*x^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^
3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((b^2*c^5 - 4*a*c^6)*x^3 + (b^3*c^4 - 4*
a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*x), -1/8*(3*((5*b^4*c - 24*a*b^2*c^2
 + 16*a^2*c^3)*x^3 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^2 + (5*a*b^4 - 2
```

```
4*a^2*b^2*c + 16*a^3*c^2)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2
)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(15*a*b^3*c - 52*a^
2*b*c^2 - 2*(b^2*c^3 - 4*a*c^4)*x^3 + 5*(b^3*c^2 - 4*a*b*c^3)*x^2 + (15*b^4
*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((b^2*c^5 -
 4*a*c^6)*x^3 + (b^3*c^4 - 4*a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*x)]
```

Sympy [F]

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^7}{(x^2(a+bx+cx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**7/(c*x**4+b*x**3+a*x**2)**(3/2),x)
[Out] Integral(x**7/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^7}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.21

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{\left(15\,b^4\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-72\,ab^2c\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+48\,a^2c^2\log\left(\left|b-2\,\sqrt{\right.\right.\right.}{8\left(b^2c^{\frac{7}{2}}-4\,ac^{\frac{9}{2}}\right)} + \frac{\left(\left(\frac{2\,(b^2c^2-4\,ac^3)x}{b^2c^3\mathrm{sgn}(x)-4\,ac^4\mathrm{sgn}(x)}-\frac{5\,(b^3c-4\,abc^2)}{b^2c^3\mathrm{sgn}(x)-4\,ac^4\mathrm{sgn}(x)}\right)x-\frac{15\,b^4-62\,ab^2c+24\,a^2c^2}{b^2c^3\mathrm{sgn}(x)-4\,ac^4\mathrm{sgn}(x)}\right)x-\frac{15\,ab^3-52\,a^2bc}{b^2c^3\mathrm{sgn}(x)-4\,ac^4\mathrm{sgn}(x)}}{4\sqrt{cx^2+bx+a}} - \frac{3\,(5\,b^2-4\,ac)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{7}{2}}\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] 1/8*(15*b^4*log(abs(b - 2*sqrt(a)*sqrt(c))) - 72*a*b^2*c*log(abs(b - 2*sqrt
(a)*sqrt(c))) + 48*a^2*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^3
```

```
*sqrt(c) - 104*a^(3/2)*b*c^(3/2))*sgn(x)/(b^2*c^(7/2) - 4*a*c^(9/2)) + 1/4*
(((2*(b^2*c^2 - 4*a*c^3)*x/(b^2*c^3*sgn(x) - 4*a*c^4*sgn(x)) - 5*(b^3*c - 4
*a*b*c^2)/(b^2*c^3*sgn(x) - 4*a*c^4*sgn(x)))*x - (15*b^4 - 62*a*b^2*c + 24*
a^2*c^2)/(b^2*c^3*sgn(x) - 4*a*c^4*sgn(x)))*x - (15*a*b^3 - 52*a^2*b*c)/(b^
2*c^3*sgn(x) - 4*a*c^4*sgn(x)))/sqrt(c*x^2 + b*x + a) - 3/8*(5*b^2 - 4*a*c)
*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/(c^(7/2)*sgn(x
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(ax^2 + bx^3 + cx^4)^{3/2}}\,dx = \int \frac{x^7}{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}\,dx$$

```
[In] int(x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
[Out] int(x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.56 $\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 201

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)} + \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{c^2(b^2-4ac)x} - \frac{3bx\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{5/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] 2*x^3*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)-3/2*b*x*arctanh(1/2*
(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(5/2)/(c*x^4+b
*x^3+a*x^2)^(1/2)-2*b*(c*x^4+b*x^3+a*x^2)^(1/2)/c/(-4*a*c+b^2)+(-8*a*c+3*b^
2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2/(-4*a*c+b^2)/x
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1937, 1963, 12, 1928, 635, 212}

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{3bx\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{5/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{c^2x(b^2-4ac)} + \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)}$$

```
[In] Int[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^3*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*b*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 +
```

```
b*x^3 + c*x^4])/(c^2*(b^2 - 4*a*c)*x) - (3*b*x*Sqrt[a + b*x + c*x^2]*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2)*Sqrt[a*x^2 + b
*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1937

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - 2*n + q + 1))*(2*a + b*x^(n - q))*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/((n - q)*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1
/((n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n
- q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n +
 c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[
p, -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
```

```
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{x^2(4a+2bx)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{b^2-4ac}\\
&= \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)} + \frac{\int \frac{x(2ab+(3b^2-8ac)x)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{c\,(b^2-4ac)}\\
&= \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)}\\
&\quad + \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{c^2\,(b^2-4ac)\,x} - \frac{\int \frac{3b(b^2-4ac)x}{2\sqrt{ax^2+bx^3+cx^4}}\,dx}{c^2\,(b^2-4ac)}\\
&= \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)}\\
&\quad + \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{c^2\,(b^2-4ac)\,x} - \frac{(3b)\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2c^2}\\
&= \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)}\\
&\quad + \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{c^2\,(b^2-4ac)\,x} - \frac{\left(3bx\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2c^2\sqrt{ax^2+bx^3+cx^4}}\\
&= \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)}\\
&\quad + \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{c^2\,(b^2-4ac)\,x}\\
&\quad - \frac{\left(3bx\sqrt{a+bx+cx^2}\right)\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c^2\sqrt{ax^2+bx^3+cx^4}}\\
&= \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)}\\
&\quad + \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{c^2\,(b^2-4ac)\,x} - \frac{3bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{5/2}\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.71

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{x\left(2\sqrt{c}(8a^2c-b^2x(3b+cx)+a(-3b^2+10bcx+4c^2x^2))-3b(b^2-4ac)\sqrt{a+}\right.}{2c^{5/2}\left(-b^2+4ac\right)\sqrt{x^2(a+x(b+}}$$

```
[In] Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*Sqrt[c]*(8*a^2*c - b^2*x*(3*b + c*x) + a*(-3*b^2 + 10*b*c*x + 4*c^2*x
^2)) - 3*b*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*Log[c^2*(b + 2*c*x - 2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])]))/(2*c^(5/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b +
 c*x))])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$-\frac{6\left(\frac{\left(\frac{1}{2}b^2x^2-5abx-4a^2\right)c^{\frac{3}{2}}}{3}-\frac{2c^{\frac{5}{2}}a\,x^2}{3}+\frac{b\left(2\left(b^2x+ab\right)\sqrt{c}+\sqrt{c\,x^2+bx+a}\,\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)\left(4ac-b^2\right)\right)}{4}\right)}{c^{\frac{5}{2}}\sqrt{c\,x^2+bx+a}\,(4ac-b^2)}$
default	$\frac{x^3\left(c\,x^2+bx+a\right)\left(8a\,c^{\frac{7}{2}}x^2-2c^{\frac{5}{2}}b^2x^2+20c^{\frac{5}{2}}abx-6c^{\frac{3}{2}}b^3x+16c^{\frac{5}{2}}a^2-6c^{\frac{3}{2}}a\,b^2-12\ln\left(\frac{2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)\sqrt{c\,x^2+bx+a}\,ab\right.}{2c^{\frac{7}{2}}\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}(4ac-b^2)}$
risch	$\frac{(c\,x^2+bx+a)x}{c^2\sqrt{x^2(c\,x^2+bx+a)}}+\frac{\left(\frac{3bx}{2c^2\sqrt{c\,x^2+bx+a}}-\frac{b^2}{4c^3\sqrt{c\,x^2+bx+a}}-\frac{b^3x}{2c^2\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{b^4}{4c^3\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{3b\ln\left(\frac{\frac{b}{2}+c}{\sqrt{c}}\right.}{}\right.}{\sqrt{x^2(c\,x^2+bx+a)}}$

```
[In] int(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -6/c^(5/2)/(c*x^2+b*x+a)^(1/2)*(1/3*(1/2*b^2*x^2-5*a*b*x-4*a^2)*c^(3/2)-2/3
*c^(5/2)*a*x^2+1/4*b*(2*(b^2*x+a*b)*c^(1/2)+(c*x^2+b*x+a)^(1/2)*ln(2*(c*x^2
+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*(4*a*c-b^2)))/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.42

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \left[\frac{3\left((b^3c-4\,abc^2)x^3+(b^4-4\,ab^2c)x^2+(ab^3-4\,a^2bc)x\right)\sqrt{c}\log\left(-\frac{8\,c^2x^3+8\,bcx^2-}{}\right.}{4\left((b^2c^4-4\right.}\right.$$

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3 - 4*a^2*b
*c)*x)*sqrt(c)*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*
(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(
3*a*b^2*c - 8*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*c^2)*x)
)/((b^2*c^4 - 4*a*c^5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (a*b^2*c^3 - 4*a^2
*c^4)*x), 1/2*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3
- 4*a^2*b*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a
*b^2*c - 8*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*c^2)*x))/(
(b^2*c^4 - 4*a*c^5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (a*b^2*c^3 - 4*a^2*c^
4)*x)]
```

Sympy [F]

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^6}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**6/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**6/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^6}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.15

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{\left(3\,b^3\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-12\,abc\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+6\,\sqrt{a}b^2\sqrt{c}-16\,a^{\frac{3}{2}}c^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{2\left(b^2c^{\frac{5}{2}}-4\,ac^{\frac{7}{2}}\right)} + \frac{\left(\frac{(b^2c-4\,ac^2)x}{b^2c^2\mathrm{sgn}(x)-4\,ac^3\mathrm{sgn}(x)}+\frac{3\,b^3-10\,abc}{b^2c^2\mathrm{sgn}(x)-4\,ac^3\mathrm{sgn}(x)}\right)x+\frac{3\,ab^2-8\,a^2c}{b^2c^2\mathrm{sgn}(x)-4\,ac^3\mathrm{sgn}(x)}}{\sqrt{cx^2+bx+a}} + \frac{3\,b\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*b^3*log(abs(b - 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(abs(b - 2*sqrt(a
)*sqrt(c))) + 6*sqrt(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))*sgn(x)/(b^2*c^(5/
2) - 4*a*c^(7/2)) + (((b^2*c - 4*a*c^2)*x/(b^2*c^2*sgn(x) - 4*a*c^3*sgn(x))
 + (3*b^3 - 10*a*b*c)/(b^2*c^2*sgn(x) - 4*a*c^3*sgn(x)))*x + (3*a*b^2 - 8*a
^2*c)/(b^2*c^2*sgn(x) - 4*a*c^3*sgn(x)))/sqrt(c*x^2 + b*x + a) + 3/2*b*log(
abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/(c^(5/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^6}{{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}}\,dx$$

```
[In] int(x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] int(x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.57 $\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 153

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)x} + \frac{x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] 2*x^2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)+x*arctanh(1/2*(2*c*x
+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(3/2)/(c*x^4+b*x^3+a
*x^2)^(1/2)-2*b*(c*x^4+b*x^3+a*x^2)^(1/2)/c/(-4*a*c+b^2)/x
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1937, 1963, 12, 1928, 635, 212}

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{cx(b^2-4ac)}$$

```
[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^2*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*b*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)*x) + (x*Sqrt[a + b*x + c*x^2]*Arc
```

```
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(c^(3/2)*Sqrt[a*x^2 +
b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1937

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - 2*n + q + 1))*(2*a + b*x^(n - q))*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/((n - q)*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1
/((n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n
- q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n +
 c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[
p, -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
```

```
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{b^2-4ac} \\
&= \frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)\,x} + \frac{2\int \frac{(b^2-4ac)x}{2\sqrt{ax^2+bx^3+cx^4}}\,dx}{c\,(b^2-4ac)} \\
&= \frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)\,x} + \frac{\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{c} \\
&= \frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)\,x} + \frac{\left(x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{c\sqrt{ax^2+bx^3+cx^4}} \\
&= \frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)\,x} \\
&\quad + \frac{\left(2x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c\sqrt{ax^2+bx^3+cx^4}} \\
&= \frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)\,x} \\
&\quad + \frac{x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.73

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{x\left(2\sqrt{c}(-ab-b^2x+2acx)+(b^2-4ac)\sqrt{a+x(b+cx)}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{c^{3/2}\,(-b^2+4ac)\sqrt{x^2(a+x(b+cx))}}$$

```
[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
[Out] -((x*(2*Sqrt[c]*(-(a*b) - b^2*x + 2*a*c*x) + (b^2 - 4*a*c)*Sqrt[a + x*(b +
c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(c^(3/2)*(-b
^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))]))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.55

method	result
pseudoelliptic	$-\frac{x}{c\sqrt{c\,x^2+bx+a}}+\frac{b(bx+2a)}{\sqrt{c\,x^2+bx+a}\,c(4ac-b^2)}+\frac{\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)}{c^{\frac{3}{2}}}$
default	$-\frac{x^3(c\,x^2+bx+a)\left(4c^{\frac{5}{2}}ax-2c^{\frac{3}{2}}b^2x-2c^{\frac{3}{2}}ab-4\ln\left(\frac{2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)\sqrt{c\,x^2+bx+a}\,a\,c^2+\ln\left(\frac{2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)\right.}{c^{\frac{5}{2}}(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}(4ac-b^2)}$

```
[In] int(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x/c/(c*x^2+b*x+a)^(1/2)+b*(b*x+2*a)/(c*x^2+b*x+a)^(1/2)/c/(4*a*c-b^2)+1/c^
(3/2)*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.71

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \left[\frac{((b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x)\sqrt{c}\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4}}{}\right.}{2\,((b^2c^3-4\,ac^4)x^3+(b^3c^2-4\,ab}\right.$$

$$-\frac{((b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)+2\,\sqrt{cx^4+bx^3+}}{(b^2c^3-4\,ac^4)x^3+(b^3c^2-4\,abc^3)x^2+(ab^2c^2-4\,a^2c^3)x}$$

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*s
qrt(c)*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x +
 b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*c +
(b^2*c - 2*a*c^2)*x))/((b^2*c^3 - 4*a*c^4)*x^3 + (b^3*c^2 - 4*a*b*c^3)*x^2
+ (a*b^2*c^2 - 4*a^2*c^3)*x), -(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^
2 + (a*b^2 - 4*a^2*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2
*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*
x^2)*(a*b*c + (b^2*c - 2*a*c^2)*x))/((b^2*c^3 - 4*a*c^4)*x^3 + (b^3*c^2 - 4
*a*b*c^3)*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*x)]
```

Sympy [F]

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^5}{\left(x^2\,(a+bx+cx^2)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**5/(c*x**4+b*x**3+a*x**2)**(3/2),x)
[Out] Integral(x**5/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^5}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(x^5/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.07

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{\left(b^2\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)-4\,ac\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+2\sqrt{a}b\sqrt{c}\right)\mathrm{sgn}(x)}{b^2c^{\frac{3}{2}}-4\,ac^{\frac{5}{2}}} - \frac{2\left(\frac{ab}{b^2c\,\mathrm{sgn}(x)-4\,ac^2\mathrm{sgn}(x)}+\frac{(b^2-2\,ac)x}{b^2c\,\mathrm{sgn}(x)-4\,ac^2\mathrm{sgn}(x)}\right)}{\sqrt{cx^2+bx+a}} - \frac{\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{3}{2}}\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] (b^2*log(abs(b - 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(b - 2*sqrt(a)*sqrt(c))
) + 2*sqrt(a)*b*sqrt(c))*sgn(x)/(b^2*c^(3/2) - 4*a*c^(5/2)) - 2*(a*b/(b^2*c
*sgn(x) - 4*a*c^2*sgn(x)) + (b^2 - 2*a*c)*x/(b^2*c*sgn(x) - 4*a*c^2*sgn(x))
)/sqrt(c*x^2 + b*x + a) - log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqr
t(c) + b))/(c^(3/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^5}{\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}\,dx$$

```
[In] int(x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
[Out] int(x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.58 $\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 40

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] 2*x*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1930}

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

```
[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
[Out] (2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1930

```
Int[(x_)^(m_.)/((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(3/
2), x_Symbol] :> Simp[x^((n - 1)/2)*((4*a + 2*b*x)/((b^2 - 4*a*c)*Sqrt[a*x^
(n - 1) + b*x^n + c*x^(n + 1)])), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, (3
*n - 1)/2] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx=\frac{2x(2a+bx)}{(b^2-4ac)\sqrt{x^2(a+x(b+cx))}}$$

```
[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
[Out] (2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
pseudoelliptic	$\frac{-2bx-4a}{\sqrt{c\,x^2+bx+a}\,(4ac-b^2)}$	34
gosper	$-\frac{2(c\,x^2+bx+a)(bx+2a)x^3}{(4ac-b^2)(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}}$	53
default	$-\frac{2(c\,x^2+bx+a)(bx+2a)x^3}{(4ac-b^2)(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}}$	53
trager	$-\frac{2(bx+2a)\sqrt{c\,x^4+b\,x^3+a\,x^2}}{(c\,x^2+bx+a)x(4ac-b^2)}$	55

```
[In] int(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] (-2*b*x-4*a)/(c*x^2+b*x+a)^(1/2)/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.82

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx=\frac{2\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)}{(b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x}$$

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)/((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4
*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)
```

Sympy [F]

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^4}{(x^2(a+bx+cx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**4/(c*x**4+b*x**3+a*x**2)**(3/2),x)
[Out] Integral(x**4/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^4}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(x^4/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.72

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2\left(\frac{bx}{b^2\mathrm{sgn}(x)-4\,ac\mathrm{sgn}(x)}+\frac{2\,a}{b^2\mathrm{sgn}(x)-4\,ac\mathrm{sgn}(x)}\right)}{\sqrt{cx^2+bx+a}} - \frac{4\,\sqrt{a}\mathrm{sgn}(x)}{b^2-4\,ac}$$

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] 2*(b*x/(b^2*sgn(x) - 4*a*c*sgn(x)) + 2*a/(b^2*sgn(x) - 4*a*c*sgn(x)))/sqrt(
c*x^2 + b*x + a) - 4*sqrt(a)*sgn(x)/(b^2 - 4*a*c)
```

Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.88

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{\left(\frac{4\,a\,c}{4\,a\,c^2-b^2\,c}+\frac{2\,b\,c\,x}{4\,a\,c^2-b^2\,c}\right)\,\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x\,\left(c\,x^2+b\,x+a\right)}$$

```
[In] int(x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
[Out] -(((4*a*c)/(4*a*c^2 - b^2*c) + (2*b*c*x)/(4*a*c^2 - b^2*c))*(a*x^2 + b*x^3
+ c*x^4)^(1/2))/(x*(a + b*x + c*x^2))
```

3.59 $\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 39

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -2*x*(2*c*x+b)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1929}

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

```
[In] Int[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
[Out] (-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1929

```
Int[(x_)^(m_.)/((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(3/
2), x_Symbol] :> Simp[-2*x^((n - 1)/2)*((b + 2*c*x)/((b^2 - 4*a*c)*Sqrt[a*x
^(n - 1) + b*x^n + c*x^(n + 1)])), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, 3
*((n - 1)/2)] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = -\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{x^2(a+x(b+cx))}}$$

```
[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
[Out] (-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
pseudoelliptic	$\frac{4cx+2b}{\sqrt{c\,x^{2}+bx+a}\,(4ac-b^{2})}$	33
gosper	$\frac{2(c\,x^{2}+bx+a)(2cx+b)x^{3}}{(4ac-b^{2})(c\,x^{4}+b\,x^{3}+a\,x^{2})^{\frac{3}{2}}}$	52
default	$\frac{2(c\,x^{2}+bx+a)(2cx+b)x^{3}}{(4ac-b^{2})(c\,x^{4}+b\,x^{3}+a\,x^{2})^{\frac{3}{2}}}$	52
trager	$\frac{2(2cx+b)\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}}{(c\,x^{2}+bx+a)x(4ac-b^{2})}$	54

```
[In] int(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/(c*x^2+b*x+a)^(1/2)*(2*c*x+b)/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.85

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{2\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)}{(b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x}$$

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] -2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)/((b^2*c - 4*a*c^2)*x^3 + (b^3 -
4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)
```

Sympy [F]

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^3}{(x^2(a+bx+cx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3/(c*x**4+b*x**3+a*x**2)**(3/2),x)
[Out] Integral(x**3/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^3}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(x^3/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.90

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2\sqrt{a}b\mathrm{sgn}(x)}{ab^2-4a^2c} - \frac{2\left(\frac{2cx}{b^2\mathrm{sgn}(x)-4ac\mathrm{sgn}(x)} + \frac{b}{b^2\mathrm{sgn}(x)-4ac\mathrm{sgn}(x)}\right)}{\sqrt{cx^2+bx+a}}$$

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] 2*sqrt(a)*b*sgn(x)/(a*b^2 - 4*a^2*c) - 2*(2*c*x/(b^2*sgn(x) - 4*a*c*sgn(x))
 + b/(b^2*sgn(x) - 4*a*c*sgn(x)))/sqrt(c*x^2 + b*x + a)
```

Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.92

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{\left(\frac{4c^2x}{4ac^2-b^2c} + \frac{2bc}{4ac^2-b^2c}\right)\sqrt{cx^4+bx^3+ax^2}}{x\,(cx^2+bx+a)}$$

```
[In] int(x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
[Out] (((4*c^2*x)/(4*a*c^2 - b^2*c) + (2*b*c)/(4*a*c^2 - b^2*c))*(a*x^2 + b*x^3 +
 c*x^4)^(1/2))/(x*(a + b*x + c*x^2))
```

3.60 $\int \frac{x^2}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \frac{x^2}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx=\frac{2x(b^2-2ac+bcx)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

```
[Out] -arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(3/2)+2*x*(b*
c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1936, 1918, 212}

$$\int \frac{x^2}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx=\frac{2x(-2ac+b^2+bcx)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

```
[In] Int[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) -
 ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/a^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1936

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Dist[(2*a*c - b^2*(p + 2))/(a*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(a*x^q
+ b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2
*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0
] && LtQ[p, -1] && RationalQ[m, p, q] && EqQ[m + p*q + 1, (-(n - q))*(2*p +
 3)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2x(b^2-2ac+bcx)}{a\,(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}+\frac{\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{a}\\ &= \frac{2x(b^2-2ac+bcx)}{a\,(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{a}\\ &= \frac{2x(b^2-2ac+bcx)}{a\,(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{2x\left(\sqrt{a}(b^2-2ac+bcx)+(b^2-4ac)\sqrt{a+x(b+cx)}\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)\right)}{a^{3/2}\,(-b^2+4ac)\sqrt{x^2(a+x(b+cx))}}$$

```
[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (-2*x*(Sqrt[a]*(b^2 - 2*a*c + b*c*x) + (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*
ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]]))/(a^(3/2)*(-b^2 + 4*a
*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$-\frac{4\left(-a^{\frac{3}{2}}c+\frac{b\sqrt{a}\,(cx+b)}{2}+\left(-\ln(2)+\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)\right)\sqrt{c\,x^{2}+bx+a}\,\left(ac-\frac{b^{2}}{4}\right)\right)}{\sqrt{c\,x^{2}+bx+a}\,a^{\frac{3}{2}}\,(4ac-b^{2})}$
default	$\frac{x^{3}\,(c\,x^{2}+bx+a)\left(-2a^{\frac{3}{2}}bcx+4a^{\frac{5}{2}}c-2a^{\frac{3}{2}}b^{2}-4\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)\sqrt{c\,x^{2}+bx+a}\,a^{2}c+\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)\right.}{(c\,x^{4}+b\,x^{3}+a\,x^{2})^{\frac{3}{2}}\,a^{\frac{5}{2}}\,(4ac-b^{2})}$

```
[In] int(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/(c*x^2+b*x+a)^(1/2)*(-a^(3/2)*c+1/2*b*a^(1/2)*(c*x+b)+(-ln(2)+ln((2*a+b*
x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))*(c*x^2+b*x+a)^(1/2)*(a*c-1/4*b
^2))/a^(3/2)/(4*a*c-b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(84) = 168.

Time = 0.31 (sec) , antiderivative size = 411, normalized size of antiderivative = 4.37

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \left[\frac{((b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x)\sqrt{a}\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3}{\cdots}\right.}{2\,((a^2b^2c-4\,a^3c^2)x^3+(a^2b^3-\cdots}\right.$$

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*s
qrt(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3
 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*c*
x + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^2*b^3 - 4*a^3*b*c)*
x^2 + (a^3*b^2 - 4*a^4*c)*x), (((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2
 + (a*b^2 - 4*a^2*c)*x)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*
x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x
^2)*(a*b*c*x + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^2*b^3 -
4*a^3*b*c)*x^2 + (a^3*b^2 - 4*a^4*c)*x)]
```

Sympy [F]

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^2}{(x^2(a+bx+cx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)
[Out] Integral(x**2/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^2}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(84) = 168$.

Time = 0.34 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.12

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{2\left(ab^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-4\,a^2c\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}b^2-2\sqrt{-a}a^{\frac{3}{2}}c\right)\mathrm{sgn}(x)}{\sqrt{-a}a^2b^2-4\sqrt{-a}a^3c} + \frac{2\left(\frac{abcx\mathrm{sgn}(x)}{a^2b^2-4\,a^3c}+\frac{ab^2\mathrm{sgn}(x)-2\,a^2c\mathrm{sgn}(x)}{a^2b^2-4\,a^3c}\right)}{\sqrt{cx^2+bx+a}} + \frac{2\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] -2*(a*b^2*arctan(sqrt(a)/sqrt(-a)) - 4*a^2*c*arctan(sqrt(a)/sqrt(-a)) + sqr
t(-a)*sqrt(a)*b^2 - 2*sqrt(-a)*a^(3/2)*c)*sgn(x)/(sqrt(-a)*a^2*b^2 - 4*sqrt
(-a)*a^3*c) + 2*(a*b*c*x*sgn(x)/(a^2*b^2 - 4*a^3*c) + (a*b^2*sgn(x) - 2*a^2
*c*sgn(x))/(a^2*b^2 - 4*a^3*c))/sqrt(c*x^2 + b*x + a) + 2*arctan(-(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^2}{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}\,dx$$

```
[In] int(x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
[Out] int(x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.61 $\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 144

$$\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{a^2(b^2-4ac)x^2} + \frac{3b\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{5/2}}$$

```
[Out] 3/2*b*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(5/2)+2*
(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)-(-8*a*c+3*b^2)*(
c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4*a*c+b^2)/x^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1938, 1965, 12, 1918, 212}

$$\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{3b\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{5/2}} - \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{a^2x^2(b^2-4ac)} + \frac{2(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

```
[In] Int[x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (
(3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a^2*(b^2 - 4*a*c)*x^2) + (3*b
```

```
*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(5/
2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1938

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n
- q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
+ (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
q] && LtQ[m + p*q + 1, n - q]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{-\frac{3b^2}{2}+4ac-bcx}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{a^2\left(b^2-4ac\right)x^2} + \frac{2\int -\frac{3b(b^2-4ac)}{4\sqrt{ax^2+bx^3+cx^4}}\,dx}{a^2\left(b^2-4ac\right)}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{a^2\left(b^2-4ac\right)x^2} - \frac{(3b)\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2a^2}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{a^2\left(b^2-4ac\right)x^2} + \frac{(3b)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{a^2}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{a^2\left(b^2-4ac\right)x^2} + \frac{3b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.93

$$\int \frac{x}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{\sqrt{a}\left(-4a^2c+3b^2x(b+cx)+a\left(b^2-10bcx-8c^2x^2\right)\right)+3b\left(b^2-4ac\right)x\sqrt{a+x(b}}{a^{5/2}\left(-b^2+4ac\right)\sqrt{x^2(a+x(b+cx))}}$$

```
[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a]*(-4*a^2*c + 3*b^2*x*(b + c*x) + a*(b^2 - 10*b*c*x - 8*c^2*x^2)) +
3*b*(b^2 - 4*a*c)*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(
b + c*x)])/Sqrt[a]])/(a^(5/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$\frac{\frac{\left(-4c^{2}x^{2}-5bcx+\frac{1}{2}b^{2}\right)a^{\frac{3}{2}}}{2}-a^{\frac{5}{2}}c+\frac{3x\left(\frac{b\sqrt{a}\,(cx+b)}{2}+\left(-\ln(2)+\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)\right)\sqrt{c\,x^{2}+bx+a}\,\left(ac-\frac{b^{2}}{4}\right)\right)b}{2}}{\sqrt{c\,x^{2}+bx+a}\,a^{\frac{5}{2}}x\left(ac-\frac{b^{2}}{4}\right)}$
default	$-\frac{x^{2}\left(c\,x^{2}+bx+a\right)\left(16a^{\frac{5}{2}}c^{2}x^{2}-6a^{\frac{3}{2}}b^{2}c\,x^{2}+20a^{\frac{5}{2}}bcx-6a^{\frac{3}{2}}b^{3}x-12\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)\sqrt{c\,x^{2}+bx+a}\,a^{2}bcx+3\ln\left(\right.}{2(c\,x^{4}+b\,x^{3}+a\,x^{2})^{\frac{3}{2}}a^{\frac{7}{2}}(4ac-b^{2})}$
risch	$-\frac{c\,x^{2}+bx+a}{a^{2}\sqrt{x^{2}(c\,x^{2}+bx+a)}}+\frac{\left(\frac{2b^{2}cx}{a^{2}\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}+\frac{b^{3}}{a^{2}\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}-\frac{4c^{2}x}{a\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}-\frac{2cb}{a\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+b}}\right.}{\sqrt{x^{2}(c\,x^{2}+bx+a)}}$

```
[In] int(x/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2/(c*x^2+b*x+a)^(1/2)/a^(5/2)*(1/3*(-4*c^2*x^2-5*b*c*x+1/2*b^2)*a^(3/2)-2
/3*a^(5/2)*c+x*(1/2*b*a^(1/2)*(c*x+b)+(-ln(2)+ln((2*a+b*x+2*a^(1/2)*(c*x^2+
b*x+a)^(1/2))/x/a^(1/2)))*(c*x^2+b*x+a)^(1/2)*(a*c-1/4*b^2))*b)/x/(a*c-1/4*
b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 496, normalized size of antiderivative = 3.44

$$\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \left[\frac{3\,((b^3c-4\,abc^2)x^4+(b^4-4\,ab^2c)x^3+(ab^3-4\,a^2bc)x^2)\sqrt{a}\log\left(-\frac{8\,abx^2+(b^2+}{}\right.}{4\,((a^3b^2c-}\right.$$

$$-\frac{3\,((b^3c-4\,abc^2)x^4+(b^4-4\,ab^2c)x^3+(ab^3-4\,a^2bc)x^2)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\sqrt{cx^4+}}{2\,((a^3b^2c-4\,a^4c^2)x^4+(a^3b^3-4\,a^4bc)x^3+(a^4b^2}$$

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*x^3 + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x
^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^
2)*(a^2*b^2 - 4*a^3*c + (3*a*b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3 - 10*a^2*b*c
)*x))/((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 + (a^4*b^2 -
 4*a^5*c)*x^2), -1/2*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*x^3 +
(a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b
*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*
```

```
x^2)*(a^2*b^2 - 4*a^3*c + (3*a*b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3 - 10*a^2*b
*c)*x))/((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 + (a^4*b^2
 - 4*a^5*c)*x^2)]
```

Sympy [F]

$$\int \frac{x}{(ax^2 + bx^3 + cx^4)^{3/2}}\,dx = \int \frac{x}{(x^2(a + bx + cx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
[Out] Integral(x/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(ax^2 + bx^3 + cx^4)^{3/2}}\,dx = \int \frac{x}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(x/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{(ax^2 + bx^3 + cx^4)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(ax^2 + bx^3 + cx^4)^{3/2}}\,dx = \int \frac{x}{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}\,dx$$

```
[In] int(x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
[Out] int(x/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.62 $\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 209

$$\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\,x\sqrt{ax^2+bx^3+cx^4}} - \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2a^2(b^2-4ac)\,x^3} + \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4a^3(b^2-4ac)\,x^2} - \frac{3(5b^2-4ac)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{7/2}}$$

```
[Out] -3/8*(-4*a*c+5*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/
2))/a^(7/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^3+a*x^2)^(1/2)-
1/2*(-12*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4*a*c+b^2)/x^3+1/4*b*(-
52*a*c+15*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/(-4*a*c+b^2)/x^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1921, 1965, 12, 1918, 212}

$$\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{3(5b^2-4ac)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{7/2}} + \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4a^3x^2(b^2-4ac)} - \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2a^2x^3(b^2-4ac)} + \frac{2(-2ac+b^2+bcx)}{ax(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(-3/2),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4]) -
 ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*x^3) +
 (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^3*(b^2 - 4*a*c)*x^2
) - (3*(5*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^
3 + c*x^4])])/(8*a^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1921

```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Simp[(-x^(-q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q + b*x^n + c*x
^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*(n -
 q)*(p + 1)*(b^2 - 4*a*c)), Int[(((p*q + 1)*(b^2 - 2*a*c) + (n - q)*(p + 1)
*(b^2 - 4*a*c) + b*c*(p*q + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^
n + c*x^(2*n - q))^(p + 1))/x^q, x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ
[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && LtQ[
p, -1]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
```

```
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)x\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{-2(b^2-2ac)+\frac{1}{2}(-b^2+4ac)-2bcx}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx}{a(b^2-4ac)} \\
&= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)x\sqrt{ax^2+bx^3+cx^4}} - \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2a^2(b^2-4ac)x^3} \\
&\quad + \frac{\int \frac{-\frac{1}{4}b(15b^2-52ac)-\frac{1}{2}c(5b^2-12ac)x}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{a^2(b^2-4ac)} \\
&= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)x\sqrt{ax^2+bx^3+cx^4}} - \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2a^2(b^2-4ac)x^3} \\
&\quad + \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4a^3(b^2-4ac)x^2} - \frac{\int -\frac{3(b^2-4ac)(5b^2-4ac)}{8\sqrt{ax^2+bx^3+cx^4}}\,dx}{a^3(b^2-4ac)} \\
&= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)x\sqrt{ax^2+bx^3+cx^4}} - \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2a^2(b^2-4ac)x^3} \\
&\quad + \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4a^3(b^2-4ac)x^2} + \frac{(3(5b^2-4ac))\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8a^3} \\
&= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)x\sqrt{ax^2+bx^3+cx^4}} - \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2a^2(b^2-4ac)x^3} \\
&\quad + \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4a^3(b^2-4ac)x^2} \\
&\quad - \frac{(3(5b^2-4ac))\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{4a^3} \\
&= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)x\sqrt{ax^2+bx^3+cx^4}} - \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2a^2(b^2-4ac)x^3} \\
&\quad + \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4a^3(b^2-4ac)x^2} - \frac{3(5b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx =$$

$$-\frac{\sqrt{a}(-8a^3c-15b^3x^2(b+cx)+2a^2(b^2+10bcx-12c^2x^2)+abx(-5b^2+62bcx+52c^2x^2))-3(5b^4-24ab^2c \cdots}{4a^{7/2}\left(b^2-4ac\right)x\sqrt{x^2(a+x(b+cx))}}$$

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-3/2),x]

[Out] -1/4*(Sqrt[a]*(-8*a^3*c - 15*b^3*x^2*(b + c*x) + 2*a^2*(b^2 + 10*b*c*x - 12 *c^2*x^2) + a*b*x*(-5*b^2 + 62*b*c*x + 52*c^2*x^2)) - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x^2*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(a^(7/2)*(b^2 - 4*a*c)*x*Sqrt[x^2*(a + x*(b + c*x))])

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{-\frac{5xb\left(-\frac{52}{5}c^2x^2-\frac{62}{5}bcx+b^2\right)a^{\frac{3}{2}}}{4}+6\left(-c^2x^2+\frac{5}{6}bcx+\frac{1}{12}b^2\right)a^{\frac{5}{2}}-2a^{\frac{7}{2}}c+6\left(-\frac{5b^3(cx+b)\sqrt{a}}{8}+\sqrt{cx^2+bx+a}\left(-\ln(2)+\ln\left(\frac{2a+bx+2\sqrt{\cdots}}{\cdots}\right)\cdots\right)\right)}{a^{\frac{7}{2}}\sqrt{cx^2+bx+a}\,(4ac-b^2)x^2}$
default	$-\frac{x(cx^2+bx+a)\left(-104a^{\frac{5}{2}}bc^2x^3+30a^{\frac{3}{2}}b^3cx^3+48a^{\frac{7}{2}}c^2x^2-124a^{\frac{5}{2}}b^2cx^2+30a^{\frac{3}{2}}b^4x^2-48\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)\sqrt{cx^2\cdots}\right)}{\cdots}$
risch	$-\frac{(cx^2+bx+a)(-7bx+2a)}{4a^3x\sqrt{x^2(cx^2+bx+a)}}+\frac{\left(-\frac{2b^3cx}{a^3\left(4ac-b^2\right)\sqrt{cx^2+bx+a}}-\frac{b^4}{a^3\left(4ac-b^2\right)\sqrt{cx^2+bx+a}}+\frac{6c^2bx}{a^2\left(4ac-b^2\right)\sqrt{cx^2+bx+a}}+\frac{3cb\cdots}{a^2\left(4ac-b^2\right)\cdots}\right)}{\cdots}$

[In] int(1/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 6/(c*x^2+b*x+a)^(1/2)*(-5/24*x*b*(-52/5*c^2*x^2-62/5*b*c*x+b^2)*a^(3/2)+(-c^2*x^2+5/6*b*c*x+1/12*b^2)*a^(5/2)-1/3*a^(7/2)*c+(-5/8*b^3*(c*x+b)*a^(1/2)+(c*x^2+b*x+a)^(1/2)*(-ln(2)+ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))*(a*c-5/4*b^2)*(a*c-1/4*b^2))*x^2)/a^(7/2)/(4*a*c-b^2)/x^2

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 630, normalized size of antiderivative = 3.01

$$\int \frac{1}{(ax^2 + bx^3 + cx^4)^{3/2}} \, dx = \left[-\frac{3\,((5\,b^4c - 24\,ab^2c^2 + 16\,a^2c^3)x^5 + (5\,b^5 - 24\,ab^3c + 16\,a^2bc^2)x^4 + (5\,ab^4 - \right.$$

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^5 + (5*b^5 - 24*a*b^3*c
+ 16*a^2*b*c^2)*x^4 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^3)*sqrt(a)*lo
g(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)
*(b*x + 2*a)*sqrt(a))/x^3) + 4*(2*a^3*b^2 - 8*a^4*c - (15*a*b^3*c - 52*a^2*
b*c^2)*x^3 - (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^2 - 5*(a^2*b^3 - 4*a^
3*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^4*b^2*c - 4*a^5*c^2)*x^5 + (a^4*
b^3 - 4*a^5*b*c)*x^4 + (a^5*b^2 - 4*a^6*c)*x^3), 1/8*(3*((5*b^4*c - 24*a*b^
2*c^2 + 16*a^2*c^3)*x^5 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^4 + (5*a*b^
4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3
+ a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(2*a^3*b^2 -
 8*a^4*c - (15*a*b^3*c - 52*a^2*b*c^2)*x^3 - (15*a*b^4 - 62*a^2*b^2*c + 24*
a^3*c^2)*x^2 - 5*(a^2*b^3 - 4*a^3*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^
4*b^2*c - 4*a^5*c^2)*x^5 + (a^4*b^3 - 4*a^5*b*c)*x^4 + (a^5*b^2 - 4*a^6*c)*
x^3)]
```

Sympy [F]

$$\int \frac{1}{(ax^2 + bx^3 + cx^4)^{3/2}} \, dx = \int \frac{1}{(ax^2 + bx^3 + cx^4)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral((a*x**2 + b*x**3 + c*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2 + bx^3 + cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(-3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(ax^2 + bx^3 + cx^4)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ax^2 + bx^3 + cx^4)^{3/2}}\,dx = \int \frac{1}{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}\,dx$$

```
[In] int(1/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
[Out] int(1/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.63 $\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 271

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx=\frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}}-\frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4}+\frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{12a^3\left(b^2-4ac\right)x^3}-\frac{\left(105b^4-460ab^2c+256a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{24a^4\left(b^2-4ac\right)x^2}+\frac{5b(7b^2-12ac)\,\mathrm{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{9/2}}$$

```
[Out] 5/16*b*(-12*a*c+7*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^
(1/2))/a^(9/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^3+a*x^2)^(
1/2)-1/3*(-16*a*c+7*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4*a*c+b^2)/x^4+1/1
2*b*(-116*a*c+35*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/(-4*a*c+b^2)/x^3-1/24*(
256*a^2*c^2-460*a*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^4/(-4*a*c+b^2)
/x^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used

$= \{1938, 1965, 12, 1918, 212\}$

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{5b(7b^2-12ac)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{9/2}} + \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{12a^3x^3\left(b^2-4ac\right)} - \frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3a^2x^4\left(b^2-4ac\right)} - \frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{24a^4x^2\left(b^2-4ac\right)} + \frac{2(-2ac+b^2+bcx)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

```
[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4])
 - ((7*b^2 - 16*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*a^2*(b^2 - 4*a*c)*x^4)
 + (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(12*a^3*(b^2 - 4*a*c)
*x^3) - ((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
/(24*a^4*(b^2 - 4*a*c)*x^2) + (5*b*(7*b^2 - 12*a*c)*ArcTanh[(x*(2*a + b*x))
/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(16*a^(9/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1938

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n
- q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
+ (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
```

```
q] && LtQ[m + p*q + 1, n - q]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int\frac{-\frac{7b^2}{2}+8ac-3bcx}{x^3\sqrt{ax^2+bx^3+cx^4}}\,dx}{a\left(b^2-4ac\right)}\\
&= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4}\\
&\quad+\frac{2\int\frac{-\frac{1}{4}b\left(35b^2-116ac\right)-c\left(7b^2-16ac\right)x}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx}{3a^2\left(b^2-4ac\right)}\\
&= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4}\\
&\quad+\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{12a^3\left(b^2-4ac\right)x^3}-\frac{\int\frac{\frac{1}{8}\left(-105b^4+460ab^2c-256a^2c^2\right)-\frac{1}{4}bc\left(35b^2-116ac\right)x}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{3a^3\left(b^2-4ac\right)}\\
&= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4}\\
&\quad+\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{12a^3\left(b^2-4ac\right)x^3}\\
&\quad-\frac{\left(105b^4-460ab^2c+256a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{24a^4\left(b^2-4ac\right)x^2}+\frac{\int-\frac{15b\left(7b^2-12ac\right)\left(b^2-4ac\right)}{16\sqrt{ax^2+bx^3+cx^4}}\,dx}{3a^4\left(b^2-4ac\right)}
\end{aligned}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{12a^3\left(b^2-4ac\right)x^3} - \frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{24a^4\left(b^2-4ac\right)x^2} - \frac{(5b(7b^2-12ac))\int\frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{16a^4}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{12a^3\left(b^2-4ac\right)x^3} - \frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{24a^4\left(b^2-4ac\right)x^2} + \frac{(5b(7b^2-12ac))\,\text{Subst}\left(\int\frac{1}{4a-x^2}\,dx,x,\frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^4}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{12a^3\left(b^2-4ac\right)x^3} - \frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{24a^4\left(b^2-4ac\right)x^2} + \frac{5b(7b^2-12ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{9/2}}$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.83

$$\int\frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{\sqrt{a}(-32a^4c+105b^4x^3(b+cx)+5ab^2x^2(7b^2-106bcx-92c^2x^2)+8a^3(b^2+7$$

```
[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a]*(-32*a^4*c + 105*b^4*x^3*(b + c*x) + 5*a*b^2*x^2*(7*b^2 - 106*b*c*
x - 92*c^2*x^2) + 8*a^3*(b^2 + 7*b*c*x + 16*c^2*x^2) + 2*a^2*x*(-7*b^3 - 86
*b^2*c*x + 244*b*c^2*x^2 + 128*c^3*x^3)) + 15*b*(7*b^4 - 40*a*b^2*c + 48*a^
2*c^2)*x^3*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)]
)/Sqrt[a]])/(24*a^(9/2)*(-b^2 + 4*a*c)*x^2*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$-\frac{15\left(-\frac{7x^{2}b^{2}\left(-\frac{92}{7}c^{2}x^{2}-\frac{106}{7}bcx+b^{2}\right)a^{\frac{3}{2}}}{72}+\frac{7x\left(-\frac{128}{7}c^{3}x^{3}-\frac{244}{7}bc^{2}x^{2}+\frac{86}{7}b^{2}cx+b^{3}\right)a^{\frac{5}{2}}}{180}+\frac{\left(-16c^{2}x^{2}-7bcx-b^{2}\right)a^{\frac{7}{2}}}{45}+\frac{4a^{\frac{9}{2}}c}{45}+\left(-\frac{7b}{}\right.\right.}{\sqrt{cx^{2}+bx+a}\,a^{\frac{9}{2}}\left(4ac-b^{2}\right)x}$
default	$-\frac{\left(cx^{2}+bx+a\right)\left(-512a^{\frac{7}{2}}c^{3}x^{4}+920a^{\frac{5}{2}}b^{2}c^{2}x^{4}-210a^{\frac{3}{2}}b^{4}cx^{4}-976a^{\frac{7}{2}}bc^{2}x^{3}+1060a^{\frac{5}{2}}b^{3}cx^{3}-210a^{\frac{3}{2}}b^{5}x^{3}+720\ln\left(\frac{2a+bx+2\sqrt{}}{}\right.\right.}{}$
risch	$-\frac{\left(cx^{2}+bx+a\right)\left(-40acx^{2}+57b^{2}x^{2}-22abx+8a^{2}\right)}{24a^{4}x^{2}\sqrt{x^{2}\left(cx^{2}+bx+a\right)}}+\frac{\left(\frac{2b^{4}cx}{a^{4}\left(4ac-b^{2}\right)\sqrt{cx^{2}+bx+a}}+\frac{b^{5}}{a^{4}\left(4ac-b^{2}\right)\sqrt{cx^{2}+bx+a}}+\frac{4c^{3}x}{a^{2}\left(4ac-b^{2}\right)\sqrt{cx}}\right.}{}$

```
[In] int(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -15*(-7/72*x^2*b^2*(-92/7*c^2*x^2-106/7*b*c*x+b^2)*a^(3/2)+7/180*x*(-128/7*
c^3*x^3-244/7*b*c^2*x^2+86/7*b^2*c*x+b^3)*a^(5/2)+1/45*(-16*c^2*x^2-7*b*c*x
-b^2)*a^(7/2)+4/45*a^(9/2)*c+(-7/24*b^3*(c*x+b)*a^(1/2)+(c*x^2+b*x+a)^(1/2)
*(-ln(2)+ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))*(a*c-7/12*b
^2)*(a*c-1/4*b^2))*x^3*b)/(c*x^2+b*x+a)^(1/2)/a^(9/2)/(4*a*c-b^2)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 716, normalized size of antiderivative = 2.64

$$\int \frac{1}{x\left(ax^{2}+bx^{3}+cx^{4}\right)^{3/2}}\,dx=\left[-\frac{15\left(\left(7\,b^{5}c-40\,ab^{3}c^{2}+48\,a^{2}bc^{3}\right)x^{6}+\left(7\,b^{6}-40\,ab^{4}c+48\,a^{2}b^{2}c^{2}\right)x^{5}+\left(7\,a\right.\right.}{}\right.$$

$$-\frac{15\left(\left(7\,b^{5}c-40\,ab^{3}c^{2}+48\,a^{2}bc^{3}\right)x^{6}+\left(7\,b^{6}-40\,ab^{4}c+48\,a^{2}b^{2}c^{2}\right)x^{5}+\left(7\,ab^{5}-40\,a^{2}b^{3}c+48\,a^{3}bc^{2}\right)x^{4}\right)\sqrt{-a}}{}$$

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6 - 40*a*b^4
*c + 48*a^2*b^2*c^2)*x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^4)*sqr
t(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*(8*a^4*b^2 - 32*a^5*c + (105*a*b^4*c
- 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^4 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3
*b*c^2)*x^3 + (35*a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^2 - 14*(a^3*b^3
- 4*a^4*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^5*b^2*c - 4*a^6*c^2)*x^6 +
```

```
(a^5*b^3 - 4*a^6*b*c)*x^5 + (a^6*b^2 - 4*a^7*c)*x^4), -1/48*(15*((7*b^5*c
- 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*
x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^4)*sqrt(-a)*arctan(1/2*sqrt
(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) +
 2*(8*a^4*b^2 - 32*a^5*c + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^
4 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^3 + (35*a^2*b^4 - 172*a^3
*b^2*c + 128*a^4*c^2)*x^2 - 14*(a^3*b^3 - 4*a^4*b*c)*x)*sqrt(c*x^4 + b*x^3
+ a*x^2))/((a^5*b^2*c - 4*a^6*c^2)*x^6 + (a^5*b^3 - 4*a^6*b*c)*x^5 + (a^6*b
^2 - 4*a^7*c)*x^4)]
```

Sympy [F]

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx=\int \frac{1}{x\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
[Out] Integral(1/(x*(x**2*(a + b*x + c*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx=\int \frac{1}{x\,{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}} \,d x$$

```
[In] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x)
[Out] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)), x)
```

3.64 $\int \frac{1}{x^2(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 343

$$\int \frac{1}{x^2(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\,x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{(9b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{4a^2(b^2-4ac)\,x^5} + \frac{b(21b^2-68ac)\sqrt{ax^2+bx^3+cx^4}}{8a^3(b^2-4ac)\,x^4} - \frac{(105b^4-448ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{32a^4(b^2-4ac)\,x^3} + \frac{b(315b^4-1680ab^2c+1808a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{64a^5(b^2-4ac)\,x^2} - \frac{15(21b^4-56ab^2c+16a^2c^2)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{11/2}}$$

```
[Out] -15/128*(16*a^2*c^2-56*a*b^2*c+21*b^4)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x
^4+b*x^3+a*x^2)^(1/2))/a^(11/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^3/(c*x
^4+b*x^3+a*x^2)^(1/2)-1/4*(-20*a*c+9*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4
*a*c+b^2)/x^5+1/8*b*(-68*a*c+21*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/(-4*a*c+
b^2)/x^4-1/32*(240*a^2*c^2-448*a*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/a
^4/(-4*a*c+b^2)/x^3+1/64*b*(1808*a^2*c^2-1680*a*b^2*c+315*b^4)*(c*x^4+b*x^3
+a*x^2)^(1/2)/a^5/(-4*a*c+b^2)/x^2
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1938, 1965, 12, 1918, 212}

$$\int \frac{1}{x^2 (ax^2+bx^3+cx^4)^{3/2}} \, dx = \frac{b(21b^2-68ac)\sqrt{ax^2+bx^3+cx^4}}{8a^3x^4(b^2-4ac)} - \frac{(9b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{4a^2x^5(b^2-4ac)} - \frac{15(16a^2c^2-56ab^2c+21b^4)\,\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{11/2}} + \frac{b(1808a^2c^2-1680ab^2c+315b^4)\sqrt{ax^2+bx^3+cx^4}}{64a^5x^2(b^2-4ac)} - \frac{(240a^2c^2-448ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{32a^4x^3(b^2-4ac)} + \frac{2(-2ac+b^2+bcx)}{ax^3(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

```
[In] Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^3*Sqrt[a*x^2 + b*x^3 + c*x^4])
 - ((9*b^2 - 20*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^2*(b^2 - 4*a*c)*x^5)
 + (b*(21*b^2 - 68*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*a^3*(b^2 - 4*a*c)*x
^4) - ((105*b^4 - 448*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(
32*a^4*(b^2 - 4*a*c)*x^3) + (b*(315*b^4 - 1680*a*b^2*c + 1808*a^2*c^2)*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(64*a^5*(b^2 - 4*a*c)*x^2) - (15*(21*b^4 - 56*a*b^
2*c + 16*a^2*c^2)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c
*x^4])])/(128*a^(11/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
```

```
- 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1938

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n
- q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
+ (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
q] && LtQ[m + p*q + 1, n - q]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int\frac{-\frac{9b^2}{2}+10ac-4bcx}{x^4\sqrt{ax^2+bx^3+cx^4}}\,dx}{a\left(b^2-4ac\right)}\\
&= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5}\\
&\quad+\frac{\int\frac{-\frac{3}{4}b\left(21b^2-68ac\right)-\frac{3}{2}c\left(9b^2-20ac\right)x}{x^3\sqrt{ax^2+bx^3+cx^4}}\,dx}{2a^2\left(b^2-4ac\right)}\\
&= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5}\\
&\quad+\frac{b\left(21b^2-68ac\right)\sqrt{ax^2+bx^3+cx^4}}{8a^3\left(b^2-4ac\right)x^4}-\frac{\int\frac{-\frac{3}{8}\left(105b^4-448ab^2c+240a^2c^2\right)-\frac{3}{2}bc\left(21b^2-68ac\right)x}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx}{6a^3\left(b^2-4ac\right)}
\end{aligned}
$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{(9b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5}$$
$$+ \frac{b(21b^2-68ac)\sqrt{ax^2+bx^3+cx^4}}{8a^3\left(b^2-4ac\right)x^4}$$
$$- \frac{(105b^4-448ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{32a^4\left(b^2-4ac\right)x^3}$$
$$+ \frac{\int \frac{-\frac{3}{16}b(315b^4-1680ab^2c+1808a^2c^2)-\frac{3}{8}c(105b^4-448ab^2c+240a^2c^2)x}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{12a^4\left(b^2-4ac\right)}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{(9b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5}$$
$$+ \frac{b(21b^2-68ac)\sqrt{ax^2+bx^3+cx^4}}{8a^3\left(b^2-4ac\right)x^4}$$
$$- \frac{(105b^4-448ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{32a^4\left(b^2-4ac\right)x^3}$$
$$+ \frac{b(315b^4-1680ab^2c+1808a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{64a^5\left(b^2-4ac\right)x^2}$$
$$- \frac{\int -\frac{45(b^2-4ac)(21b^4-56ab^2c+16a^2c^2)}{32\sqrt{ax^2+bx^3+cx^4}}\,dx}{12a^5\left(b^2-4ac\right)}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{(9b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5}$$
$$+ \frac{b(21b^2-68ac)\sqrt{ax^2+bx^3+cx^4}}{8a^3\left(b^2-4ac\right)x^4}$$
$$- \frac{(105b^4-448ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{32a^4\left(b^2-4ac\right)x^3}$$
$$+ \frac{b(315b^4-1680ab^2c+1808a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{64a^5\left(b^2-4ac\right)x^2}$$
$$+ \frac{(15(21b^4-56ab^2c+16a^2c^2))\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{128a^5}$$

$$= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{(9b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{4a^2(b^2-4ac)x^5} + \frac{b(21b^2-68ac)\sqrt{ax^2+bx^3+cx^4}}{8a^3(b^2-4ac)x^4} - \frac{(105b^4-448ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{32a^4(b^2-4ac)x^3} + \frac{b(315b^4-1680ab^2c+1808a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{64a^5(b^2-4ac)x^2} - \frac{(15(21b^4-56ab^2c+16a^2c^2))\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{64a^5}$$

$$= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{(9b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{4a^2(b^2-4ac)x^5} + \frac{b(21b^2-68ac)\sqrt{ax^2+bx^3+cx^4}}{8a^3(b^2-4ac)x^4} - \frac{(105b^4-448ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{32a^4(b^2-4ac)x^3} + \frac{b(315b^4-1680ab^2c+1808a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{64a^5(b^2-4ac)x^2} - \frac{15(21b^4-56ab^2c+16a^2c^2)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{11/2}}$$

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^2(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{\sqrt{a}(-64a^5c-315b^5x^4(b+cx)-105ab^3x^3(b^2-18bcx-16c^2x^2)+16a^4(b^2+$$

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a]*(-64*a^5*c - 315*b^5*x^4*(b + c*x) - 105*a*b^3*x^3*(b^2 - 18*b*c*x
 - 16*c^2*x^2) + 16*a^4*(b^2 + 6*b*c*x + 10*c^2*x^2) + 2*a^2*b*x^2*(21*b^3
+ 308*b^2*c*x - 1352*b*c^2*x^2 - 904*c^3*x^3) - 8*a^3*x*(3*b^3 + 26*b^2*c*x
 + 98*b*c^2*x^2 - 60*c^3*x^3)) - 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2
 - 64*a^3*c^3)*x^4*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b
 + c*x)])/Sqrt[a]])/(64*a^(11/2)*(-b^2 + 4*a*c)*x^3*Sqrt[x^2*(a + x*(b + c*
x))])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$-\frac{15\left(\frac{2a^{\frac{11}{2}}c}{15}+\frac{7b^{3}x^{3}\left(-16c^{2}x^{2}-18bcx+b^{2}\right)a^{\frac{3}{2}}}{32}-\frac{7\left(-\frac{904}{21}c^{3}x^{3}-\frac{1352}{21}b\,c^{2}x^{2}+\frac{44}{3}b^{2}cx+b^{3}\right)x^{2}b\,a^{\frac{5}{2}}}{80}+\frac{x\left(-20c^{3}x^{3}+\frac{98}{3}b\,c^{2}x^{2}+\frac{26}{3}b^{2}cx+\cdots\right.}{20}\cdots\right.}{8a\cdots}$
risch	$-\frac{\left(c\,x^{2}+bx+a\right)\left(292abc\,x^{3}-187b^{3}x^{3}-56a^{2}c\,x^{2}+82a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)}{64a^{5}x^{3}\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}+\frac{\left(\frac{374b^{5}(2cx+b)}{\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}-\frac{288a^{2}b\,c^{2}(2cx+b)}{\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+\cdots}}\cdots\right.}{\cdots}$
default	$-\frac{\left(c\,x^{2}+bx+a\right)\left(3616a^{\frac{7}{2}}b\,c^{3}x^{5}-3360a^{\frac{5}{2}}b^{3}c^{2}x^{5}+630a^{\frac{3}{2}}b^{5}c\,x^{5}-960a^{\frac{9}{2}}c^{3}x^{4}+5408a^{\frac{7}{2}}b^{2}c^{2}x^{4}-3780a^{\frac{5}{2}}b^{4}c\,x^{4}+630a^{\frac{3}{2}}b^{6}x^{4}+\cdots\right.}{\cdots}$

```
[In] int(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -15/8/a^(11/2)*(2/15*a^(11/2)*c+7/32*b^3*x^3*(-16*c^2*x^2-18*b*c*x+b^2)*a^(
3/2)-7/80*(-904/21*c^3*x^3-1352/21*b*c^2*x^2+44/3*b^2*c*x+b^3)*x^2*b*a^(5/2
)+1/20*x*(-20*c^3*x^3+98/3*b*c^2*x^2+26/3*b^2*c*x+b^3)*a^(7/2)+(-1/5*b*c*x-
1/3*c^2*x^2-1/30*b^2)*a^(9/2)+(21/32*b^5*(c*x+b)*a^(1/2)+(-ln(2)+ln((2*a+b*
x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))*(a^2*c^2-7/2*a*b^2*c+21/16*b^4
)*(c*x^2+b*x+a)^(1/2)*(a*c-1/4*b^2))*x^4)/(c*x^2+b*x+a)^(1/2)/x^4/(a*c-1/4*
b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 866, normalized size of antiderivative = 2.52

$$\int \frac{1}{x^{2}\left(ax^{2}+bx^{3}+cx^{4}\right)^{3/2}}\,dx=\left[\frac{15\left(\left(21\,b^{6}c-140\,ab^{4}c^{2}+240\,a^{2}b^{2}c^{3}-64\,a^{3}c^{4}\right)x^{7}+\left(21\,b^{7}-140\,ab^{5}c+24\cdots\right.\right.}{\cdots}\right.$$

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(15*((21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*x^7 +
 (21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*x^6 + (21*a*b^6 -
140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*x^5)*sqrt(a)*log(-(8*a*b*x^2
+ (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*s
qrt(a))/x^3) - 4*(16*a^5*b^2 - 64*a^6*c - (315*a*b^5*c - 1680*a^2*b^3*c^2 +
 1808*a^3*b*c^3)*x^5 - (315*a*b^6 - 1890*a^2*b^4*c + 2704*a^3*b^2*c^2 - 480
*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*b^3*c + 112*a^4*b*c^2)*x^3 + 2*(21*a
^3*b^4 - 104*a^4*b^2*c + 80*a^5*c^2)*x^2 - 24*(a^4*b^3 - 4*a^5*b*c)*x)*sqrt
```

```
(c*x^4 + b*x^3 + a*x^2))/((a^6*b^2*c - 4*a^7*c^2)*x^7 + (a^6*b^3 - 4*a^7*b*
c)*x^6 + (a^7*b^2 - 4*a^8*c)*x^5), 1/128*(15*((21*b^6*c - 140*a*b^4*c^2 + 2
40*a^2*b^2*c^3 - 64*a^3*c^4)*x^7 + (21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2
- 64*a^3*b*c^3)*x^6 + (21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*
c^3)*x^5)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(
-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(16*a^5*b^2 - 64*a^6*c - (315*a*b^5*c
- 1680*a^2*b^3*c^2 + 1808*a^3*b*c^3)*x^5 - (315*a*b^6 - 1890*a^2*b^4*c + 27
04*a^3*b^2*c^2 - 480*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*b^3*c + 112*a^4*
b*c^2)*x^3 + 2*(21*a^3*b^4 - 104*a^4*b^2*c + 80*a^5*c^2)*x^2 - 24*(a^4*b^3
- 4*a^5*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^6*b^2*c - 4*a^7*c^2)*x^7 +
 (a^6*b^3 - 4*a^7*b*c)*x^6 + (a^7*b^2 - 4*a^8*c)*x^5)]
```

Sympy [F]

$$\int \frac{1}{x^2 (ax^2 + bx^3 + cx^4)^{3/2}}\, dx = \int \frac{1}{x^2 \left(x^2 (a + bx + cx^2)\right)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)
[Out] Integral(1/(x**2*(x**2*(a + b*x + c*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (ax^2 + bx^3 + cx^4)^{3/2}}\, dx = \int \frac{1}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}} x^2}\, dx$$

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (ax^2 + bx^3 + cx^4)^{3/2}}\, dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}\,dx$$

```
[In] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x)
[Out] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)), x)
```

3.65 $\int x^m(ax + bx^3 + cx^5)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int x^m\left(ax + bx^3 + cx^5\right)\, dx = \frac{ax^{2+m}}{2+m} + \frac{bx^{4+m}}{4+m} + \frac{cx^{6+m}}{6+m}$$

```
[Out] a*x^(2+m)/(2+m)+b*x^(4+m)/(4+m)+c*x^(6+m)/(6+m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\int x^m\left(ax + bx^3 + cx^5\right)\, dx = \frac{ax^{m+2}}{m+2} + \frac{bx^{m+4}}{m+4} + \frac{cx^{m+6}}{m+6}$$

```
[In] Int[x^m*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^(2 + m))/(2 + m) + (b*x^(4 + m))/(4 + m) + (c*x^(6 + m))/(6 + m)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ax^{1+m} + bx^{3+m} + cx^{5+m}\right)\, dx \\ &= \frac{ax^{2+m}}{2+m} + \frac{bx^{4+m}}{4+m} + \frac{cx^{6+m}}{6+m}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x^m\left(ax+bx^3+cx^5\right)\,dx=x^{2+m}\left(\frac{a}{2+m}+\frac{bx^2}{4+m}+\frac{cx^4}{6+m}\right)$$

```
[In] Integrate[x^m*(a*x + b*x^3 + c*x^5),x]
[Out] x^(2 + m)*(a/(2 + m) + (b*x^2)/(4 + m) + (c*x^4)/(6 + m))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

method	result	size
norman	$\frac{a\,x^{2} {\mathrm e}^{m \ln \left(x \right)}}{2+m}+\frac{b\,x^{4} {\mathrm e}^{m \ln \left(x \right)}}{4+m}+\frac{c\,x^{6} {\mathrm e}^{m \ln \left(x \right)}}{6+m}$	47
gosper	$\frac{x^{2+m} \left(c\,m^{2} x^{4}+6 c m\,x^{4}+b\,m^{2} x^{2}+8 c\,x^{4}+8 b m\,x^{2}+a\,m^{2}+12 b\,x^{2}+10 a m+24 a \right)}{\left(2+m \right) \left(4+m \right) \left(6+m \right)}$	77
risch	$\frac{x^{m} \left(c\,m^{2} x^{4}+6 c m\,x^{4}+b\,m^{2} x^{2}+8 c\,x^{4}+8 b m\,x^{2}+a\,m^{2}+12 b\,x^{2}+10 a m+24 a \right) x^{2}}{\left(6+m \right) \left(4+m \right) \left(2+m \right)}$	78
parallelrisch	$\frac{x^{6} x^{m} c\,m^{2}+6 x^{6} x^{m} c m+8 x^{6} x^{m} c+x^{4} x^{m} b\,m^{2}+8 x^{4} x^{m} b m+12 x^{4} x^{m} b+x^{2} x^{m} a\,m^{2}+10 x^{2} x^{m} a m+24 x^{2} x^{m} a}{\left(6+m \right) \left(4+m \right) \left(2+m \right)}$	108

```
[In] int(x^m*(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] a/(2+m)*x^2*exp(m*ln(x))+b/(4+m)*x^4*exp(m*ln(x))+c/(6+m)*x^6*exp(m*ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.92

$$\int x^m\left(ax+bx^3+cx^5\right)\,dx = \frac{\left((cm^{2}+6\,cm+8\,c)x^{6}+(bm^{2}+8\,bm+12\,b)x^{4}+(am^{2}+10\,am+24\,a)x^{2}\right)x^{m}}{m^{3}+12\,m^{2}+44\,m+48}$$

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="fricas")
[Out] ((c*m^2 + 6*c*m + 8*c)*x^6 + (b*m^2 + 8*b*m + 12*b)*x^4 + (a*m^2 + 10*a*m +
 24*a)*x^2)*x^m/(m^3 + 12*m^2 + 44*m + 48)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(29) = 58.

Time = 0.34 (sec) , antiderivative size = 280, normalized size of antiderivative = 7.57

$$\int x^m(ax+bx^3+cx^5)\ dx$$
$$= \begin{cases} -\frac{a}{4x^4}-\frac{b}{2x^2}+c\log(x) \\ -\frac{a}{2x^2}+b\log(x)+\frac{cx^2}{2} \\ a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4} \\ \frac{am^2x^2x^m}{m^3+12m^2+44m+48}+\frac{10amx^2x^m}{m^3+12m^2+44m+48}+\frac{24ax^2x^m}{m^3+12m^2+44m+48}+\frac{bm^2x^4x^m}{m^3+12m^2+44m+48}+\frac{8bmx^4x^m}{m^3+12m^2+44m+48}+\frac{12bx^4x^m}{m^3+12m^2+44} \end{cases}$$

```
[In] integrate(x**m*(c*x**5+b*x**3+a*x),x)
```

```
[Out] Piecewise((-a/(4*x**4) - b/(2*x**2) + c*log(x), Eq(m, -6)), (-a/(2*x**2) +
b*log(x) + c*x**2/2, Eq(m, -4)), (a*log(x) + b*x**2/2 + c*x**4/4, Eq(m, -2)
), (a*m**2*x**2*x**m/(m**3 + 12*m**2 + 44*m + 48) + 10*a*m*x**2*x**m/(m**3
+ 12*m**2 + 44*m + 48) + 24*a*x**2*x**m/(m**3 + 12*m**2 + 44*m + 48) + b*m*
*2*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) + 8*b*m*x**4*x**m/(m**3 + 12*m**2
 + 44*m + 48) + 12*b*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) + c*m**2*x**6*x
**m/(m**3 + 12*m**2 + 44*m + 48) + 6*c*m*x**6*x**m/(m**3 + 12*m**2 + 44*m +
 48) + 8*c*x**6*x**m/(m**3 + 12*m**2 + 44*m + 48), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x^m(ax+bx^3+cx^5)\ dx = \frac{cx^{m+6}}{m+6}+\frac{bx^{m+4}}{m+4}+\frac{ax^{m+2}}{m+2}$$

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] c*x^(m + 6)/(m + 6) + b*x^(m + 4)/(m + 4) + a*x^(m + 2)/(m + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(37) = 74$.

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.89

$$\int x^m \left(ax + bx^3 + cx^5\right)\, dx$$
$$= \frac{cm^2x^6x^m + 6\,cmx^6x^m + bm^2x^4x^m + 8\,cx^6x^m + 8\,bmx^4x^m + am^2x^2x^m + 12\,bx^4x^m + 10\,amx^2x^m + 24\,ax^2}{m^3 + 12\,m^2 + 44\,m + 48}$$

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] (c*m^2*x^6*x^m + 6*c*m*x^6*x^m + b*m^2*x^4*x^m + 8*c*x^6*x^m + 8*b*m*x^4*x^
m + a*m^2*x^2*x^m + 12*b*x^4*x^m + 10*a*m*x^2*x^m + 24*a*x^2*x^m)/(m^3 + 12
*m^2 + 44*m + 48)
```

Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.41

$$\int x^m \left(ax + bx^3 + cx^5\right)\, dx = x^m \left(\frac{a\,x^2\,(m^2 + 10\,m + 24)}{m^3 + 12\,m^2 + 44\,m + 48} + \frac{b\,x^4\,(m^2 + 8\,m + 12)}{m^3 + 12\,m^2 + 44\,m + 48} + \frac{c\,x^6\,(m^2 + 6\,m + 8)}{m^3 + 12\,m^2 + 44\,m + 48}\right)$$

```
[In] int(x^m*(a*x + b*x^3 + c*x^5),x)
```

```
[Out] x^m*((a*x^2*(10*m + m^2 + 24))/(44*m + 12*m^2 + m^3 + 48) + (b*x^4*(8*m + m
^2 + 12))/(44*m + 12*m^2 + m^3 + 48) + (c*x^6*(6*m + m^2 + 8))/(44*m + 12*m
^2 + m^3 + 48))
```

3.66 $\int x^2(ax+bx^3+cx^5)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 25

$$\int x^2(ax+bx^3+cx^5)\,dx=\frac{ax^4}{4}+\frac{bx^6}{6}+\frac{cx^8}{8}$$

```
[Out] 1/4*a*x^4+1/6*b*x^6+1/8*c*x^8
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\int x^2(ax+bx^3+cx^5)\,dx=\frac{ax^4}{4}+\frac{bx^6}{6}+\frac{cx^8}{8}$$

```
[In] Int[x^2*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int(ax^3+bx^5+cx^7)\,dx\\&=\frac{ax^4}{4}+\frac{bx^6}{6}+\frac{cx^8}{8}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^2(ax + bx^3 + cx^5)\ dx = \frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

```
[In] Integrate[x^2*(a*x + b*x^3 + c*x^5),x]
[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{1}{4}a\,x^4 + \frac{1}{6}b\,x^6 + \frac{1}{8}c\,x^8$	20
norman	$\frac{1}{4}a\,x^4 + \frac{1}{6}b\,x^6 + \frac{1}{8}c\,x^8$	20
risch	$\frac{1}{4}a\,x^4 + \frac{1}{6}b\,x^6 + \frac{1}{8}c\,x^8$	20
parallelrisch	$\frac{1}{4}a\,x^4 + \frac{1}{6}b\,x^6 + \frac{1}{8}c\,x^8$	20
gosper	$\frac{x^4(3c\,x^4+4b\,x^2+6a)}{24}$	22

```
[In] int(x^2*(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] 1/4*a*x^4+1/6*b*x^6+1/8*c*x^8
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(ax + bx^3 + cx^5)\ dx = \frac{1}{8}\,cx^8 + \frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="fricas")
[Out] 1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(ax+bx^3+cx^5)\ dx=\frac{ax^4}{4}+\frac{bx^6}{6}+\frac{cx^8}{8}$$

```
[In] integrate(x**2*(c*x**5+b*x**3+a*x),x)
[Out] a*x**4/4 + b*x**6/6 + c*x**8/8
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(ax+bx^3+cx^5)\ dx=\frac{1}{8}\,cx^8+\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="maxima")
[Out] 1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(ax+bx^3+cx^5)\ dx=\frac{1}{8}\,cx^8+\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="giac")
[Out] 1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(ax+bx^3+cx^5)\ dx=\frac{c\,x^8}{8}+\frac{b\,x^6}{6}+\frac{a\,x^4}{4}$$

```
[In] int(x^2*(a*x + b*x^3 + c*x^5),x)
[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

3.67 $\int x(ax+bx^3+cx^5)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int x\left(ax+bx^3+cx^5\right)\,dx=\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

```
[Out] 1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$\int x\left(ax+bx^3+cx^5\right)\,dx=\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

```
[In] Int[x*(a*x + b*x^3 + c*x^5),x]
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(ax^2+bx^4+cx^6\right)\,dx\\&=\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x(ax + bx^3 + cx^5) \, dx = \frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

```
[In] Integrate[x*(a*x + b*x^3 + c*x^5),x]
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{1}{3}a\,x^3 + \frac{1}{5}b\,x^5 + \frac{1}{7}c\,x^7$	20
norman	$\frac{1}{3}a\,x^3 + \frac{1}{5}b\,x^5 + \frac{1}{7}c\,x^7$	20
risch	$\frac{1}{3}a\,x^3 + \frac{1}{5}b\,x^5 + \frac{1}{7}c\,x^7$	20
parallelrisch	$\frac{1}{3}a\,x^3 + \frac{1}{5}b\,x^5 + \frac{1}{7}c\,x^7$	20
gosper	$\frac{x^3(15c\,x^4+21b\,x^2+35a)}{105}$	22

```
[In] int(x*(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] 1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax + bx^3 + cx^5) \, dx = \frac{1}{7}\,cx^7 + \frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

```
[In] integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="fricas")
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax + bx^3 + cx^5) \, dx = \frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

```
[In] integrate(x*(c*x**5+b*x**3+a*x),x)
[Out] a*x**3/3 + b*x**5/5 + c*x**7/7
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax + bx^3 + cx^5) \, dx = \frac{1}{7} cx^7 + \frac{1}{5} bx^5 + \frac{1}{3} ax^3$$

```
[In] integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="maxima")
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax + bx^3 + cx^5) \, dx = \frac{1}{7} cx^7 + \frac{1}{5} bx^5 + \frac{1}{3} ax^3$$

```
[In] integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="giac")
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax + bx^3 + cx^5) \, dx = \frac{c\,x^7}{7} + \frac{b\,x^5}{5} + \frac{a\,x^3}{3}$$

```
[In] int(x*(a*x + b*x^3 + c*x^5),x)
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

3.68 $\int (ax + bx^3 + cx^5)\ dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int (ax + bx^3 + cx^5)\ dx = \frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

```
[Out] 1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ax + bx^3 + cx^5)\ dx = \frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

```
[In] Int[a*x + b*x^3 + c*x^5,x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Rubi steps

$$\text{integral} = \frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left(ax+bx^3+cx^5\right)\,dx=\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

```
[In] Integrate[a*x + b*x^3 + c*x^5,x]
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{1}{2}a\,x^2+\frac{1}{4}b\,x^4+\frac{1}{6}c\,x^6$	20
norman	$\frac{1}{2}a\,x^2+\frac{1}{4}b\,x^4+\frac{1}{6}c\,x^6$	20
risch	$\frac{1}{2}a\,x^2+\frac{1}{4}b\,x^4+\frac{1}{6}c\,x^6$	20
parallelrisch	$\frac{1}{2}a\,x^2+\frac{1}{4}b\,x^4+\frac{1}{6}c\,x^6$	20
parts	$\frac{1}{2}a\,x^2+\frac{1}{4}b\,x^4+\frac{1}{6}c\,x^6$	20
gosper	$\frac{x^2\left(2c\,x^4+3b\,x^2+6a\right)}{12}$	22

```
[In] int(c*x^5+b*x^3+a*x,x,method=_RETURNVERBOSE)
[Out] 1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax+bx^3+cx^5\right)\,dx=\frac{1}{6}\,cx^6+\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

```
[In] integrate(c*x^5+b*x^3+a*x,x, algorithm="fricas")
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax + bx^3 + cx^5\right) \, dx = \frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

```
[In] integrate(c*x**5+b*x**3+a*x,x)
[Out] a*x**2/2 + b*x**4/4 + c*x**6/6
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax + bx^3 + cx^5\right) \, dx = \frac{1}{6} \, cx^6 + \frac{1}{4} \, bx^4 + \frac{1}{2} \, ax^2$$

```
[In] integrate(c*x^5+b*x^3+a*x,x, algorithm="maxima")
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax + bx^3 + cx^5\right) \, dx = \frac{1}{6} \, cx^6 + \frac{1}{4} \, bx^4 + \frac{1}{2} \, ax^2$$

```
[In] integrate(c*x^5+b*x^3+a*x,x, algorithm="giac")
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax + bx^3 + cx^5\right) \, dx = \frac{c\,x^6}{6} + \frac{b\,x^4}{4} + \frac{a\,x^2}{2}$$

```
[In] int(a*x + b*x^3 + c*x^5,x)
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

3.69 $\int \frac{ax+bx^3+cx^5}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 20

$$\int \frac{ax+bx^3+cx^5}{x}\,dx = ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

```
[Out] a*x+1/3*b*x^3+1/5*c*x^5
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{14\}$

$$\int \frac{ax+bx^3+cx^5}{x}\,dx = ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

```
[In] Int[(a*x + b*x^3 + c*x^5)/x,x]
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(a+bx^2+cx^4\right)\,dx \\ &= ax+\frac{bx^3}{3}+\frac{cx^5}{5} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{ax + bx^3 + cx^5}{x} \, dx = ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

[In] Integrate[(a*x + b*x^3 + c*x^5)/x,x]

[Out] a*x + (b*x^3)/3 + (c*x^5)/5

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
default	$ax + \frac{1}{3}b\,x^3 + \frac{1}{5}c\,x^5$	17
norman	$ax + \frac{1}{3}b\,x^3 + \frac{1}{5}c\,x^5$	17
risch	$ax + \frac{1}{3}b\,x^3 + \frac{1}{5}c\,x^5$	17
parallelrisch	$ax + \frac{1}{3}b\,x^3 + \frac{1}{5}c\,x^5$	17
parts	$ax + \frac{1}{3}b\,x^3 + \frac{1}{5}c\,x^5$	17
gosper	$\frac{x\left(3c\,x^4+5b\,x^2+15a\right)}{15}$	20

[In] int((c*x^5+b*x^3+a*x)/x,x,method=_RETURNVERBOSE)

[Out] a*x+1/3*b*x^3+1/5*c*x^5

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax + bx^3 + cx^5}{x} \, dx = \frac{1}{5} \, cx^5 + \frac{1}{3} \, bx^3 + ax$$

[In] integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="fricas")

[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{ax + bx^3 + cx^5}{x} \, dx = ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

```
[In] integrate((c*x**5+b*x**3+a*x)/x,x)
[Out] a*x + b*x**3/3 + c*x**5/5
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax + bx^3 + cx^5}{x} \, dx = \frac{1}{5} \, cx^5 + \frac{1}{3} \, bx^3 + ax$$

```
[In] integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="maxima")
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax + bx^3 + cx^5}{x} \, dx = \frac{1}{5} \, cx^5 + \frac{1}{3} \, bx^3 + ax$$

```
[In] integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="giac")
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax + bx^3 + cx^5}{x} \, dx = \frac{c\,x^5}{5} + \frac{b\,x^3}{3} + a\,x$$

```
[In] int((a*x + b*x^3 + c*x^5)/x,x)
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

3.70 $\int \frac{ax+bx^3+cx^5}{x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{ax+bx^3+cx^5}{x^2}\,dx=\frac{bx^2}{2}+\frac{cx^4}{4}+a\log(x)$$

```
[Out] 1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\int \frac{ax+bx^3+cx^5}{x^2}\,dx=a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

```
[In] Int[(a*x + b*x^3 + c*x^5)/x^2,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{a}{x}+bx+cx^3\right)\,dx\\&=\frac{bx^2}{2}+\frac{cx^4}{4}+a\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{ax+bx^3+cx^5}{x^2}\,dx=\frac{bx^2}{2}+\frac{cx^4}{4}+a\log(x)$$

```
[In] Integrate[(a*x + b*x^3 + c*x^5)/x^2,x]
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{b\,x^{2}}{2}+\frac{c\,x^{4}}{4}+a\ln\left(x\right)$	18
parallelrisch	$\frac{b\,x^{2}}{2}+\frac{c\,x^{4}}{4}+a\ln\left(x\right)$	18
norman	$\frac{\frac{1}{2}b\,x^{3}+\frac{1}{4}c\,x^{5}}{x}+a\ln\left(x\right)$	23
risch	$\frac{c\,x^{4}}{4}+\frac{b\,x^{2}}{2}+\frac{b^{2}}{4c}+a\ln\left(x\right)$	26

```
[In] int((c*x^5+b*x^3+a*x)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{ax+bx^3+cx^5}{x^2}\,dx=\frac{1}{4}\,cx^4+\frac{1}{2}\,bx^2+a\log\left(x\right)$$

```
[In] integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="fricas")
[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{ax + bx^3 + cx^5}{x^2}\,dx = a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

```
[In] integrate((c*x**5+b*x**3+a*x)/x**2,x)
[Out] a*log(x) + b*x**2/2 + c*x**4/4
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{ax + bx^3 + cx^5}{x^2}\,dx = \frac{1}{4}\,cx^4 + \frac{1}{2}\,bx^2 + a\log(x)$$

```
[In] integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="maxima")
[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{ax + bx^3 + cx^5}{x^2}\,dx = \frac{1}{4}\,cx^4 + \frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log\left(x^2\right)$$

```
[In] integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="giac")
[Out] 1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{ax + bx^3 + cx^5}{x^2}\,dx = \frac{b\,x^2}{2} + \frac{c\,x^4}{4} + a\,\ln(x)$$

```
[In] int((a*x + b*x^3 + c*x^5)/x^2,x)
[Out] (b*x^2)/2 + (c*x^4)/4 + a*log(x)
```

3.71 $\int \frac{ax+bx^3+cx^5}{x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{ax+bx^3+cx^5}{x^3} dx = -\frac{a}{x}+bx+\frac{cx^3}{3}$$

```
[Out] -a/x+b*x+1/3*c*x^3
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\int \frac{ax+bx^3+cx^5}{x^3} dx = -\frac{a}{x}+bx+\frac{cx^3}{3}$$

```
[In] Int[(a*x + b*x^3 + c*x^5)/x^3,x]
```

```
[Out] -(a/x) + b*x + (c*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(b+\frac{a}{x^2}+cx^2\right) dx \\ &= -\frac{a}{x}+bx+\frac{cx^3}{3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{ax + bx^3 + cx^5}{x^3} \, dx = -\frac{a}{x} + bx + \frac{cx^3}{3}$$

```
[In] Integrate[(a*x + b*x^3 + c*x^5)/x^3,x]
[Out] -(a/x) + b*x + (c*x^3)/3
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{a}{x}+bx+\frac{c\,x^{3}}{3}$	17
risch	$-\frac{a}{x}+bx+\frac{c\,x^{3}}{3}$	17
norman	$\frac{b\,x^{3}-ax+\frac{1}{3}c\,x^{5}}{x^{2}}$	21
parallelrisch	$\frac{c\,x^{4}+3b\,x^{2}-3a}{3x}$	21
gosper	$-\frac{-c\,x^{4}-3b\,x^{2}+3a}{3x}$	22

```
[In] int((c*x^5+b*x^3+a*x)/x^3,x,method=_RETURNVERBOSE)
[Out] -a/x+b*x+1/3*c*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{ax + bx^3 + cx^5}{x^3} \, dx = \frac{cx^4 + 3\,bx^2 - 3\,a}{3\,x}$$

```
[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="fricas")
[Out] 1/3*(c*x^4 + 3*b*x^2 - 3*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{ax+bx^3+cx^5}{x^3}\,dx = -\frac{a}{x}+bx+\frac{cx^3}{3}$$

```
[In] integrate((c*x**5+b*x**3+a*x)/x**3,x)
[Out] -a/x + b*x + c*x**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{ax+bx^3+cx^5}{x^3}\,dx = \frac{1}{3}\,cx^3+bx-\frac{a}{x}$$

```
[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="maxima")
[Out] 1/3*c*x^3 + b*x - a/x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{ax+bx^3+cx^5}{x^3}\,dx = \frac{1}{3}\,cx^3+bx-\frac{a}{x}$$

```
[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="giac")
[Out] 1/3*c*x^3 + b*x - a/x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{ax+bx^3+cx^5}{x^3}\,dx = b\,x-\frac{a}{x}+\frac{c\,x^3}{3}$$

```
[In] int((a*x + b*x^3 + c*x^5)/x^3,x)
[Out] b*x - a/x + (c*x^3)/3
```

3.72 $\int x^m(ax+bx^3+cx^5)^2\, dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int x^m(ax+bx^3+cx^5)^2\, dx = \frac{a^2x^{3+m}}{3+m}+\frac{2abx^{5+m}}{5+m}+\frac{(b^2+2ac)\,x^{7+m}}{7+m}+\frac{2bcx^{9+m}}{9+m}+\frac{c^2x^{11+m}}{11+m}$$

```
[Out] a^2*x^(3+m)/(3+m)+2*a*b*x^(5+m)/(5+m)+(2*a*c+b^2)*x^(7+m)/(7+m)+2*b*c*x^(9+
m)/(9+m)+c^2*x^(11+m)/(11+m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1599, 1122}

$$\int x^m(ax+bx^3+cx^5)^2\, dx = \frac{a^2x^{m+3}}{m+3}+\frac{x^{m+7}(2ac+b^2)}{m+7}+\frac{2abx^{m+5}}{m+5}+\frac{2bcx^{m+9}}{m+9}+\frac{c^2x^{m+11}}{m+11}$$

```
[In] Int[x^m*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^(3 + m))/(3 + m) + (2*a*b*x^(5 + m))/(5 + m) + ((b^2 + 2*a*c)*x^(7 +
 m))/(7 + m) + (2*b*c*x^(9 + m))/(9 + m) + (c^2*x^(11 + m))/(11 + m)
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^{2+m}\left(a+bx^2+cx^4\right)^2\,dx\\
&= \int \left(a^2x^{2+m}+2abx^{4+m}+\left(b^2+2ac\right)x^{6+m}+2bcx^{8+m}+c^2x^{10+m}\right)\,dx\\
&= \frac{a^2x^{3+m}}{3+m}+\frac{2abx^{5+m}}{5+m}+\frac{\left(b^2+2ac\right)x^{7+m}}{7+m}+\frac{2bcx^{9+m}}{9+m}+\frac{c^2x^{11+m}}{11+m}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$
\int x^m\left(ax+bx^3+cx^5\right)^2\,dx = x^{3+m}\left(\frac{a^2}{3+m}+\frac{2abx^2}{5+m}+\frac{\left(b^2+2ac\right)x^4}{7+m}+\frac{2bcx^6}{9+m}+\frac{c^2x^8}{11+m}\right)
$$

```
[In] Integrate[x^m*(a*x + b*x^3 + c*x^5)^2,x]
[Out] x^(3 + m)*(a^2/(3 + m) + (2*a*b*x^2)/(5 + m) + ((b^2 + 2*a*c)*x^4)/(7 + m)
+ (2*b*c*x^6)/(9 + m) + (c^2*x^8)/(11 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(76) = 152.

Time = 0.14 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.95

method	result
gosper	$\frac{x^{3+m}\left(c^2m^4x^8+24c^2m^3x^8+2bc\,m^4x^6+206c^2m^2x^8+52bc\,m^3x^6+744m\,x^8c^2+2ac\,m^4x^4+b^2m^4x^4+472bc\,m^2x^6+945c^2x^8+56a\right.}{}$
risch	$\frac{x^m\left(c^2m^4x^8+24c^2m^3x^8+2bc\,m^4x^6+206c^2m^2x^8+52bc\,m^3x^6+744m\,x^8c^2+2ac\,m^4x^4+b^2m^4x^4+472bc\,m^2x^6+945c^2x^8+56ac\right.}{}$
parallelrisch	$\frac{24x^{11}x^mc^2m^3+206x^{11}x^mc^2m^2+744x^{11}x^mc^2m+x^7x^mb^2m^4+28x^7x^mb^2m^3+2310x^9x^mbc+4158x^5x^mab+374x^3x^ma^2m^2+1}{}$

```
[In] int(x^m*(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
[Out] x^(3+m)/(3+m)/(5+m)/(7+m)/(9+m)/(11+m)*(c^2*m^4*x^8+24*c^2*m^3*x^8+2*b*c*m^
4*x^6+206*c^2*m^2*x^8+52*b*c*m^3*x^6+744*c^2*m*x^8+2*a*c*m^4*x^4+b^2*m^4*x^
4+472*b*c*m^2*x^6+945*c^2*x^8+56*a*c*m^3*x^4+28*b^2*m^3*x^4+1772*b*c*m*x^6+
```

```
2*a*b*m^4*x^2+548*a*c*m^2*x^4+274*b^2*m^2*x^4+2310*b*c*x^6+60*a*b*m^3*x^2+2
184*a*c*m*x^4+1092*b^2*m*x^4+a^2*m^4+640*a*b*m^2*x^2+2970*a*c*x^4+1485*b^2*
x^4+32*a^2*m^3+2820*a*b*m*x^2+374*a^2*m^2+4158*a*b*x^2+1888*a^2*m+3465*a^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(76) = 152.

Time = 0.27 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.17

$$\int x^m \left(ax + bx^3 + cx^5\right)^2 dx = \frac{((c^2m^4 + 24\,c^2m^3 + 206\,c^2m^2 + 744\,c^2m + 945\,c^2)x^{11} + 2\,(bcm^4 + 26\,bcm^3 + 236\,bcm^2 + 886\,bcm + 1155\,b}{}$$

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 24*c^2*m^3 + 206*c^2*m^2 + 744*c^2*m + 945*c^2)*x^11 + 2*(b*c*m
^4 + 26*b*c*m^3 + 236*b*c*m^2 + 886*b*c*m + 1155*b*c)*x^9 + ((b^2 + 2*a*c)*
m^4 + 28*(b^2 + 2*a*c)*m^3 + 274*(b^2 + 2*a*c)*m^2 + 1485*b^2 + 2970*a*c +
1092*(b^2 + 2*a*c)*m)*x^7 + 2*(a*b*m^4 + 30*a*b*m^3 + 320*a*b*m^2 + 1410*a*
b*m + 2079*a*b)*x^5 + (a^2*m^4 + 32*a^2*m^3 + 374*a^2*m^2 + 1888*a^2*m + 34
65*a^2)*x^3)*x^m/(m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9129*m + 10395)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1377 vs. 2(66) = 132.

Time = 0.65 (sec) , antiderivative size = 1377, normalized size of antiderivative = 18.12

$$\int x^m \left(ax + bx^3 + cx^5\right)^2 dx = \text{Too large to display}$$

```
[In] integrate(x**m*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Piecewise((-a**2/(8*x**8) - a*b/(3*x**6) - a*c/(2*x**4) - b**2/(4*x**4) - b
*c/x**2 + c**2*log(x), Eq(m, -11)), (-a**2/(6*x**6) - a*b/(2*x**4) - a*c/x*
*2 - b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2, Eq(m, -9)), (-a**2/(4*x**4
) - a*b/x**2 + 2*a*c*log(x) + b**2*log(x) + b*c*x**2 + c**2*x**4/4, Eq(m, -
7)), (-a**2/(2*x**2) + 2*a*b*log(x) + a*c*x**2 + b**2*x**2/2 + b*c*x**4/2 +
 c**2*x**6/6, Eq(m, -5)), (a**2*log(x) + a*b*x**2 + a*c*x**4/2 + b**2*x**4/
4 + b*c*x**6/3 + c**2*x**8/8, Eq(m, -3)), (a**2*m**4*x**3*x**m/(m**5 + 35*m
**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 32*a**2*m**3*x**3*x**m/(m**5
 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 374*a**2*m**2*x**3*x*
*m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1888*a**2*m*x
**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 3465*a*
```

```
*2*x**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2*a
*b*m**4*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395)
+ 60*a*b*m**3*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 1
0395) + 640*a*b*m**2*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 912
9*m + 10395) + 2820*a*b*m*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2
+ 9129*m + 10395) + 4158*a*b*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m*
*2 + 9129*m + 10395) + 2*a*c*m**4*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 30
10*m**2 + 9129*m + 10395) + 56*a*c*m**3*x**7*x**m/(m**5 + 35*m**4 + 470*m**
3 + 3010*m**2 + 9129*m + 10395) + 548*a*c*m**2*x**7*x**m/(m**5 + 35*m**4 +
470*m**3 + 3010*m**2 + 9129*m + 10395) + 2184*a*c*m*x**7*x**m/(m**5 + 35*m*
*4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2970*a*c*x**7*x**m/(m**5 + 35
*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + b**2*m**4*x**7*x**m/(m**5
+ 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 28*b**2*m**3*x**7*x**m
/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 274*b**2*m**2*x
**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1092*b*
*2*m*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1
485*b**2*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395)
 + 2*b*c*m**4*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 1
0395) + 52*b*c*m**3*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129
*m + 10395) + 472*b*c*m**2*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2
 + 9129*m + 10395) + 1772*b*c*m*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010
*m**2 + 9129*m + 10395) + 2310*b*c*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3
010*m**2 + 9129*m + 10395) + c**2*m**4*x**11*x**m/(m**5 + 35*m**4 + 470*m**
3 + 3010*m**2 + 9129*m + 10395) + 24*c**2*m**3*x**11*x**m/(m**5 + 35*m**4 +
 470*m**3 + 3010*m**2 + 9129*m + 10395) + 206*c**2*m**2*x**11*x**m/(m**5 +
35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 744*c**2*m*x**11*x**m/(m
**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 945*c**2*x**11*x**
m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int x^m \left(ax+bx^3+cx^5\right)^2 \, dx = \frac{c^2x^{m+11}}{m+11} + \frac{2\,bcx^{m+9}}{m+9} + \frac{b^2x^{m+7}}{m+7} + \frac{2\,acx^{m+7}}{m+7} + \frac{2\,abx^{m+5}}{m+5} + \frac{a^2x^{m+3}}{m+3}$$

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] c^2*x^(m + 11)/(m + 11) + 2*b*c*x^(m + 9)/(m + 9) + b^2*x^(m + 7)/(m + 7) +
 2*a*c*x^(m + 7)/(m + 7) + 2*a*b*x^(m + 5)/(m + 5) + a^2*x^(m + 3)/(m + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(76) = 152.

Time = 0.29 (sec) , antiderivative size = 399, normalized size of antiderivative = 5.25

$$\int x^m \left(ax+bx^3+cx^5\right)^2\,dx$$
$$=\frac{c^2m^4x^{11}x^m+24\,c^2m^3x^{11}x^m+2\,bcm^4x^9x^m+206\,c^2m^2x^{11}x^m+52\,bcm^3x^9x^m+744\,c^2mx^{11}x^m+b^2m^4x^7x^m}{}$$

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] (c^2*m^4*x^11*x^m + 24*c^2*m^3*x^11*x^m + 2*b*c*m^4*x^9*x^m + 206*c^2*m^2*x
^11*x^m + 52*b*c*m^3*x^9*x^m + 744*c^2*m*x^11*x^m + b^2*m^4*x^7*x^m + 2*a*c
*m^4*x^7*x^m + 472*b*c*m^2*x^9*x^m + 945*c^2*x^11*x^m + 28*b^2*m^3*x^7*x^m
+ 56*a*c*m^3*x^7*x^m + 1772*b*c*m*x^9*x^m + 2*a*b*m^4*x^5*x^m + 274*b^2*m^2
*x^7*x^m + 548*a*c*m^2*x^7*x^m + 2310*b*c*x^9*x^m + 60*a*b*m^3*x^5*x^m + 10
92*b^2*m*x^7*x^m + 2184*a*c*m*x^7*x^m + a^2*m^4*x^3*x^m + 640*a*b*m^2*x^5*x
^m + 1485*b^2*x^7*x^m + 2970*a*c*x^7*x^m + 32*a^2*m^3*x^3*x^m + 2820*a*b*m*
x^5*x^m + 374*a^2*m^2*x^3*x^m + 4158*a*b*x^5*x^m + 1888*a^2*m*x^3*x^m + 346
5*a^2*x^3*x^m)/(m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9129*m + 10395)
```

Mupad [B] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.57

$$\int x^m \left(ax+bx^3+cx^5\right)^2\,dx=\frac{a^2\,x^m\,x^3\,\left(m^4+32\,m^3+374\,m^2+1888\,m+3465\right)}{m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395}+\frac{c^2\,x^m\,x^{11}\,\left(m^4+24\,m^3+206\,m^2+744\,m+945\right)}{m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395}+\frac{x^m\,x^7\,\left(b^2+2\,a\,c\right)\,\left(m^4+28\,m^3+274\,m^2+1092\,m+1485\right)}{m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395}+\frac{2\,a\,b\,x^m\,x^5\,\left(m^4+30\,m^3+320\,m^2+1410\,m+2079\right)}{m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395}+\frac{2\,b\,c\,x^m\,x^9\,\left(m^4+26\,m^3+236\,m^2+886\,m+1155\right)}{m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395}$$

```
[In] int(x^m*(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] (a^2*x^m*x^3*(1888*m + 374*m^2 + 32*m^3 + m^4 + 3465))/(9129*m + 3010*m^2 +
 470*m^3 + 35*m^4 + m^5 + 10395) + (c^2*x^m*x^11*(744*m + 206*m^2 + 24*m^3
+ m^4 + 945))/(9129*m + 3010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395) + (x^m*x
^7*(2*a*c + b^2)*(1092*m + 274*m^2 + 28*m^3 + m^4 + 1485))/(9129*m + 3010*m
^2 + 470*m^3 + 35*m^4 + m^5 + 10395) + (2*a*b*x^m*x^5*(1410*m + 320*m^2 + 3
0*m^3 + m^4 + 2079))/(9129*m + 3010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395) +
 (2*b*c*x^m*x^9*(886*m + 236*m^2 + 26*m^3 + m^4 + 1155))/(9129*m + 3010*m^2
 + 470*m^3 + 35*m^4 + m^5 + 10395)
```

3.73 $\int x^2(ax+bx^3+cx^5)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 54

$$\int x^2\left(ax+bx^3+cx^5\right)^2\,dx=\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}$$

```
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*(2*a*c+b^2)*x^9+2/11*b*c*x^11+1/13*c^2*x^13
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1599, 1122}

$$\int x^2\left(ax+bx^3+cx^5\right)^2\,dx=\frac{a^2x^5}{5}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}$$

```
[In] Int[x^2*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11)/11 + (c^
2*x^13)/13
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
```

```
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^4\left(a+bx^2+cx^4\right)^2\,dx\\ &= \int \left(a^2x^4+2abx^6+\left(b^2+2ac\right)x^8+2bcx^{10}+c^2x^{12}\right)\,dx\\ &= \frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x^2\left(ax+bx^3+cx^5\right)^2\,dx=\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}$$

```
[In] Integrate[x^2*(a*x + b*x^3 + c*x^5)^2,x]
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11)/11 + (c^
2*x^13)/13
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^5}{5}+\frac{2ab\,x^7}{7}+\frac{(2ac+b^2)x^9}{9}+\frac{2bc\,x^{11}}{11}+\frac{c^2x^{13}}{13}$	45
norman	$\frac{c^2x^{13}}{13}+\frac{2bc\,x^{11}}{11}+\left(\frac{2ac}{9}+\frac{b^2}{9}\right)x^9+\frac{2ab\,x^7}{7}+\frac{a^2x^5}{5}$	46
risch	$\frac{1}{5}a^2x^5+\frac{2}{7}ab\,x^7+\frac{2}{9}x^9ac+\frac{1}{9}b^2x^9+\frac{2}{11}bc\,x^{11}+\frac{1}{13}c^2x^{13}$	47
parallelrisch	$\frac{1}{5}a^2x^5+\frac{2}{7}ab\,x^7+\frac{2}{9}x^9ac+\frac{1}{9}b^2x^9+\frac{2}{11}bc\,x^{11}+\frac{1}{13}c^2x^{13}$	47
gosper	$\frac{x^5\left(3465c^2x^8+8190bc\,x^6+10010ac\,x^4+5005b^2x^4+12870ab\,x^2+9009a^2\right)}{45045}$	49

```
[In] int(x^2*(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*(2*a*c+b^2)*x^9+2/11*b*c*x^11+1/13*c^2*x^13
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^2 (ax + bx^3 + cx^5)^2 \, dx = \frac{1}{13} c^2 x^{13} + \frac{2}{11} bcx^{11} + \frac{1}{9} (b^2 + 2\, ac) x^9 + \frac{2}{7} abx^7 + \frac{1}{5} a^2 x^5$$

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/13*c^2*x^13 + 2/11*b*c*x^11 + 1/9*(b^2 + 2*a*c)*x^9 + 2/7*a*b*x^7 + 1/5*a
^2*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int x^2 (ax + bx^3 + cx^5)^2 \, dx = \frac{a^2 x^5}{5} + \frac{2abx^7}{7} + \frac{2bcx^{11}}{11} + \frac{c^2 x^{13}}{13} + x^9 \cdot \left(\frac{2ac}{9} + \frac{b^2}{9} \right)$$

```
[In] integrate(x**2*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**5/5 + 2*a*b*x**7/7 + 2*b*c*x**11/11 + c**2*x**13/13 + x**9*(2*a*c/9
 + b**2/9)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^2 (ax + bx^3 + cx^5)^2 \, dx = \frac{1}{13} c^2 x^{13} + \frac{2}{11} bcx^{11} + \frac{1}{9} (b^2 + 2\, ac) x^9 + \frac{2}{7} abx^7 + \frac{1}{5} a^2 x^5$$

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/13*c^2*x^13 + 2/11*b*c*x^11 + 1/9*(b^2 + 2*a*c)*x^9 + 2/7*a*b*x^7 + 1/5*a
^2*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x^2(ax+bx^3+cx^5)^2\,dx = \frac{1}{13}\,c^2x^{13} + \frac{2}{11}\,bcx^{11} + \frac{1}{9}\,b^2x^9 + \frac{2}{9}\,acx^9 + \frac{2}{7}\,abx^7 + \frac{1}{5}\,a^2x^5$$

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/13*c^2*x^13 + 2/11*b*c*x^11 + 1/9*b^2*x^9 + 2/9*a*c*x^9 + 2/7*a*b*x^7 + 1
/5*a^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int x^2(ax+bx^3+cx^5)^2\,dx = x^9\left(\frac{b^2}{9}+\frac{2\,a\,c}{9}\right)+\frac{a^2\,x^5}{5}+\frac{c^2\,x^{13}}{13}+\frac{2\,a\,b\,x^7}{7}+\frac{2\,b\,c\,x^{11}}{11}$$

```
[In] int(x^2*(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] x^9*((2*a*c)/9 + b^2/9) + (a^2*x^5)/5 + (c^2*x^13)/13 + (2*a*b*x^7)/7 + (2*
b*c*x^11)/11
```

3.74 $\int x(ax+bx^3+cx^5)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 54

$$\int x\left(ax+bx^3+cx^5\right)^2\,dx=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{1}{8}\left(b^2+2ac\right)x^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{12}}{12}$$

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*(2*a*c+b^2)*x^8+1/5*b*c*x^10+1/12*c^2*x^12
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1599, 1128, 645}

$$\int x\left(ax+bx^3+cx^5\right)^2\,dx=\frac{a^2x^4}{4}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{5}bcx^{10}+\frac{c^2x^{12}}{12}$$

```
[In] Int[x*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^8)/8 + (b*c*x^10)/5 + (c^2*x^1
2)/12
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^3 (a + bx^2 + cx^4)^2 \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int x (a + bx + cx^2)^2 \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2 x + 2abx^2 + (b^2 + 2ac) x^3 + 2bcx^4 + c^2 x^5\right) dx, x, x^2\right) \\
&= \frac{a^2 x^4}{4} + \frac{1}{3}abx^6 + \frac{1}{8}(b^2 + 2ac) x^8 + \frac{1}{5}bcx^{10} + \frac{c^2 x^{12}}{12}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int x (ax + bx^3 + cx^5)^2 \, dx = \frac{1}{120} x^4 (30a^2 + 40abx^2 + 15(b^2 + 2ac) x^4 + 24bcx^6 + 10c^2 x^8)$$

```
[In] Integrate[x*(a*x + b*x^3 + c*x^5)^2,x]
[Out] (x^4*(30*a^2 + 40*a*b*x^2 + 15*(b^2 + 2*a*c)*x^4 + 24*b*c*x^6 + 10*c^2*x^8)
)/120
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^{2} x^{4}}{4}+\frac{a b \,x^{6}}{3}+\frac{\left(2 a c +b^{2}\right) x^{8}}{8}+\frac{b c \,x^{10}}{5}+\frac{c^{2} x^{12}}{12}$	45
norman	$\frac{c^{2} x^{12}}{12}+\frac{b c \,x^{10}}{5}+\left(\frac{a c}{4}+\frac{b^{2}}{8}\right) x^{8}+\frac{a b \,x^{6}}{3}+\frac{a^{2} x^{4}}{4}$	46
risch	$\frac{1}{4} a^{2} x^{4}+\frac{1}{3} a b \,x^{6}+\frac{1}{4} x^{8} a c +\frac{1}{8} b^{2} x^{8}+\frac{1}{5} b c \,x^{10}+\frac{1}{12} c^{2} x^{12}$	47
parallelrisch	$\frac{1}{4} a^{2} x^{4}+\frac{1}{3} a b \,x^{6}+\frac{1}{4} x^{8} a c +\frac{1}{8} b^{2} x^{8}+\frac{1}{5} b c \,x^{10}+\frac{1}{12} c^{2} x^{12}$	47
gosper	$\frac{x^{4} \left(10 c^{2} x^{8}+24 b c \,x^{6}+30 a c \,x^{4}+15 b^{2} x^{4}+40 a b \,x^{2}+30 a^{2}\right)}{120}$	49

```
[In] int(x*(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*(2*a*c+b^2)*x^8+1/5*b*c*x^10+1/12*c^2*x^12
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x\left(ax+bx^3+cx^5\right)^2\,dx=\frac{1}{12}\,c^2x^{12}+\frac{1}{5}\,bcx^{10}+\frac{1}{8}\,\left(b^2+2\,ac\right)x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

```
[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*(b^2 + 2*a*c)*x^8 + 1/3*a*b*x^6 + 1/4*a^
2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x\left(ax+bx^3+cx^5\right)^2\,dx=\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{bcx^{10}}{5}+\frac{c^2x^{12}}{12}+x^8\left(\frac{ac}{4}+\frac{b^2}{8}\right)$$

```
[In] integrate(x*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**4/4 + a*b*x**6/3 + b*c*x**10/5 + c**2*x**12/12 + x**8*(a*c/4 + b**2
/8)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x(ax+bx^3+cx^5)^2\,dx=\frac{1}{12}\,c^2x^{12}+\frac{1}{5}\,bcx^{10}+\frac{1}{8}\,(b^2+2\,ac)x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

```
[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*(b^2 + 2*a*c)*x^8 + 1/3*a*b*x^6 + 1/4*a^
2*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x(ax+bx^3+cx^5)^2\,dx=\frac{1}{12}\,c^2x^{12}+\frac{1}{5}\,bcx^{10}+\frac{1}{8}\,b^2x^8+\frac{1}{4}\,acx^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

```
[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*b^2*x^8 + 1/4*a*c*x^8 + 1/3*a*b*x^6 + 1/
4*a^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int x(ax+bx^3+cx^5)^2\,dx=x^8\left(\frac{b^2}{8}+\frac{a\,c}{4}\right)+\frac{a^2\,x^4}{4}+\frac{c^2\,x^{12}}{12}+\frac{a\,b\,x^6}{3}+\frac{b\,c\,x^{10}}{5}$$

```
[In] int(x*(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] x^8*((a*c)/4 + b^2/8) + (a^2*x^4)/4 + (c^2*x^12)/12 + (a*b*x^6)/3 + (b*c*x^
10)/5
```

3.75 $\int (ax+bx^3+cx^5)^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 54

$$\int (ax+bx^3+cx^5)^2\,dx=\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1608, 1122}

$$\int (ax+bx^3+cx^5)^2\,dx=\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

```
[In] Int[(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*
x^11)/11
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
```

```
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^2\left(a+bx^2+cx^4\right)^2\,dx\\&= \int\left(a^2x^2+2abx^4+\left(b^2+2ac\right)x^6+2bcx^8+c^2x^{10}\right)dx\\&= \frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int\left(ax+bx^3+cx^5\right)^2\,dx=\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^2,x]
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*
x^11)/11
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^3}{3}+\frac{2ab\,x^5}{5}+\frac{(2ac+b^2)x^7}{7}+\frac{2bc\,x^9}{9}+\frac{c^2x^{11}}{11}$	45
norman	$\frac{c^2x^{11}}{11}+\frac{2bc\,x^9}{9}+\left(\frac{2ac}{7}+\frac{b^2}{7}\right)x^7+\frac{2ab\,x^5}{5}+\frac{a^2x^3}{3}$	46
risch	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{2}{7}x^7ac+\frac{1}{7}b^2x^7+\frac{2}{9}bc\,x^9+\frac{1}{11}c^2x^{11}$	47
parallelrisch	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{2}{7}x^7ac+\frac{1}{7}b^2x^7+\frac{2}{9}bc\,x^9+\frac{1}{11}c^2x^{11}$	47
gosper	$\frac{x^3\left(315c^2x^8+770bc\,x^6+990ac\,x^4+495b^2x^4+1386ab\,x^2+1155a^2\right)}{3465}$	49

```
[In] int((c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(ax+bx^3+cx^5\right)^2\,dx=\frac{1}{11}\,c^2x^{11}+\frac{2}{9}\,bcx^9+\frac{1}{7}\left(b^2+2\,ac\right)x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

```
[In] integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*(b^2 + 2*a*c)*x^7 + 2/5*a*b*x^5 + 1/3*a^2
*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \left(ax+bx^3+cx^5\right)^2\,dx=\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{2bcx^9}{9}+\frac{c^2x^{11}}{11}+x^7\cdot\left(\frac{2ac}{7}+\frac{b^2}{7}\right)$$

```
[In] integrate((c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + 2*b*c*x**9/9 + c**2*x**11/11 + x**7*(2*a*c/7 +
 b**2/7)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \left(ax+bx^3+cx^5\right)^2\,dx=\frac{1}{11}\,c^2x^{11}+\frac{2}{9}\,bcx^9+\frac{1}{7}\,b^2x^7+\frac{1}{3}\,a^2x^3+\frac{2}{35}\left(5\,cx^7+7\,bx^5\right)a$$

```
[In] integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 1/3*a^2*x^3 + 2/35*(5*c*x^7 + 7
*b*x^5)*a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \left(ax+bx^3+cx^5\right)^2\,dx=\frac{1}{11}\,c^2x^{11}+\frac{2}{9}\,bcx^9+\frac{1}{7}\,b^2x^7+\frac{2}{7}\,acx^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

```
[In] integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 2/5*a*b*x^5 + 1/3
*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \left(ax+bx^3+cx^5\right)^2\,dx=x^7\left(\frac{b^2}{7}+\frac{2\,a\,c}{7}\right)+\frac{a^2\,x^3}{3}+\frac{c^2\,x^{11}}{11}+\frac{2\,a\,b\,x^5}{5}+\frac{2\,b\,c\,x^9}{9}$$

```
[In] int((a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] x^7*((2*a*c)/7 + b^2/7) + (a^2*x^3)/3 + (c^2*x^11)/11 + (2*a*b*x^5)/5 + (2*
b*c*x^9)/9
```

3.76 $\int \frac{(ax+bx^3+cx^5)^2}{x} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{(ax+bx^3+cx^5)^2}{x} \, dx = \frac{a^2x^2}{2} + \frac{1}{2}abx^4 + \frac{1}{6}(b^2+2ac)\,x^6 + \frac{1}{4}bcx^8 + \frac{c^2x^{10}}{10}$$

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1599, 1121, 625}

$$\int \frac{(ax+bx^3+cx^5)^2}{x} \, dx = \frac{a^2x^2}{2} + \frac{1}{6}x^6(2ac+b^2) + \frac{1}{2}abx^4 + \frac{1}{4}bcx^8 + \frac{c^2x^{10}}{10}$$

```
[In] Int[(a*x + b*x^3 + c*x^5)^2/x,x]
```

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^6)/6 + (b*c*x^8)/4 + (c^2*x^10
)/10
```

Rule 625

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x\left(a+bx^2+cx^4\right)^2\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int\left(a+bx+cx^2\right)^2\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(a^2+2abx+b^2\left(1+\frac{2ac}{b^2}\right)x^2+2bcx^3+c^2x^4\right)dx,x,x^2\right)\\
&= \frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{1}{6}\left(b^2+2ac\right)x^6+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int\frac{\left(ax+bx^3+cx^5\right)^2}{x}\,dx=\frac{1}{60}x^2\left(30a^2+30abx^2+10\left(b^2+2ac\right)x^4+15bcx^6+6c^2x^8\right)$$

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^2/x,x]
[Out] (x^2*(30*a^2 + 30*a*b*x^2 + 10*(b^2 + 2*a*c)*x^4 + 15*b*c*x^6 + 6*c^2*x^8))
/60
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^2}{2}+\frac{ab\,x^4}{2}+\frac{(2ac+b^2)x^6}{6}+\frac{bc\,x^8}{4}+\frac{c^2x^{10}}{10}$	45
norman	$\frac{c^2x^{10}}{10}+\frac{bc\,x^8}{4}+\left(\frac{ac}{3}+\frac{b^2}{6}\right)x^6+\frac{ab\,x^4}{2}+\frac{a^2x^2}{2}$	46
risch	$\frac{1}{2}a^2x^2+\frac{1}{2}ab\,x^4+\frac{1}{3}x^6ac+\frac{1}{6}b^2x^6+\frac{1}{4}bc\,x^8+\frac{1}{10}c^2x^{10}$	47
parallelrisch	$\frac{1}{2}a^2x^2+\frac{1}{2}ab\,x^4+\frac{1}{3}x^6ac+\frac{1}{6}b^2x^6+\frac{1}{4}bc\,x^8+\frac{1}{10}c^2x^{10}$	47
gosper	$\frac{x^2(6c^2x^8+15bc\,x^6+20ac\,x^4+10b^2x^4+30ab\,x^2+30a^2)}{60}$	49

```
[In] int((c*x^5+b*x^3+a*x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(ax+bx^3+cx^5)^2}{x}\,dx=\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\,(b^2+2\,ac)x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="fricas")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^2
*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(ax+bx^3+cx^5)^2}{x}\,dx=\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{bcx^8}{4}+\frac{c^2x^{10}}{10}+x^6\left(\frac{ac}{3}+\frac{b^2}{6}\right)$$

```
[In] integrate((c*x**5+b*x**3+a*x)**2/x,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b*c*x**8/4 + c**2*x**10/10 + x**6*(a*c/3 + b**2/
6)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(ax+bx^3+cx^5)^2}{x}\,dx = \frac{1}{10}\,c^2x^{10} + \frac{1}{4}\,bcx^8 + \frac{1}{6}\,(b^2+2\,ac)x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="maxima")
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^2
*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(ax+bx^3+cx^5)^2}{x}\,dx = \frac{1}{10}\,c^2x^{10} + \frac{1}{4}\,bcx^8 + \frac{1}{6}\,b^2x^6 + \frac{1}{3}\,acx^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="giac")
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 1/2*a*b*x^4 + 1/2
*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{(ax+bx^3+cx^5)^2}{x}\,dx = x^6\left(\frac{b^2}{6}+\frac{a\,c}{3}\right)+\frac{a^2\,x^2}{2}+\frac{c^2\,x^{10}}{10}+\frac{a\,b\,x^4}{2}+\frac{b\,c\,x^8}{4}$$

```
[In] int((a*x + b*x^3 + c*x^5)^2/x,x)
[Out] x^6*((a*c)/3 + b^2/6) + (a^2*x^2)/2 + (c^2*x^10)/10 + (a*b*x^4)/2 + (b*c*x^
8)/4
```

3.77 $\int \frac{(ax+bx^3+cx^5)^2}{x^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{(ax+bx^3+cx^5)^2}{x^2} \, dx = a^2x + \frac{2}{3}abx^3 + \frac{1}{5}\left(b^2+2ac\right)x^5 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

```
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1599, 1104}

$$\int \frac{(ax+bx^3+cx^5)^2}{x^2} \, dx = a^2x + \frac{1}{5}x^5\left(2ac+b^2\right) + \frac{2}{3}abx^3 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

```
[In] Int[(a*x + b*x^3 + c*x^5)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rule 1104

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
```

```
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a+bx^2+cx^4\right)^2 \, dx \\ &= \int \left(a^2+2abx^2+b^2\left(1+\frac{2ac}{b^2}\right)x^4+2bcx^6+c^2x^8\right) dx \\ &= a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{\left(ax+bx^3+cx^5\right)^2}{x^2} \, dx = a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^2/x^2,x]
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
default	$a^2x+\frac{2ab\,x^3}{3}+\frac{(2ac+b^2)x^5}{5}+\frac{2bc\,x^7}{7}+\frac{c^2x^9}{9}$	42
risch	$a^2x+\frac{2}{3}ab\,x^3+\frac{2}{5}x^5ac+\frac{1}{5}b^2x^5+\frac{2}{7}bc\,x^7+\frac{1}{9}c^2x^9$	44
parallelrisch	$a^2x+\frac{2}{3}ab\,x^3+\frac{2}{5}x^5ac+\frac{1}{5}b^2x^5+\frac{2}{7}bc\,x^7+\frac{1}{9}c^2x^9$	44
gosper	$\frac{x(35c^2x^8+90bc\,x^6+126ac\,x^4+63b^2x^4+210ab\,x^2+315a^2)}{315}$	47
norman	$\frac{a^2x^2+\left(\frac{2ac}{5}+\frac{b^2}{5}\right)x^6+\frac{c^2x^{10}}{9}+\frac{2ab\,x^4}{3}+\frac{2bc\,x^8}{7}}{x}$	49

```
[In] int((c*x^5+b*x^3+a*x)^2/x^2,x,method=_RETURNVERBOSE)
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{(ax + bx^3 + cx^5)^2}{x^2} \, dx = \frac{1}{9} \, c^2 x^9 + \frac{2}{7} \, bcx^7 + \frac{1}{5} \, (b^2 + 2 \, ac)x^5 + \frac{2}{3} \, abx^3 + a^2 x$$

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="fricas")
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{(ax + bx^3 + cx^5)^2}{x^2} \, dx = a^2 x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2 x^9}{9} + x^5 \cdot \left(\frac{2ac}{5} + \frac{b^2}{5} \right)$$

```
[In] integrate((c*x**5+b*x**3+a*x)**2/x**2,x)
[Out] a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/5
)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{(ax + bx^3 + cx^5)^2}{x^2} \, dx = \frac{1}{9} \, c^2 x^9 + \frac{2}{7} \, bcx^7 + \frac{1}{5} \, (b^2 + 2 \, ac)x^5 + \frac{2}{3} \, abx^3 + a^2 x$$

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="maxima")
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{(ax+bx^3+cx^5)^2}{x^2}\,dx = \frac{1}{9}\,c^2x^9 + \frac{2}{7}\,bcx^7 + \frac{1}{5}\,b^2x^5 + \frac{2}{5}\,acx^5 + \frac{2}{3}\,abx^3 + a^2x$$

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="giac")
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{(ax+bx^3+cx^5)^2}{x^2}\,dx = a^2\,x + x^5\left(\frac{b^2}{5} + \frac{2\,a\,c}{5}\right) + \frac{c^2\,x^9}{9} + \frac{2\,a\,b\,x^3}{3} + \frac{2\,b\,c\,x^7}{7}$$

```
[In] int((a*x + b*x^3 + c*x^5)^2/x^2,x)
[Out] a^2*x + x^5*((2*a*c)/5 + b^2/5) + (c^2*x^9)/9 + (2*a*b*x^3)/3 + (2*b*c*x^7)
/7
```

3.78 $\int \frac{x^8}{ax+bx^3+cx^5} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 100

$$\int \frac{x^8}{ax+bx^3+cx^5} \, dx = -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b(b^2-3ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

```
[Out] -1/2*b*x^2/c^2+1/4*x^4/c+1/4*(-a*c+b^2)*ln(c*x^4+b*x^2+a)/c^3+1/2*b*(-3*a*c
+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1599, 1128, 715, 648, 632, 212, 642}

$$\int \frac{x^8}{ax+bx^3+cx^5} \, dx = \frac{b(b^2-3ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

```
[In] Int[x^8/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -1/2*(b*x^2)/c^2 + x^4/(4*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x^2 + c*x
^4])/(4*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^7}{a+bx^2+cx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+(b^2-ac)\,x}{c^2\,(a+bx+cx^2)}\right)\,dx,x,x^2\right)\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{\text{Subst}\left(\int \frac{ab+(b^2-ac)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c^2}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}-\frac{(b(b^2-3ac))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3}+\frac{(b^2-ac)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}+\frac{(b(b^2-3ac))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c^3}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{ax+bx^3+cx^5}\,dx = \frac{cx^2(-2b+cx^2)-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

```
[In] Integrate[x^8/(a*x + b*x^3 + c*x^5),x]
[Out] (c*x^2*(-2*b + c*x^2) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

method	result
default	$-\frac{-\frac{1}{2}c\,x^4+b\,x^2}{2c^2}+\frac{\frac{\left(-ac+b^2\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(ab-\frac{\left(-ac+b^2\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c^2}$
risch	$\frac{x^4}{4c}-\frac{b\,x^2}{2c^2}+\frac{b^2}{4c^3}-\frac{\ln\left(\left(12a^2b\,c^2-7a\,b^3c+b^5+\sqrt{-b^2(4ac-b^2)(3ac-b^2)^2}\,b\right)x^2+2\sqrt{-b^2(4ac-b^2)(3ac-b^2)^2}\,a\right)a^2}{c(4ac-b^2)}+\frac{5\ln\left(\left(12a^2\right.\right.}{}$

[In] int(x^8/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)

[Out] -1/2/c^2*(-1/2*c*x^4+b*x^2)+1/2/c^2*(1/2*(-a*c+b^2)/c*ln(c*x^4+b*x^2+a)+2*(a*b-1/2*(-a*c+b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.13

$$\int \frac{x^8}{ax+bx^3+cx^5}\,dx = \left[\frac{(b^2c^2-4\,ac^3)x^4-2\,(b^3c-4\,abc^2)x^2-(b^3-3\,abc)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+ \cdots}{4\,(b^2c^3-4\,ac^4)}\right.$$

[In] integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="fricas")

[Out] [1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2*(b^3*c - 4*a*b*c^2)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2*(b^3*c - 4*a*b*c^2)*x^2 + 2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(92) = 184$.

Time = 1.75 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.91

$$\int \frac{x^8}{ax+bx^3+cx^5}\,dx = -\frac{bx^2}{2c^2} + \left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{4c^3}\right)\log\left(x^2 + \frac{2a^2c-ab^2+8ac^3\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)-2b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac-}{4c^}\right.}{3abc-b^3}\right.$$

$$+\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{4c^3}\right)\log\left(x^2 + \frac{2a^2c-ab^2+8ac^3\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)-2b^2c^2\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)}{3abc-b^3}\right.$$

$$+\frac{x^4}{4c}$$

```
[In] integrate(x**8/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -b*x**2/(2*c**2) + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c -
b**2)) - (a*c - b**2)/(4*c**3))*log(x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(-
b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)
/(4*c**3)) - 2*b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*
a*c - b**2)) - (a*c - b**2)/(4*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c +
 b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3))*log(
x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/
(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)) - 2*b**2*c**2*(b*sqrt(-4*a
*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)))
/(3*a*b*c - b**3)) + x**4/(4*c)
```

Maxima [F]

$$\int \frac{x^8}{ax+bx^3+cx^5}\,dx = \int \frac{x^8}{cx^5+bx^3+ax}\,dx$$

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^4 - 2*b*x^2)/c^2 - integrate(-((b^2 - a*c)*x^3 + a*b*x)/(c*x^4 + b
*x^2 + a), x)/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{x^8}{ax+bx^3+cx^5}\,dx = \frac{cx^4-2\,bx^2}{4\,c^2}+\frac{(b^2-ac)\log\left(cx^4+bx^2+a\right)}{4\,c^3} - \frac{(b^3-3\,abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^3}$$

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/4*(c*x^4 - 2*b*x^2)/c^2 + 1/4*(b^2 - a*c)*log(c*x^4 + b*x^2 + a)/c^3 - 1/
2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a
*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 842, normalized size of antiderivative = 8.42

$$\int \frac{x^8}{ax+bx^3+cx^5}\,dx = \frac{x^4}{4\,c}-\frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{2\,\left(16\,a\,c^4-4\,b^2\,c^3\right)}-\frac{b\,x^2}{2\,c^2}$$

$$+\frac{b\,\mathrm{atan}\left(\frac{2\,c^4\,\left(4\,a\,c-b^2\right)\left(\frac{\frac{b\,\left(3\,a\,c-b^2\right)\,\left(\frac{8\,a^2\,c^4-8\,a\,b^2\,c^3}{c^4}-\frac{8\,a\,c^2\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{16\,a\,c^4-4\,b^2\,c^3}\right)}{8\,c^3\,\sqrt{4\,a\,c-b^2}}-\frac{a\,b\,\left(3\,a\,c-b^2\right)\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{c\,\sqrt{4\,a\,c-b^2}\,\left(16\,a\,c^4-4\,b^2\,c^3\right)}}{a}-x^2\left(\frac{b\,\left(\frac{6\,b^3\,c^3 \cdots}{\cdots}\right)}{\cdots}\right.\right.}{\cdots}\right)}{\cdots}$$

```
[In] int(x^8/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] x^4/(4*c) - (log(a + b*x^2 + c*x^4)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(1
6*a*c^4 - 4*b^2*c^3)) - (b*x^2)/(2*c^2) + (b*atan((2*c^4*(4*a*c - b^2)*(((b
*(3*a*c - b^2)*((8*a^2*c^4 - 8*a*b^2*c^3)/c^4 - (8*a*c^2*(2*b^4 + 8*a^2*c^2
 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3)))/(8*c^3*(4*a*c - b^2)^(1/2)) - (a*b
*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(c*(4*a*c - b^2)^(1/2)*(16
*a*c^4 - 4*b^2*c^3)))/a - x^2*(((b*((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4*b*c^2
```

```
*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(3*a*c - b^2))/(
8*c^3*(4*a*c - b^2)^(1/2)) + (b^2*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b
^2*c))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a + (b*((b^5 + 2*a
^2*b*c^2 - 3*a*b^3*c)/c^4 + (((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4*b*c^2*(2*b^
4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4 + 8*a^2*c^2 - 1
0*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b^3*(3*a*c - b^2)^2)/(2*c^4*(4*a*
c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (b*((((8*a^2*c^4 - 8*a*b^2*c^3)/c^
4 - (8*a*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*b
^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (a*b^4 + a^3*c^2
 - 2*a^2*b^2*c)/c^4 + (a*b^2*(3*a*c - b^2)^2)/(c^4*(4*a*c - b^2))))/(2*a*(4
*a*c - b^2)^(1/2))))/(b^6 + 9*a^2*b^2*c^2 - 6*a*b^4*c))*(3*a*c - b^2))/(2*c
^3*(4*a*c - b^2)^(1/2))
```

3.79 $\int \frac{x^7}{ax+bx^3+cx^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 203

$$\int \frac{x^7}{ax+bx^3+cx^5}\,dx = -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -b*x/c^2+1/3*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2
))*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(-4*a*c+b
^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))
*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1599, 1136, 1293, 1180, 211}

$$\int \frac{x^7}{ax+bx^3+cx^5}\,dx = \frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

```
[In] Int[x^7/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*
c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^6}{a+bx^2+cx^4}\,dx\\
&= \frac{x^3}{3c} - \frac{\int \frac{x^2(3a+3bx^2)}{a+bx^2+cx^4}\,dx}{3c}\\
&= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\int \frac{3ab+3(b^2-ac)x^2}{a+bx^2+cx^4}\,dx}{3c^2}\\
&= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2}\\
&\quad + \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2}\\
&= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}\\
&\quad + \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.23

$$\begin{aligned}
\int \frac{x^7}{ax+bx^3+cx^5}\,dx = &-\frac{bx}{c^2} + \frac{x^3}{3c}\\
&+ \frac{\left(-b^3+3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}\\
&+ \frac{\left(b^3-3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

```
[In] Integrate[x^7/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3 - 3
*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.36

method	result
risch	$\frac{x^{3}}{3c}-\frac{bx}{c^{2}}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(\left(-ac+b^{2}\right)\_R^{2}+ab\right)\ln\left(x-\_R\right)}{2c\_R^{3}+\_Rb}}{2c^{2}}$
default	$-\frac{-\frac{1}{3}c\,x^{3}+bx}{c^{2}}+\frac{\frac{\left(-ac\sqrt{-4ac+b^{2}}+b^{2}\sqrt{-4ac+b^{2}}-3abc+b^{3}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,c\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\left(-ac\sqrt{-4ac+b^{2}}+b^{2}\sqrt{-4ac+b^{2}}+3abc-b^{3}\right)\sqrt{\ldots}}{2\sqrt{-4ac+b^{2}}\,c\sqrt{\left(-b+\ldots\right.}}}{c}$

```
[In] int(x^7/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3/c-b*x/c^2+1/2/c^2*sum(((-a*c+b^2)*_R^2+a*b)/(2*_R^3*c+_R*b)*ln(x-_R
),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1564 vs. $2(167) = 334$.

Time = 0.31 (sec) , antiderivative size = 1564, normalized size of antiderivative = 7.70

$$\int \frac{x^7}{ax+bx^3+cx^5}\,dx = \text{Too large to display}$$

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*x^3 - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*
c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b
^2*c + a^4*c^2)*x + sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c
^3 - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^
3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))
) + 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c
^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*
c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*
c^2)*x - sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 - (b^4*c
^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2
*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
```

```
*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(
1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*
c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x + sq
rt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^
2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b
^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sq
rt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^
2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x - sqrt(1/2)*(b^
7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^2*c^6 + 8*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*
a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b
^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 6*b*x)/c^2
```

Sympy [A] (verification not implemented)

Time = 11.60 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{ax+bx^3+cx^5}\,dx = -\frac{bx}{c^2} + \mathrm{RootSum}\left(t^4\cdot\left(256a^2c^7-128ab^2c^6+16b^4c^5\right)+t^2\left(-80a^3bc^3+100a^2b^3c^2-36ab^5c+4b^7\right)+a^5,\left(t\mapsto t\right.\right. + \frac{x^3}{3c}$$

```
[In] integrate(x**7/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -b*x/c**2 + RootSum(_t**4*(256*a**2*c**7 - 128*a*b**2*c**6 + 16*b**4*c**5)
+ _t**2*(-80*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**7) + a**
5, Lambda(_t, _t*log(x + (-64*_t**3*a**2*c**7 + 48*_t**3*a*b**2*c**6 - 8*_t
**3*b**4*c**5 + 14*_t*a**3*b*c**3 - 28*_t*a**2*b**3*c**2 + 14*_t*a*b**5*c -
 2*_t*b**7)/(a**4*c**2 - 3*a**3*b**2*c + a**2*b**4)))) + x**3/(3*c)
```

Maxima [F]

$$\int \frac{x^7}{ax+bx^3+cx^5}\,dx = \int \frac{x^7}{cx^5+bx^3+ax}\,dx$$

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^3 - 3*b*x)/c^2 - integrate(-((b^2 - a*c)*x^2 + a*b)/(c*x^4 + b*x^2
 + a), x)/c^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2457 vs. $2(167) = 334$.

Time = 0.61 (sec) , antiderivative size = 2457, normalized size of antiderivative = 12.10

$$\int \frac{x^7}{ax+bx^3+cx^5}\,dx = \text{Too large to display}$$

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5 -
(2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 -
4*a*c)*a^2*c^4)*c^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2
- 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*
```

```
c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 +
2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(c))*arctan(2*sqr
t(1/2)*x/sqrt((b*c^3 + sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*a^2*b
^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^
2) + 1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c
^5 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b
^2 - 4*a*c)*a^2*c^4)*c^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5
*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2
*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c
^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(c))*arctan(
2*sqrt(1/2)*x/sqrt((b*c^3 - sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*
a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^
7)*c^2) + 1/3*(c^2*x^3 - 3*b*c*x)/c^3
```

Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 4127, normalized size of antiderivative = 20.33

$$\int \frac{x^7}{ax+bx^3+cx^5}\,dx = \text{Too large to display}$$

```
[In] int(x^7/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] x^3/(3*c) - atan(((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16
*a*b*c^6)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3
```

```
*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7
 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(
-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*
a^2*b^2*c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a
^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c -
3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2)*1i - (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b
*c^6)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2
 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b
^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*
b^2*c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b
*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*
b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(
1/2)*1i)/((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6
)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a
^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^
(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b^4*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*
c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3
 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)
 + (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^
7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b^4*(-(4*a*c
 - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 +
 b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 - 6
*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a
^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(
a^4*c - a^3*b^2))/c^3))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^
3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2
*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2
)*2i - atan(((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*
c^6)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 +
 a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a
```

```
*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2
*c^2 - 6*a*b^4*c))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3
 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)
*1i - (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*((
b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^
2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a*c - b^
2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7
 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 -
6*a*b^4*c))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a
^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((
((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*((b^4*(-(
4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(
1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/
2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c
^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 - 6*a*b^4
*c))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*
c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((4*a*b^3*
c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*((b^4*(-(4*a*c - b^
2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7
 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7
 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b
^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b
^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 - 6*a*b^4*c))/c^3)*
((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(a^4*c - a^3*b^2))
/c^3))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2
 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - (b*x)/c^2
```

3.80 $\int \frac{x^6}{ax+bx^3+cx^5} dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{x^6}{ax+bx^3+cx^5} dx = \frac{x^2}{2c} - \frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} - \frac{b\log(a+bx^2+cx^4)}{4c^2}$$

```
[Out] 1/2*x^2/c-1/4*b*ln(c*x^4+b*x^2+a)/c^2-1/2*(-2*a*c+b^2)*arctanh((2*c*x^2+b)/
(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1599, 1128, 717, 648, 632, 212, 642}

$$\int \frac{x^6}{ax+bx^3+cx^5} dx = -\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} - \frac{b\log(a+bx^2+cx^4)}{4c^2} + \frac{x^2}{2c}$$

```
[In] Int[x^6/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x^2/(2*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^5}{a+bx^2+cx^4}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^2\right)\end{aligned}$$

$$= \frac{x^2}{2c} + \frac{\text{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^2\right)}{2c}$$

$$= \frac{x^2}{2c} - \frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} + \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2}$$

$$= \frac{x^2}{2c} - \frac{b\log\left(a+bx^2+cx^4\right)}{4c^2} - \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c^2}$$

$$= \frac{x^2}{2c} - \frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} - \frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{x^6}{ax+bx^3+cx^5}\,dx = \frac{2cx^2 + \frac{2(b^2-2ac)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - b\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[In] Integrate[x^6/(a*x + b*x^3 + c*x^5),x]
[Out] (2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

method	result
default	$\frac{x^2}{2c} + \frac{-\frac{b\ln\left(c\,x^4+b\,x^2+a\right)}{2c} + \frac{2\left(-a+\frac{b^2}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$
risch	$\frac{x^2}{2c} - \frac{\ln\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,b\right)x^2+2\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,a\right)ab}{c(4ac-b^2)} + \ln\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{-(}\right.\right.$

```
[In] int(x^6/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2/c+1/2/c*(-1/2*b/c*ln(c*x^4+b*x^2+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(1
/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.14

$$\int \frac{x^6}{ax+bx^3+cx^5}\,dx = \left[\frac{2\,(b^2c-4\,ac^2)x^2-(b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-(b^3-4\,abc)\log\left(cx^4\right.}{4\,(b^2c^2-4\,ac^3)}\right.$$

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*x^2 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b
*x^2 + a)) - (b^3 - 4*a*b*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1
/4*(2*(b^2*c - 4*a*c^2)*x^2 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^4 +
 b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(71) = 142.

Time = 1.11 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.90

$$\int \frac{x^6}{ax+bx^3+cx^5}\,dx = \left(-\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2+\frac{-ab-8ac^2\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)+2b^2c\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac}{4c^2\cdot(4ac-b^2}\right.}{2ac-b^2}\right.$$
$$+\left(-\frac{b}{4c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2+\frac{-ab-8ac^2\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)+2b^2c\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac}{4c^2\cdot(4ac-b^2}\right.}{2ac-b^2}\right.$$
$$+\frac{x^2}{2c}$$

```
[In] integrate(x**6/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*
log(x**2 + (-a*b - 8*a*c**2*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**
```

```
2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(
2*a*c - b**2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(4*c**2) + sq
rt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (-a*b
- 8*a*c**2*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c
 - b**2))) + 2*b**2*c*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*
c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

Maxima [F]

$$\int \frac{x^6}{ax+bx^3+cx^5}\,dx = \int \frac{x^6}{cx^5+bx^3+ax}\,dx$$

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - integrate((b*x^3 + a*x)/(c*x^4 + b*x^2 + a), x)/c
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{x^6}{ax+bx^3+cx^5}\,dx = \frac{x^2}{2\,c} - \frac{b\log\left(cx^4+bx^2+a\right)}{4\,c^2} + \frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^2}$$

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c - 1/4*b*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*arctan((2*
c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 655, normalized size of antiderivative = 8.09

$$\int \frac{x^6}{ax+bx^3+cx^5}\,dx = \frac{x^2}{2\,c} + \frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(2\,b^3-8\,a\,b\,c\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}$$

$$+\ \operatorname{atan}\left(\frac{2\,c^2\,\left(4\,a\,c-b^2\right)\left(\frac{\frac{\left(8\,a\,b+\frac{8\,a\,c^2\,\left(2\,b^3-8\,a\,b\,c\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)\,\left(2\,a\,c-b^2\right)}{8\,c^2\,\sqrt{4\,a\,c-b^2}}+\frac{a\,\left(2\,b^3-8\,a\,b\,c\right)\,\left(2\,a\,c-b^2\right)}{\sqrt{4\,a\,c-b^2}\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}}{a}-x^2\left(\frac{\frac{\left(2\,a\,c-b^2\right)\,\left(\frac{4\,a\,c^3-6\,b^2\,c^2}{c^2}-\frac{4\,b\,c^2\,\left(2\,b^3-8\,a\,b\,c\right)}{16\,a\,c^3-4\,b^2\,c^2}\right.}{8\,c^2\,\sqrt{4\,a\,c-b^2}}}{a}\right.\right.}{}\right.$$

[In] int(x^6/(a*x + b*x^3 + c*x^5),x)

[Out] x^2/(2*c) + (log(a + b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) + (atan((2*c^2*(4*a*c - b^2)*((((8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (a*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a - x^2*((((2*a*c - b^2)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/2)) - (b*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a + (b*(((2*b^3 - 8*a*b*c)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b^3 - a*b*c)/c^2 + (b*(2*a*c - b^2)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (b*((a*b^2)/c^2 + ((2*b^3 - 8*a*b*c)*(8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*(2*a*c - b^2)^2)/(c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^4 + 4*a^2*c^2 - 4*a*b^2*c))*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(1/2))

3.81 $\int \frac{x^5}{ax+bx^3+cx^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 179

$$\int \frac{x^5}{ax+bx^3+cx^5}\,dx=\frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] x/c-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b^
2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arc
tan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c
+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1599, 1136, 1180, 211}

$$\int \frac{x^5}{ax+bx^3+cx^5}\,dx=-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

```
[In] Int[x^5/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^4}{a+bx^2+cx^4}\,dx\\
&= \frac{x}{c}-\frac{\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx}{c}\\
&= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}\\
&= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.13

$$\int \frac{x^5}{ax+bx^3+cx^5}\,dx=\frac{x}{c}-\frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[In] Integrate[x^5/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.32

method	result
risch	$\frac{x}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\_R^2b-a\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{x}{c}+\frac{\left(-b\sqrt{-4ac+b^2}+2ac-b^2\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(b^2-2ac-b\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$

```
[In] int(x^5/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] x/c+1/2/c*sum((-_R^2*b-a)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+
a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. $2(143) = 286$.

Time = 0.27 (sec) , antiderivative size = 1059, normalized size of antiderivative = 5.92

$$\int \frac{x^5}{ax+bx^3+cx^5}\,dx = -\frac{\sqrt{\frac{1}{2}}c\sqrt{-\frac{b^3-3\,abc+(b^2c^3-4\,ac^4)\sqrt{\frac{b^4-2\,ab^2c+a^2c^2}{b^2c^6-4\,ac^7}}}{b^2c^3-4\,ac^4}}\log\left(-2\,(ab^2-a^2c)x+\sqrt{\frac{1}{2}}\left(b^4-5\,ab^2c+4\,a^2c^2-(b^3c^3-4\,ab\right.\right.}{}$$

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2
 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b
*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 -
4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2
 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b^3
- 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4
- 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2
*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (
b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(
b^2*c^3 - 4*a*c^4))) - 2*x)/c
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.72

$$\int \frac{x^5}{ax+bx^3+cx^5}\,dx$$
$$= \text{RootSum}\left(t^4\cdot\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\cdot\left(48a^2bc^2-28ab^3c+4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{32t^3\right.\right.\right.$$
$$+\frac{x}{c}$$

```
[In] integrate(x**5/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t**3
*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**4)
/(a**2*c - a*b**2)))) + x/c
```

Maxima [F]

$$\int \frac{x^5}{ax+bx^3+cx^5}\,dx = \int \frac{x^5}{cx^5+bx^3+ax}\,dx$$

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2109 vs. 2(143) = 286.

Time = 0.56 (sec) , antiderivative size = 2109, normalized size of antiderivative = 11.78

$$\int \frac{x^5}{ax+bx^3+cx^5}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] x/c - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
```

```
4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*
b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 -
2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 + 2*(sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a
*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*
c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2)
)/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*
b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2
- 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^
2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3
+ 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^
2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*
c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 3026, normalized size of antiderivative = 16.91

$$\int \frac{x^5}{ax + bx^3 + cx^5}\,dx = \text{Too large to display}$$

```
[In] int(x^5/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] x/c - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*
(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x
*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4
*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3 - 4
*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c
))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)
 + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2
*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c -
 (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c
^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^
```

```
3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 -
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2
*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4
*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))^(1/2)*1i)/((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4
)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)
*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2
*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*
b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5
- b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
```

3.82 $\int \frac{x^4}{ax+bx^3+cx^5} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 63

$$\int \frac{x^4}{ax+bx^3+cx^5} \, dx = \frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[Out] 1/4*ln(c*x^4+b*x^2+a)/c+1/2*b*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c/(-4
*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1599, 1128, 648, 632, 212, 642}

$$\int \frac{x^4}{ax+bx^3+cx^5} \, dx = \frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[In] Int[x^4/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^2 + c*x^4]/(4*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^3}{a+bx^2+cx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c}-\frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c}\\
&= \frac{\log\left(a+bx^2+cx^4\right)}{4c}+\frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{x^4}{ax+bx^3+cx^5}\,dx=\frac{-\frac{2b\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[In] Integrate[x^4/(a*x + b*x^3 + c*x^5),x]
[Out] ((-2*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^2 + c*x^4])/(4*c)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
default	$\frac{\ln(c\,x^4+b\,x^2+a)}{4c}-\frac{b\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2c\sqrt{4ac-b^2}}$
risch	$\frac{\ln\left(\left(-4abc+b^3+\sqrt{-b^2(4ac-b^2)}\,b\right)x^2+2\sqrt{-b^2(4ac-b^2)}\,a\right)a}{4ac-b^2}-\frac{\ln\left(\left(-4abc+b^3+\sqrt{-b^2(4ac-b^2)}\,b\right)x^2+2\sqrt{-b^2(4ac-b^2)}\,a\right)b^2}{4c(4ac-b^2)}+$

```
[In] int(x^4/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(c*x^4+b*x^2+a)/c-1/2*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.13

$$\int \frac{x^4}{ax+bx^3+cx^5}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+(b^2-4\,ac)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c-4\,ac^2)},\frac{2\sqrt{-b^2+4\,ac}b\,\mathrm{ar}}{}\right.$$

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^2 - 4*a*c)*log(c*x^4 +
 b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^4 + b*x^2
+ a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(54) = 108$.

Time = 0.50 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.54

$$\int \frac{x^4}{ax + bx^3 + cx^5}\, dx = \left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)} + \frac{1}{4c}\right)\log\left(x^2 + \frac{-8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)} + \frac{1}{4c}\right) + 2a + 2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)} + \frac{1}{4c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)} + \frac{1}{4c}\right)\log\left(x^2 + \frac{-8ac\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)} + \frac{1}{4c}\right) + 2a + 2b^2\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)} + \frac{1}{4c}\right)}{b}\right)$$

```
[In] integrate(x**4/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*
(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(-b*
sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b) + (b*sqrt(-4*a*c +
b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*(b*sqrt(-4*a*c + b
**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(b*sqrt(-4*a*c + b**2)/
(4*c*(4*a*c - b**2)) + 1/(4*c)))/b)
```

Maxima [F]

$$\int \frac{x^4}{ax + bx^3 + cx^5}\, dx = \int \frac{x^4}{cx^5 + bx^3 + ax}\, dx$$

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^5 + b*x^3 + a*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{ax + bx^3 + cx^5}\, dx = -\frac{b \arctan\left(\frac{2\, cx^2 + b}{\sqrt{-b^2 + 4\, ac}}\right)}{2\, \sqrt{-b^2 + 4\, ac} c} + \frac{\log\left(cx^4 + bx^2 + a\right)}{4\, c}$$

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
4*log(c*x^4 + b*x^2 + a)/c
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.87

$$\int \frac{x^4}{ax + bx^3 + cx^5}\, dx = \frac{4\, a\, c\, \ln\left(c\, x^4 + b\, x^2 + a\right)}{16\, a\, c^2 - 4\, b^2\, c} - \frac{b^2\, \ln\left(c\, x^4 + b\, x^2 + a\right)}{16\, a\, c^2 - 4\, b^2\, c} - \frac{b\, \mathrm{atan}\left(\frac{b}{\sqrt{4\, a\, c - b^2}} + \frac{2\, c\, x^2}{\sqrt{4\, a\, c - b^2}}\right)}{2\, c\, \sqrt{4\, a\, c - b^2}}$$

```
[In] int(x^4/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] (4*a*c*log(a + b*x^2 + c*x^4))/(16*a*c^2 - 4*b^2*c) - (b^2*log(a + b*x^2 +
c*x^4))/(16*a*c^2 - 4*b^2*c) - (b*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x^2)/(4
*a*c - b^2)^(1/2)))/(2*c*(4*a*c - b^2)^(1/2))
```

3.83 $\int \frac{x^3}{ax+bx^3+cx^5} dx$

Optimal result

Integrand size = 20, antiderivative size = 150

$$\int \frac{x^3}{ax+bx^3+cx^5} dx = -\frac{\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/
2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1599, 1144, 211}

$$\int \frac{x^3}{ax+bx^3+cx^5} dx = \frac{\sqrt{\sqrt{b^2-4ac}+b}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[In] Int[x^3/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sqrt[b + Sqrt[b^2 - 4*
```

```
a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b^2 - 4*a*c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^2}{a+bx^2+cx^4}\,dx \\
&= -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx\right) \\
&\quad +\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx \\
&= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.10

$$
\int \frac{x^3}{ax+bx^3+cx^5}\,dx = \frac{\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}
$$

```
[In] Integrate[x^3/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.27

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2c\_R^{3}+\_Rb}\right)}{2}$	41
default	$4c\left(\frac{\left(b+\sqrt{-4ac+b^{2}}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8c\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\left(-b+\sqrt{-4ac+b^{2}}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,c\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)$	149

```
[In] int(x^3/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum(_R^2/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(115) = 230.

Time = 0.28 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.73

$$\int \frac{x^3}{ax+bx^3+cx^5}\,dx = \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right)$$

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sq
rt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/
```

```
2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c -
4*a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqr
t(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/2
*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4
*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(
b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) + 1/2*s
qrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a
*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b
^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.50

$$\int \frac{x^3}{ax+bx^3+cx^5}\,dx$$
$$= \text{RootSum}\left(t^4\cdot\left(256a^2c^3-128ab^2c^2+16b^4c\right)+t^2\left(-16abc+4b^3\right)+a,\ \left(t\mapsto t\log\left(64t^3ac^2-16t^3b^2c-2tb+\right.\right.\right.$$

```
[In] integrate(x**3/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t**2*(-16*a*
b*c + 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c - 2*
_t*b + x)))
```

Maxima [F]

$$\int \frac{x^3}{ax+bx^3+cx^5}\,dx = \int \frac{x^3}{cx^5+bx^3+ax}\,dx$$

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^5 + b*x^3 + a*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. 2(115) = 230.

Time = 0.64 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.35

$$\int \frac{x^3}{ax + bx^3 + cx^5}\,dx$$

$$= \frac{\left(2\,b^2c^2 - 8\,ac^3 - \sqrt{2}\sqrt{b^2 - 4\,ac}\sqrt{bc + \sqrt{b^2 - 4\,ac}c}b^2 + 4\,\sqrt{2}\sqrt{b^2 - 4\,ac}\sqrt{bc + \sqrt{b^2 - 4\,ac}c}ac + 2\,\sqrt{2}\sqrt{b^2 \cdots}\right.}{2\,(b^4 - 8\,ab^2c - 2\,b^3c + 16\,a^2 \cdots}$$

$$+ \frac{\left(2\,b^2c^2 - 8\,ac^3 - \sqrt{2}\sqrt{b^2 - 4\,ac}\sqrt{bc - \sqrt{b^2 - 4\,ac}c}b^2 + 4\,\sqrt{2}\sqrt{b^2 - 4\,ac}\sqrt{bc - \sqrt{b^2 - 4\,ac}c}ac + 2\,\sqrt{2}\sqrt{b \cdots}\right.}{2\,(b^4 - 8\,ab^2c - 2\,b^3c + 16 \cdots}$$

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*
c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((b^4 - 8*a*b
^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c)) + 1/2*
(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c
^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((b^4 - 8*a*b^2*c
 - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.77

$$\int \frac{x^3}{ax + bx^3 + cx^5}\,dx =$$

$$-2\,\mathrm{atanh}\left(\frac{\left(x\,(4\,a\,c^2 - 2\,b^2\,c) + \frac{x\,(8\,b^3\,c^2 - 32\,a\,b\,c^3)\,\left(b^3 + \sqrt{-(4\,a\,c - b^2)^3} - 4\,a\,b\,c\right)}{8\,(16\,a^2\,c^3 - 8\,a\,b^2\,c^2 + b^4\,c)}\right)\sqrt{-\frac{b^3 + \sqrt{-(4\,a\,c - b^2)^3} - 4\,a\,b\,c}{8\,(16\,a^2\,c^3 - 8\,a\,b^2\,c^2 + b^4\,c)}}}{a\,c}\right)\sqrt{- \cdots}$$

$$-2\,\mathrm{atanh}\left(\frac{\left(x\,(4\,a\,c^2 - 2\,b^2\,c) - \frac{x\,(8\,b^3\,c^2 - 32\,a\,b\,c^3)\,\left(\sqrt{-(4\,a\,c - b^2)^3} - b^3 + 4\,a\,b\,c\right)}{8\,(16\,a^2\,c^3 - 8\,a\,b^2\,c^2 + b^4\,c)}\right)\sqrt{\frac{\sqrt{-(4\,a\,c - b^2)^3} - b^3 + 4\,a\,b\,c}{8\,(16\,a^2\,c^3 - 8\,a\,b^2\,c^2 + b^4\,c)}}}{a\,c}\right)\sqrt{\frac{\sqrt{- \cdots}}{8\,(\cdots}}$$

```
[In] int(x^3/(a*x + b*x^3 + c*x^5),x)

[Out] - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) + (x*(8*b^3*c^2 - 32*a*b*c^3)*(b^3 + (-(4
*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-
(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2
*c^2)))^(1/2))/(a*c))*(-(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*
c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) - (
x*(8*b^3*c^2 - 32*a*b*c^3)*((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c))/(8*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*
b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(a*c))*(((-(4*a*c - b^2
)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

3.84 $\int \frac{x^2}{ax+bx^3+cx^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 36

$$\int \frac{x^2}{ax+bx^3+cx^5}\,dx=-\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1599, 1121, 632, 212}

$$\int \frac{x^2}{ax+bx^3+cx^5}\,dx=-\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[In] Int[x^2/(a*x + b*x^3 + c*x^5),x]
[Out] -(ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x}{a+bx^2+cx^4}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)\\ &= -\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)\\ &= -\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{ax+bx^3+cx^5}\,dx = \frac{\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

```
[In] Integrate[x^2/(a*x + b*x^3 + c*x^5),x]
[Out] ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}$	36
risch	$-\frac{\ln\left(\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}-2a\right)}{2\sqrt{-4ac+b^{2}}}+\frac{\ln\left(\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}+2a\right)}{2\sqrt{-4ac+b^{2}}}$	70

```
[In] int(x^2/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] 1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.58

$$\int \frac{x^2}{ax+bx^3+cx^5}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\,\sqrt{b^2-4\,ac}},\; -\frac{\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
[Out] [1/2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*arctan(-(
2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(34) = 68.

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.64

$$\int \frac{x^2}{ax+bx^3+cx^5}\,dx = -\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2} + \frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}$$

```
[In] integrate(x**2/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2 + sqrt(-1/(4*a*c - b**2))*log(x**2 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2
```

Maxima [F]

$$\int \frac{x^2}{ax + bx^3 + cx^5}\,dx = \int \frac{x^2}{cx^5 + bx^3 + ax}\,dx$$

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^5 + b*x^3 + a*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{ax + bx^3 + cx^5}\,dx = \frac{\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B] (verification not implemented)

Time = 8.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{ax + bx^3 + cx^5}\,dx = \frac{\mathrm{atan}\left(\frac{2\,a\,c\,x^2+a\,b}{a\,\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

```
[In] int(x^2/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] atan((a*b + 2*a*c*x^2)/(a*(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)^(1/2)
```

3.85 $\int \frac{x}{ax+bx^3+cx^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 150

$$\int \frac{x}{ax+bx^3+cx^5}\,dx=\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*
a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*2^(1/2)*c^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1599, 1107, 211}

$$\int \frac{x}{ax+bx^3+cx^5}\,dx=\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[In] Int[x/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{a+bx^2+cx^4}\,dx\\ &= \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}\\ &= \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.86

$$\int \frac{x}{ax+bx^3+cx^5}\,dx = \frac{\sqrt{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

```
[In] Integrate[x/(a*x + b*x^3 + c*x^5),x]
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/S
qrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.25

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\ln\left(x-\_R\right)}{2c\_R^{3}+\_Rb}\right)}{2}$	38
default	$4c\left(-\frac{\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{4\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{4\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)$	117

```
[In] int(x/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] 1/2*sum(1/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(114) = 228.

Time = 0.28 (sec) , antiderivative size = 613, normalized size of antiderivative = 4.09

$$\int \frac{x}{ax+bx^3+cx^5}\,dx = -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx + \sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx - \sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx + \sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx - \sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$

```
[In] integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(
a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^
2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^
3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a
^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 -
4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*
(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*
sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*lo
g(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3
*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c
)))
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.58

$$\int \frac{x}{ax + bx^3 + cx^5}\,dx$$
$$= \text{RootSum}\left(t^4 \cdot \left(256a^3c^2 - 128a^2b^2c + 16ab^4\right) + t^2\left(-16abc + 4b^3\right) + c, \left(t \mapsto t\log\left(x + \frac{32t^3a^2bc - 8t^3ab^3}{c}\right.\right.\right.$$

```
[In] integrate(x/(c*x**5+b*x**3+a*x),x)
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a*
b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b**
3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

Maxima [F]

$$\int \frac{x}{ax + bx^3 + cx^5}\,dx = \int \frac{x}{cx^5 + bx^3 + ax}\,dx$$

```
[In] integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
[Out] integrate(x/(c*x^5 + b*x^3 + a*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1024 vs. 2(114) = 228.

Time = 0.59 (sec) , antiderivative size = 1024, normalized size of antiderivative = 6.83

$$\int \frac{x}{ax + bx^3 + cx^5}\,dx$$
$$= \frac{\left(\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4\,ac}c}b^4 - 8\,\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4\,ac}c}ab^2c - 2\,\sqrt{2}\sqrt{bc + \sqrt{b^2 - 4\,ac}c}b^3c - 2\,b^4c + 16\,\sqrt{2}\sqrt{\cdots}\right.}{\cdots}$$
$$+ \frac{\left(\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4\,ac}c}b^4 - 8\,\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4\,ac}c}ab^2c - 2\,\sqrt{2}\sqrt{bc - \sqrt{b^2 - 4\,ac}c}b^3c + 2\,b^4c + 16\,\sqrt{2}\cdots\right.}{\cdots}$$

```
[In] integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="giac")
[Out] 1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
```

```
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*
b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^
2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs
(c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 -
4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4
 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c
^3)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 763, normalized size of antiderivative = 5.09

$$\int \frac{x}{ax+bx^3+cx^5}\,dx =$$

$$-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i}+b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c^2\,\cdots}{4\,a\,b^4\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}\cdots}\right.$$
$$-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i}-b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c^2\,x\,16\cdots}{4\,a\,b^4\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-32\cdots}\right.$$

```
[In] int(x/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] - atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/
2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*(-(b^3 + (b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*
a^2*b^2*c))^(1/2) + 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
- 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2
) - 32*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^
```

```
(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(-(b^3 + (
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 +
 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i
)/(4*a*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 +
 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^3*c^2*(((b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^
4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(((b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2
 - 64*a^2*b^2*c))^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i
```

3.86 $\int \frac{1}{ax+bx^3+cx^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 69

$$\int \frac{1}{ax+bx^3+cx^5}\,dx=\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}$$

```
[Out] ln(x)/a-1/4*ln(c*x^4+b*x^2+a)/a+1/2*b*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2
))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1608, 1128, 719, 29, 648, 632, 212, 642}

$$\int \frac{1}{ax+bx^3+cx^5}\,dx=\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

```
[In] Int[(a*x + b*x^3 + c*x^5)^(-1),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^2 + c*x^4]/(4*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\,(a + bx^2 + cx^4)}\,dx$$

$$
\begin{aligned}
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2a} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, x^2\right)}{2a} \\
&= \frac{\log(x)}{a} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a} \\
&= \frac{\log(x)}{a} - \frac{\log\left(a+bx^2+cx^4\right)}{4a} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2a} \\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log\left(a+bx^2+cx^4\right)}{4a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.64

$$
\int \frac{1}{ax+bx^3+cx^5}\,dx
= \frac{4\sqrt{b^2-4ac}\log(x) - \left(b+\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right) + \left(b-\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+\right.}{4a\sqrt{b^2-4ac}}
$$

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^(-1),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*
a*c] + 2*c*x^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\frac{\ln\left(c\,x^4+b\,x^2+a\right)}{2}+\frac{b\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a}$	65
risch	$\frac{\ln(x)}{a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4c\,a^2-b^2a\right)\_Z^2+\left(4ac-b^2\right)\_Z+c\right)}\_R\ln\left(\left(\left(10ac-3b^2\right)\_R+5c\right)x^2-ab\_R+2b\right)\right)}{2}$	77

```
[In] int(1/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/a-1/2/a*(1/2*ln(c*x^4+b*x^2+a)+b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.23

$$\int \frac{1}{ax + bx^3 + cx^5} \, dx$$
$$= \left[\frac{\sqrt{b^2 - 4\,ac}b \log\left(\frac{2\,c^2x^4 + 2\,bcx^2 + b^2 - 2\,ac + (2\,cx^2 + b)\sqrt{b^2 - 4\,ac}}{cx^4 + bx^2 + a}\right) - (b^2 - 4\,ac)\log\left(cx^4 + bx^2 + a\right) + 4\,(b^2 - 4\,ac)\log}{4\,(ab^2 - 4\,a^2c)} \right.$$

```
[In] integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^2 - 4*a*c)*log(c*x^4 +
 b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/4*(2*sqrt(-b^2 +
 4*a*c)*b*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 -
4*a*c)*log(c*x^4 + b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.

Time = 8.64 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.67

$$\int \frac{1}{ax + bx^3 + cx^5} \, dx$$
$$= \left(-\frac{b\sqrt{-4ac + b^2}}{4a\,(4ac - b^2)} - \frac{1}{4a}\right)\log\left(x^2 + \frac{-8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)} - \frac{1}{4a}\right) + 2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)} - \frac{1}{4a}\right) - 2ac + b^2}{bc}\right)$$
$$+ \left(\frac{b\sqrt{-4ac + b^2}}{4a\,(4ac - b^2)} - \frac{1}{4a}\right)\log\left(x^2 + \frac{-8a^2c\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)} - \frac{1}{4a}\right) + 2ab^2\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)} - \frac{1}{4a}\right) - 2ac + b^2}{bc}\right)$$
$$+ \frac{\log(x)}{a}$$

```
[In] integrate(1/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2
*c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(-b*s
```

```
qrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) +
 (b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2
*c*(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(b*sqr
t(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) + l
og(x)/a
```

Maxima [F]

$$\int \frac{1}{ax + bx^3 + cx^5} \, dx = \int \frac{1}{cx^5 + bx^3 + ax} \, dx$$

```
[In] integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
[Out] -integrate((c*x^3 + b*x)/(c*x^4 + b*x^2 + a), x)/a + log(x)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{1}{ax + bx^3 + cx^5} \, dx = -\frac{b \arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a} - \frac{\log\left(cx^4+bx^2+a\right)}{4\,a} + \frac{\log\left(x^2\right)}{2\,a}$$

```
[In] integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="giac")
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
4*log(c*x^4 + b*x^2 + a)/a + 1/2*log(x^2)/a
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 1014, normalized size of antiderivative = 14.70

$$\int \frac{1}{ax+bx^3+cx^5}\,dx = \frac{\ln(x)}{a} + \frac{\ln(c\,x^4+b\,x^2+a)\,(8\,a\,c-2\,b^2)}{2\,(4\,a\,b^2-16\,a^2\,c)} + \frac{b\,\mathrm{atan}\left(\frac{16\,a^3\,x^2\left(\frac{(3\,b^3-8\,a\,b\,c)\left(\frac{(8\,a\,c-2\,b^2)^2\left(10\,b\,c^3-\frac{(12\,b^3\,c^2-40\,a\,b\,c^3)\,(8\,a\,c-2\,b^2)}{2\,(4\,a\,b^2-16\,a^2\,c)}\right)}{4\,(4\,a\,b^2-16\,a^2\,c)^2}-\frac{b^2\left(10\,b\,c^3-\frac{(12\,b^3\,c^2-40\,a\,b\,c^3)\,(8\,a\,c-2\,b^2)}{2\,(4\,a\,b^2-16\,a^2\,c)}\right)}{16\,a^2\,(4\,a\,c-b^2)}+\frac{b^2\,\cdots}{16\,\cdots}\right)}{8\,a^3\,c^2\,(25\,a\,c-6\,b^2)}\cdots\right)}{\cdots}\right)}{\cdots}$$

```
[In] int(1/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] log(x)/a + (log(a + b*x^2 + c*x^4)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c)
) + (b*atan((16*a^3*x^2*(((3*b^3 - 8*a*b*c)*(((8*a*c - 2*b^2)^2*(10*b*c^3 -
 ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*
(4*a*b^2 - 16*a^2*c)^2) - (b^2*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*
c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(16*a^2*(4*a*c - b^2)) + (b^2*(12*b^
3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(16*a^2*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*(
(b^3*(12*b^3*c^2 - 40*a*b*c^3))/(64*a^3*(4*a*c - b^2)^(3/2)) - (b*(12*b^3*c
^2 - 40*a*b*c^3)*(8*a*c - 2*b^2)^2)/(16*a*(4*a*b^2 - 16*a^2*c)^2*(4*a*c - b
^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a
*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^
2)^(3/2))/(b^2*c^2) + (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)*(((8*a*c - 2
*b^2)^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))/(4*a*b^2 - 16*a^2*c)))/(
4*(4*a*b^2 - 16*a^2*c)^2) - (b^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))
/(4*a*b^2 - 16*a^2*c)))/(16*a^2*(4*a*c - b^2)) + (b^4*c^2*(8*a*c - 2*b^2))/
(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2))))/(b^2*c^4*(25*a*c - 6*b^2)) - (2*
(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b^5*c^2)/(16*a^2*(4*a*c -
 b^2)^(3/2)) - (b^3*c^2*(8*a*c - 2*b^2)^2)/(4*(4*a*b^2 - 16*a^2*c)^2*(4*a*c
 - b^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^
2))/(4*a*b^2 - 16*a^2*c)))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))
/(b^2*c^4*(25*a*c - 6*b^2))))/(2*a*(4*a*c - b^2)^(1/2))
```

3.87 $\int \frac{1}{x(ax+bx^3+cx^5)} dx$

Optimal result

Integrand size = 20, antiderivative size = 174

$$\int \frac{1}{x(ax+bx^3+cx^5)} dx = -\frac{1}{ax} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/a/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(
1+b/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x
*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(1-b/(-4*a*c+b^2)^(1
/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1599, 1137, 1180, 211}

$$\int \frac{1}{x(ax+bx^3+cx^5)} dx = -\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}} - \frac{1}{ax}$$

```
[In] Int[1/(x*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sq
rt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

$$= -\frac{1}{ax}+\frac{\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx}{a}$$

$$= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}$$

$$= -\frac{1}{ax}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\,(ax+bx^3+cx^5)}\,dx = -\frac{\frac{2}{x}+\frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2a}$$

```
[In] Integrate[1/(x*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] -1/2*(2/x + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*
a*c]]))/a
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

method	result
default	$\frac{4c\left(\frac{\left(b-\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-b-\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{a}-\frac{1}{ax}$
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^5c^2-8a^4b^2c+b^4a^3\right)\_Z^4+\left(12a^2b\,c^2-7a\,b^3c+b^5\right)\_Z^2+c^3\right)}\_R\ln\left(\left(\left(40a^5c^2-22a^4b^2c+3b^4a^3\right)\_R^4+\left(25a^2b\right.\right.\right.\right.}{2}$

```
[In] int(1/x/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 4/a*c*(1/8*(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8
*(-b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/a/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1116 vs. 2(137) = 274.

Time = 0.27 (sec) , antiderivative size = 1116, normalized size of antiderivative = 6.41

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)}\,dx = -\frac{\sqrt{\frac{1}{2}}ax\sqrt{-\frac{b^3-3\,abc+\left(a^3b^2-4\,a^4c\right)\sqrt{\frac{b^4-2\,ab^2c+a^2c^2}{a^6b^2-4\,a^7c}}}{a^3b^2-4\,a^4c}}\log\left(-2\left(b^2c^2-ac^3\right)x+\sqrt{\frac{1}{2}}\left(b^5-5\,ab^3c+4\,a^2bc^2-\left(a^3b^4-\right.\right.\right.}{}$$

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2
*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a
^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))
)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(b^3 -
 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 -
4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*(b^5
 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^
2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^
2 - 4*a^4*c))) + sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*
log(-2*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^
3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2
- 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*s
qrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqr
t(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*
c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c
)))/(a^3*b^2 - 4*a^4*c))) + 2)/(a*x)
```

Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)}\,dx$$
$$= \operatorname{RootSum}\left(t^4\cdot\left(256a^5c^2-128a^4b^2c+16a^3b^4\right)+t^2\cdot\left(48a^2bc^2-28ab^3c+4b^5\right)+c^3,\left(t\mapsto t\log\left(x+\frac{-64t\right.\right.\right.$$
$$-\frac{1}{ax}$$

```
[In] integrate(1/x/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + c**3, Lambda(_t, _t*log(x + (-64*_t**
3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*a**3*b**4 - 10*_t*a**2*b*c**2
+ 10*_t*a*b**3*c - 2*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(a*x)
```

Maxima [F]

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)}\,dx = \int \frac{1}{(cx^5+bx^3+ax)x}\,dx$$

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b)/(c*x^4 + b*x^2 + a), x)/a - 1/(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1839 vs. 2(137) = 274.

Time = 0.73 (sec) , antiderivative size = 1839, normalized size of antiderivative = 10.57

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*b^2*c
^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
```

```
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*
(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5
 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 -
 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^
2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(1/2)*
x/sqrt((a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*
a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(a)*abs(
c)) - 1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*
b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c -
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2
 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*
c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^3 -
2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(
1/2)*x/sqrt((a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4 - 8*a^4*b^2*c
 - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(a)
*abs(c)) - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 2997, normalized size of antiderivative = 17.22

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)}\,dx = \text{Too large to display}$$

```
[In] int(1/(x*(a*x + b*x^3 + c*x^5)),x)
```

```
[Out] - atan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a
^4*b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^
3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b
^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b
*c^3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2
*c^3) + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/
2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5
+ b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - a
tan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*
b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a
```

```
^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^3*c
^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a
^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5
- b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^
3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b
^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2*c^
3) + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*
(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*(-(b
^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - 1/(a
*x)
```

3.88 $\int \frac{1}{x^2(ax+bx^3+cx^5)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{1}{x^2(ax+bx^3+cx^5)}\,dx = -\frac{1}{2ax^2} - \frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^2+cx^4)}{4a^2}$$

```
[Out] -1/2/a/x^2-b*ln(x)/a^2+1/4*b*ln(c*x^4+b*x^2+a)/a^2-1/2*(-2*a*c+b^2)*arctanh
((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1599, 1128, 723, 814, 648, 632, 212, 642}

$$\int \frac{1}{x^2(ax+bx^3+cx^5)}\,dx = -\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} + \frac{b\log(a+bx^2+cx^4)}{4a^2} - \frac{b\log(x)}{a^2} - \frac{1}{2ax^2}$$

```
[In] Int[1/(x^2*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] -1/2*1/(a*x^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(
2*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^2 + c*x^4])/(4*a
^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^3\left(a+bx^2+cx^4\right)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right) \\
&= -\frac{1}{2ax^2}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx, x, x^2\right)}{2a} \\
&= -\frac{1}{2ax^2}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)\,dx, x, x^2\right)}{2a} \\
&= -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2} \\
&= -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2}+\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2} \\
&= -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2a^2} \\
&= -\frac{1}{2ax^2}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.52

$$
\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)}\,dx = \frac{-\frac{2a}{x^2}-4b\log(x)+\frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{4a^2}
$$

```
[In] Integrate[1/(x^2*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] ((-2*a)/x^2 - 4*b*Log[x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b - Sqr
t[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-b^2 + 2*a*c + b*Sqrt[b^2
- 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
default	$-\frac{1}{2a\,x^{2}}-\frac{b\ln\left(x\right)}{a^{2}}-\frac{-\frac{b\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2}+\frac{2\left(ac-\frac{b^{2}}{2}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a^{2}}$
risch	$-\frac{1}{2a\,x^{2}}-\frac{b\ln\left(x\right)}{a^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{3}c-a^{2}b^{2}\right)\_Z^{2}+\left(-4abc+b^{3}\right)\_Z+c^{2}\right)}\_R\ln\left(\left(\left(10a^{3}c-3a^{2}b^{2}\right)\_R^{2}-4\_Rabc+2c^{2}\right)x^{2}-a\right.\right.}{2}$

```
[In] int(1/x^2/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/x^2-b*ln(x)/a^2-1/2/a^2*(-1/2*b*ln(c*x^4+b*x^2+a)+2*(a*c-1/2*b^2)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.29

$$\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)}\,dx$$

$$=\left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}x^2\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-(b^3-4\,abc)x^2\log\left(cx^4+bx^2+a\right)+}{4\,(a^2b^2-4\,a^3c)x^2}\right.$$

$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}x^2\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^3-4\,abc)x^2\log\left(cx^4+bx^2+a\right)+4\,(b^3-}{4\,(a^2b^2-4\,a^3c)x^2}$$

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2
 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^3 - 4
*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2), -1/4*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4
*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**2/(c*x**5+b*x**3+a*x),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)}\,dx = \int \frac{1}{(cx^5+bx^3+ax)x^2}\,dx$$

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
[Out] -b*log(x)/a^2 + integrate((b*c*x^3 + (b^2 - a*c)*x)/(c*x^4 + b*x^2 + a), x)
/a^2 - 1/2/(a*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)}\,dx = \frac{b\log\left(cx^4+bx^2+a\right)}{4\,a^2} - \frac{b\log\left(x^2\right)}{2\,a^2} + \frac{(b^2-2\,ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2} + \frac{bx^2-a}{2\,a^2x^2}$$

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x),x, algorithm="giac")
[Out] 1/4*b*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 + 1/2*(b^2 - 2*a*c)*a
rctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*x
^2 - a)/(a^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 2033, normalized size of antiderivative = 22.84

$$\int \frac{1}{x^2 (ax + bx^3 + cx^5)} dx = \text{Too large to display}$$

```
[In] int(1/(x^2*(a*x + b*x^3 + c*x^5)),x)
```

```
[Out] (atan((16*a^6*x^2*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(c^5/a^3 + ((2*b^3 - 8*a*
b*c)*((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3
+ ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*
a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) - ((((2
*a*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*
b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b
^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^
2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^2))/(4*a
^2*(4*a*c - b^2)^(1/2)) - ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2
)*(2*a*c - b^2)^2)/(32*a^7*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2))))/(8*a^3*c
^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((((2*b^3 - 8*a*b*c)*(((2*a*c - b^2)*(
(20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^
3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)) +
((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2))/(8*a^5*(4
*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) - ((
40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2)^3)/(64*a^9*(4*a*c - b^2)^(3/2)
) + (((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3
+ ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*
a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2)
^(1/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2
)*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4 + b^4*c^
2 - 4*a*b^2*c^3) - (2*a^3*(4*a*c - b^2)*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c)
*(((2*b^3 - 8*a*b*c)*((((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(2
*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2
)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2*a*(4*a*c - b^2)^(1/
2)*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) + ((2*a*c - b^2)*((
a^2*c^4 - 4*a*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*((4*a^3*b*c^3 - 4*a^2*b^3*c
^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2)))/(2*(16*a
^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(2*a*c - b^2)^3
)/(16*a^5*(4*a*c - b^2)^(3/2))))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2
*c^4 + b^4*c^2 - 4*a*b^2*c^3)) + (2*a^3*(4*a*c - b^2)^(3/2)*(3*b^4 + a^2*c^
2 - 9*a*b^2*c)*((b*c^4)/a^3 - ((2*b^3 - 8*a*b*c)*((a^2*c^4 - 4*a*b^2*c^3)/a
^3 + ((2*b^3 - 8*a*b*c)*((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(
2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2)))/(2*(16*a^3*c - 4*a^2*b^2))))/(2*
(16*a^3*c - 4*a^2*b^2)) + ((2*a*c - b^2)*((((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a
^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*a*c - b^2))
```

```
/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2
*a*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)
) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2)^2)/(8*a^3*(4*a*c - b^2)*(16*a^
3*c - 4*a^2*b^2))))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4 + b^4*c^
2 - 4*a*b^2*c^3)))*(2*a*c - b^2))/(2*a^2*(4*a*c - b^2)^(1/2)) - (b*log(x))/
a^2 - (log(a + b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2))
 - 1/(2*a*x^2)
```

3.89 $\int \frac{x^{11}}{(ax+bx^3+cx^5)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 166

$$\int \frac{x^{11}}{(ax+bx^3+cx^5)^2}\,dx = \frac{(b^2-3ac)\,x^2}{c^2\,(b^2-4ac)} - \frac{bx^4}{2c\,(b^2-4ac)} + \frac{x^6(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{(b^4-6ab^2c+6a^2c^2)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{3/2}} - \frac{b\log(a+bx^2+cx^4)}{2c^3}$$

```
[Out] (-3*a*c+b^2)*x^2/c^2/(-4*a*c+b^2)-1/2*b*x^4/c/(-4*a*c+b^2)+1/2*x^6*(b*x^2+2
*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x^2
+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)-1/2*b*ln(c*x^4+b*x^2+a)/c^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1599, 1128, 752, 814, 648, 632, 212, 642}

$$\int \frac{x^{11}}{(ax+bx^3+cx^5)^2}\,dx = -\frac{(6a^2c^2-6ab^2c+b^4)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{3/2}} + \frac{x^2(b^2-3ac)}{c^2\,(b^2-4ac)} - \frac{bx^4}{2c\,(b^2-4ac)} + \frac{x^6(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{b\log(a+bx^2+cx^4)}{2c^3}$$

```
[In] Int[x^11/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((b^2 - 3*a*c)*x^2)/(c^2*(b^2 - 4*a*c)) - (b*x^4)/(2*c*(b^2 - 4*a*c)) + (x^
6*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c
+ 6*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(
3/2)) - (b*Log[a + b*x^2 + c*x^4])/(2*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^9}{(a+bx^2+cx^4)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx+cx^2)^2}\,dx, x, x^2\right) \\
&= \frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{x^2(6a+2bx)}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2-4ac)} \\
&= \frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \left(-\frac{2(b^2-3ac)}{c^2}+\frac{2bx}{c}+\frac{2(a(b^2-3ac)+b(b^2-4ac)x)}{c^2(a+bx+cx^2)}\right)dx, x, x^2\right)}{2(b^2-4ac)} \\
&= \frac{(b^2-3ac)x^2}{c^2(b^2-4ac)} - \frac{bx^4}{2c(b^2-4ac)} + \frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\
&\quad - \frac{\text{Subst}\left(\int \frac{a(b^2-3ac)+b(b^2-4ac)x}{a+bx+cx^2}\,dx, x, x^2\right)}{c^2(b^2-4ac)} \\
&= \frac{(b^2-3ac)x^2}{c^2(b^2-4ac)} - \frac{bx^4}{2c(b^2-4ac)} + \frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\
&\quad - \frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{2c^3} + \frac{(b^4-6ab^2c+6a^2c^2)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2c^3(b^2-4ac)} \\
&= \frac{(b^2-3ac)x^2}{c^2(b^2-4ac)} - \frac{bx^4}{2c(b^2-4ac)} + \frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\
&\quad - \frac{b\log(a+bx^2+cx^4)}{2c^3} - \frac{(b^4-6ab^2c+6a^2c^2)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{c^3(b^2-4ac)} \\
&= \frac{(b^2-3ac)x^2}{c^2(b^2-4ac)} - \frac{bx^4}{2c(b^2-4ac)} + \frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\
&\quad - \frac{(b^4-6ab^2c+6a^2c^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}} - \frac{b\log(a+bx^2+cx^4)}{2c^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{(ax+bx^3+cx^5)^2}\,dx = \frac{cx^2+\frac{-b^4x^2-ab^2(b-4cx^2)+a^2c(3b-2cx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2(b^4-6ab^2c+6a^2c^2)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}-b\log\left(a+bx^2+cx^4\right)}{2c^3}$$

```
[In] Integrate[x^11/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (c*x^2 + (-(b^4*x^2) - a*b^2*(b - 4*c*x^2) + a^2*c*(3*b - 2*c*x^2))/((b^2 -
 4*a*c)*(a + b*x^2 + c*x^4)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b +
 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + b*x^2 + c*x
^4])/(2*c^3)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{x^{2}}{2c^{2}}-\frac{\frac{-\frac{\left(2a^{2}c^{2}-4a\,b^{2}c+b^{4}\right)x^{2}}{c\left(4ac-b^{2}\right)}+\frac{ba\left(3ac-b^{2}\right)}{c\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{\frac{\left(4abc-b^{3}\right)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{c}+\frac{4\left(3c\,a^{2}-b^{2}a-\frac{\left(4abc-b^{3}\right)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{4ac-b^{2}}}{2c^{2}}$	209
risch	Expression too large to display	1217

```
[In] int(x^11/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/c^2-1/2/c^2*((-(2*a^2*c^2-4*a*b^2*c+b^4)/c/(4*a*c-b^2)*x^2+b*a/c*(3
*a*c-b^2)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*(1/2*(4*a*b*c-b^3)/c*l
n(c*x^4+b*x^2+a)+2*(3*c*a^2-b^2*a-1/2*(4*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(156) = 312.

Time = 0.29 (sec) , antiderivative size = 868, normalized size of antiderivative = 5.23

$$\int \frac{x^{11}}{(ax+bx^3+cx^5)^2}\,dx = \left[\frac{(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^6-ab^5+7\,a^2b^3c-12\,a^3bc^2+(b^5c-8\,ab^3c^2+16\,a^2bc^3)x^4-(b^6-9\,ab^4c+26}{}\right.$$

```
[In] integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^6 - a*b^5 + 7*a^2*b^3*c - 12*a
^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^4 - (b^6 - 9*a*b^4*c + 26
*a^2*b^2*c^2 - 24*a^3*c^3)*x^2 - (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + (b^4*c
- 6*a*b^2*c^2 + 6*a^2*c^3)*x^4 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(
b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 +
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^4 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^
2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (
b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b
*c^5)*x^2), 1/2*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^6 - a*b^5 + 7*a^2*b
^3*c - 12*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^4 - (b^6 - 9*a
*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x^2 - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*
c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^4 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^
2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
4*a*c)) - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*x^4 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2)*log(c*x^4 + b*x^2 +
 a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*
a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(ax + bx^3 + cx^5)^2}\, dx = \text{Timed out}$$

```
[In] integrate(x**11/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{11}}{(ax + bx^3 + cx^5)^2}\, dx = \int \frac{x^{11}}{(cx^5 + bx^3 + ax)^2}\, dx$$

```
[In] integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*b^3 - 3*a^2*b*c + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^2)/(a*b^2*c^3 - 4
*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2) + 1/2*x^2/c
^2 + 2*integrate(-((b^3 - 4*a*b*c)*x^3 + (a*b^2 - 3*a^2*c)*x)/(c*x^4 + b*x^
2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

Giac [A] (verification not implemented)

none

Time = 0.61 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}}{(ax+bx^3+cx^5)^2}\,dx = \frac{(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^3-4\,ac^4)\sqrt{-b^2+4\,ac}} + \frac{x^2}{2\,c^2} + \frac{b^3x^4-4\,abcx^4-2\,a^2cx^2-a^2b}{2\,(cx^4+bx^2+a)(b^2c^2-4\,ac^3)} - \frac{b\log\left(cx^4+bx^2+a\right)}{2\,c^3}$$

```
[In] integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^
2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + 1/2*x^2/c^2 + 1/2*(b^3*x^4 - 4*a*b*c
*x^4 - 2*a^2*c*x^2 - a^2*b)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/2
*b*log(c*x^4 + b*x^2 + a)/c^3
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1473, normalized size of antiderivative = 8.87

$$\int \frac{x^{11}}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

```
[In] int(x^11/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] ((a*(b^3 - 3*a*b*c))/(2*c*(4*a*c - b^2)) + (x^2*(b^4 + 2*a^2*c^2 - 4*a*b^2*
c))/(2*c*(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) + x^2/(2*c^2) + (log
(a + b*x^2 + c*x^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*
(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (atan(((4*a*c^5*(
4*a*c - b^2)^3 - b^2*c^4*(4*a*c - b^2)^3)*(((((16*a*b)/c + (8*a*c^2*(b^7 -
64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(64*a^3*c^6 - b^6*c^3 + 12*a*b
^4*c^4 - 48*a^2*b^2*c^5))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*c^3*(4*a*c - b^
2)^(3/2)) + (4*a*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2
*b^3*c^2 - 12*a*b^5*c))/(c*(4*a*c - b^2)^(3/2)*(64*a^3*c^6 - b^6*c^3 + 12*a
*b^4*c^4 - 48*a^2*b^2*c^5)))/(2*a*(4*a*c - b^2)) - x^2*(((((4*(6*a^2*c^5 +
3*b^4*c^3 - 14*a*b^2*c^4))/(4*a*c^5 - b^2*c^4) + (2*(2*b^3*c^6 - 8*a*b*c^7)
*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((4*a*c^5 - b^2*c^4)*(
64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)))*(b^4 + 6*a^2*c^2 -
6*a*b^2*c))/(2*c^3*(4*a*c - b^2)^(3/2)) + ((2*b^3*c^6 - 8*a*b*c^7)*(b^4 + 6
*a^2*c^2 - 6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(
c^3*(4*a*c - b^2)^(3/2)*(4*a*c^5 - b^2*c^4)*(64*a^3*c^6 - b^6*c^3 + 12*a*b^
4*c^4 - 48*a^2*b^2*c^5)))/(2*a*(4*a*c - b^2)) + (b*((4*(b^5 + 3*a^2*b*c^2 -
 5*a*b^3*c))/(4*a*c^5 - b^2*c^4) + (((4*(6*a^2*c^5 + 3*b^4*c^3 - 14*a*b^2*c
```

```
^4))/(4*a*c^5 - b^2*c^4) + (2*(2*b^3*c^6 - 8*a*b*c^7)*(b^7 - 64*a^3*b*c^3 +
 48*a^2*b^3*c^2 - 12*a*b^5*c))/((4*a*c^5 - b^2*c^4)*(64*a^3*c^6 - b^6*c^3 +
 12*a*b^4*c^4 - 48*a^2*b^2*c^5)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12
*a*b^5*c))/(2*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - ((2
*b^3*c^6 - 8*a*b*c^7)*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^3
*(4*a*c^5 - b^2*c^4))))/(2*a*(4*a*c - b^2)^(3/2))) + (b*((4*a*b^2)/c^4 + ((
(16*a*b)/c + (8*a*c^2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(
64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5))*(b^7 - 64*a^3*b*c^3
+ 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 4
8*a^2*b^2*c^5)) - (4*a*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2)/(c^4*(4*a*c - b^2)^
3)))/(2*a*(4*a*c - b^2)^(3/2))))/(2*b^8 + 72*a^4*c^4 + 96*a^2*b^4*c^2 - 144
*a^3*b^2*c^3 - 24*a*b^6*c))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(c^3*(4*a*c - b^
2)^(3/2))
```

3.90 $\int \frac{x^{10}}{(ax+bx^3+cx^5)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 331

$$\int \frac{x^{10}}{(ax+bx^3+cx^5)^2} dx = \frac{(3b^2-10ac)\,x}{2c^2(b^2-4ac)} - \frac{bx^3}{2c(b^2-4ac)} + \frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(3b^3-13abc-\frac{3b^4-19ab^2c+20a^2c^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(3b^3-13abc+\frac{3b^4-19ab^2c+20a^2c^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*(-10*a*c+3*b^2)*x/c^2/(-4*a*c+b^2)-1/2*b*x^3/c/(-4*a*c+b^2)+1/2*x^5*(b*
x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2))*(3*b^3-13*a*b*c+(-20*a^2*c^2+19*a*b^2*c-3*b^4)/(-4*a*
c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4
*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3-13*a*b*c+(20
*a^2*c^2-19*a*b^2*c+3*b^4)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)
/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{1599, 1134, 1293, 1180, 211\}$

$$\int \frac{x^{10}}{(ax+bx^3+cx^5)^2}\,dx = -\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x(3b^2-10ac)}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[In] Int[x^10/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((3*b^2 - 10*a*c)*x)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(2*c*(b^2 - 4*a*c)) +
(x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3 - 13*a*
b*c - (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3 - 13*a*b*c + (3*b^4 - 19*a*b^2*c + 20
*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
```

```
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^8}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{x^4(10a+3bx^2)}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)}$$

$$= -\frac{bx^3}{2c(b^2-4ac)} + \frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\int \frac{x^2(9ab+3(3b^2-10ac)x^2)}{a+bx^2+cx^4}\,dx}{6c(b^2-4ac)}$$

$$= \frac{(3b^2-10ac)\,x}{2c^2(b^2-4ac)} - \frac{bx^3}{2c(b^2-4ac)} + \frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{3a(3b^2-10ac)+3b(3b^2-13ac)x^2}{a+bx^2+cx^4}\,dx}{6c^2(b^2-4ac)}$$

$$= \frac{(3b^2-10ac)\,x}{2c^2(b^2-4ac)} - \frac{bx^3}{2c(b^2-4ac)} + \frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(3b^3-13abc-\frac{3b^4-19ab^2c+20a^2c^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4c^2(b^2-4ac)} - \frac{\left(3b^3-13abc+\frac{3b^4-19ab^2c+20a^2c^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4c^2(b^2-4ac)}$$

$$= \frac{(3b^2-10ac)\,x}{2c^2(b^2-4ac)} - \frac{bx^3}{2c(b^2-4ac)} + \frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(3b^3-13abc-\frac{3b^4-19ab^2c+20a^2c^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(3b^3-13abc+\frac{3b^4-19ab^2c+20a^2c^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.99

$$\int \frac{x^{10}}{(ax+bx^3+cx^5)^2}\,dx$$
$$= \frac{4\sqrt{c}x - \frac{2\sqrt{c}x(2a^2c-b^3x^2-ab(b-3cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{2}\left(-3b^4+19ab^2c-20a^2c^2+3b^3\sqrt{b^2-4ac}-13abc\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}(3b}{}}{4c^{5/2}}$$

```
[In] Integrate[x^10/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (4*Sqrt[c]*x - (2*Sqrt[c]*x*(2*a^2*c - b^3*x^2 - a*b*(b - 3*c*x^2)))/((b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4 + 19*a*b^2*c - 20*a^2*c^2
+ 3*b^3*Sqrt[b^2 - 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (Sqrt[2]*(3*b^4 - 19*a*b^2*c + 20*a^2*c^2 + 3*b^3*Sqrt[b^2 -
 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c
^(5/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.53

method	result
risch	$\frac{x}{c^2} + \frac{\frac{b(3ac-b^2)x^3}{8ac-2b^2} + \frac{a(2ac-b^2)x}{8ac-2b^2}}{c^2(cx^4+bx^2+a)} + \frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)} \frac{\left(-\frac{b(13ac-3b^2)\_R^2}{4ac-b^2} - \frac{a(10ac-3b^2)}{4ac-b^2}\right)\ln(x-\_R)}{2c\_R^3+\_Rb}}{4c^2}$
default	$\frac{x}{c^2} - \frac{-\frac{b(3ac-b^2)x^3}{2(4ac-b^2)} - \frac{a(2ac-b^2)x}{2(4ac-b^2)}}{cx^4+bx^2+a} + \frac{2c\left(\frac{\left(13\sqrt{-4ac+b^2}\,abc-3\sqrt{-4ac+b^2}\,b^3-20a^2c^2+19ab^2c-3b^4\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{8c\sqrt{-4ac+b^2}\sqrt{(b+\sqrt{-4ac+b^2})c}} - \frac{(13}{}\right.}{4ac-b^2}}{c^2}$

```
[In] int(x^10/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^2+(1/2*b*(3*a*c-b^2)/(4*a*c-b^2)*x^3+1/2*a*(2*a*c-b^2)/(4*a*c-b^2)*x)/c
^2/(c*x^4+b*x^2+a)+1/4/c^2*sum((-b*(13*a*c-3*b^2)/(4*a*c-b^2)*_R^2-a*(10*a*
c-3*b^2)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2856 vs. 2(285) = 570.

Time = 0.48 (sec) , antiderivative size = 2856, normalized size of antiderivative = 8.63

$$\int \frac{x^{10}}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^2*c - 4*a*c^2)*x^5 + 2*(3*b^3 - 11*a*b*c)*x^3 + sqrt(1/2)*(a*b^2*
c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt
(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*
b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^
2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64
*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5
*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^
3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6 +
336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a*b^
6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385
*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 6
4*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3
 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))
)/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2)*(a*b
^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*s
qrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12
*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051
*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 -
 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*
a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818
*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6
 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a
*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c +
385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7
- 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^1
3)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqrt(1/2)*(
a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2
)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 -
 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3
051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
```

```
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^
7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 25
00*a^5*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8
818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b^7*
c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c
 + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c
^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b
^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*
c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2
)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*
x^2)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^
5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c
^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2
*c^7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 -
 2500*a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2
- 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b
^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 -
 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10
 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^
5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^
2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^
3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a
^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + 2*(3*
a*b^2 - 10*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^
3*c^2 - 4*a*b*c^3)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(ax + bx^3 + cx^5)^2} dx = \text{Timed out}$$

```
[In] integrate(x**10/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{10}}{(ax + bx^3 + cx^5)^2} \, dx = \int \frac{x^{10}}{(cx^5 + bx^3 + ax)^2} \, dx$$

```
[In] integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3 - 3*a*b*c)*x^3 + (a*b^2 - 2*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b
^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + 1/2*integrate(-(3*a*b^
2 - 10*a^2*c + (3*b^3 - 13*a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4
*a*c^3) + x/c^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3335 vs. $2(285) = 570$.

Time = 0.99 (sec) , antiderivative size = 3335, normalized size of antiderivative = 10.08

$$\int \frac{x^{10}}{(ax + bx^3 + cx^5)^2} \, dx = \text{Too large to display}$$

```
[In] integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b*c*x^3 + a*b^2*x - 2*a^2*c*x)/((c*x^4 + b*x^2 + a)*(b^2
*c^2 - 4*a*c^3)) + x/c^2 + 1/16*(6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8
 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 19
2*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9 - (6*b^5*c^2 - 50
*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
```

*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 +
13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b
^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2 - 2*(
3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^5*c^4 - 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b^2*c^5 + 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 68*a^2*b^4*c^5 - 160*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*
c^6 - 256*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^7
+ 320*a^4*c^7 + 6*(b^2 - 4*a*c)*a*b^4*c^4 - 44*(b^2 - 4*a*c)*a^2*b^2*c^5 +
80*(b^2 - 4*a*c)*a^3*c^6)*abs(b^2*c^2 - 4*a*c^3))*arctan(2*sqrt(1/2)*x/sqrt
((b^3*c^2 - 4*a*b*c^3 + sqrt((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^2
c^3)(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2*b^4*
c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a^4*c^
8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)*abs(b^2*c^2 - 4*a*c^3)*abs(c
)) - 1/16*(6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 6
40*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^
5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c
^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6
 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 46
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 1
92*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 +
31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 32
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b
^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^
3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9 - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b
*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 25
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 +
26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2 + 2*(3*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 6*a*b^6
*c^4 + 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(2)

```
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^4*c^5 - 68*a^2*b^4*c^5 - 160*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6
- 22*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 256*a^3*b^2*c^6
+ 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7 - 320*a^4*c^7 - 6*(b^2
 - 4*a*c)*a*b^4*c^4 + 44*(b^2 - 4*a*c)*a^2*b^2*c^5 - 80*(b^2 - 4*a*c)*a^3*c
^6)*abs(b^2*c^2 - 4*a*c^3))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2 - 4*a*b*c^3
- sqrt((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*c^3 - 4*a*c
^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b^5*c^6 + 48
*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*b*c^8 - 8*a
^2*b^2*c^8 + 16*a^3*c^9)*abs(b^2*c^2 - 4*a*c^3)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 7599, normalized size of antiderivative = 22.96

$$\int \frac{x^{10}}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

```
[In] int(x^10/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] ((b*x^3*(3*a*c - b^2))/(2*(4*a*c - b^2)) + (a*x*(2*a*c - b^2))/(2*(4*a*c -
b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) - atan(((((10240*a^5*c^7 + 48*a*b^8*c^
3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6
- b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*b^13 + 9*b^4*(-(4*a*c
 - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 +
 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2
) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11
+ b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*
b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3
*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b
^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 -
 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3
*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8 + 200*
a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5
+ b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 268
80*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4
4800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*
a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10
*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2
*c^10)))^(1/2)*1i - (((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 422
4*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4
- 48*a^2*b^2*c^5)) + (x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*
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a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4480
0*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^
6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^
10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7)
)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^
2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 3024
0*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 2
13*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^1
2*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c
^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2
 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b
^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2
c^2(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9
)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 -
1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/((((10
240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4
*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(
-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*
c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^
2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 128
0*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 -
 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b
*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5
*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 2
40*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))
^(1/2) - (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*
a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^
3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^
11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a
^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (((10240*a^5*c^7 + 48*a*b^8*c^3 -
 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b
^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*b^13 + 9*b^4*(-(4*a*c -
b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30
240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b
^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4
*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*
c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13

+ 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (63*a^3*b^5 - 573*a^4*b^3*c + 1300*a^5*b*c^2)/(4*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5))))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*2i - atan(((((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i - (((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7)

)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/((((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4

```
*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*
a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4480
0*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^
6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^
10)))^(1/2) + (63*a^3*b^5 - 573*a^4*b^3*c + 1300*a^5*b*c^2)/(4*(64*a^3*c^6
- b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5))))*(-(9*b^13 - 9*b^4*(-(4*a*c -
b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30
240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b
^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4
*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*2i + x/c^2
```

3.91 $\int \frac{x^9}{(ax+bx^3+cx^5)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 132

$$\int \frac{x^9}{(ax+bx^3+cx^5)^2} dx = -\frac{bx^2}{2c(b^2-4ac)} + \frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{b(b^2-6ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2-4ac)^{3/2}} + \frac{\log(a+bx^2+cx^4)}{4c^2}$$

```
[Out] -1/2*b*x^2/c/(-4*a*c+b^2)+1/2*x^4*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+
1/2*b*(-6*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)
^(3/2)+1/4*ln(c*x^4+b*x^2+a)/c^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1599, 1128, 752, 787, 648, 632, 212, 642}

$$\int \frac{x^9}{(ax+bx^3+cx^5)^2} dx = \frac{b(b^2-6ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2-4ac)^{3/2}} - \frac{bx^2}{2c(b^2-4ac)} + \frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\log(a+bx^2+cx^4)}{4c^2}$$

```
[In] Int[x^9/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -1/2*(b*x^2)/(c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]
)/(2*c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x^2 + c*x^4]/(4*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
```

```
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^7}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx+cx^2)^2}\,dx, x, x^2\right)$$

$$= \frac{x^4(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{x(4a+bx)}{a+bx+cx^2}\,dx, x, x^2\right)}{2\,(b^2-4ac)}$$

$$= -\frac{bx^2}{2c\,(b^2-4ac)} + \frac{x^4(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{-ab+(-b^2+4ac)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c\,(b^2-4ac)}$$

$$= -\frac{bx^2}{2c\,(b^2-4ac)} + \frac{x^4(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} - \frac{(b(b^2-6ac))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2\,(b^2-4ac)}$$

$$= -\frac{bx^2}{2c\,(b^2-4ac)} + \frac{x^4(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\log\left(a+bx^2+cx^4\right)}{4c^2} + \frac{(b(b^2-6ac))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c^2\,(b^2-4ac)}$$

$$= -\frac{bx^2}{2c\,(b^2-4ac)} + \frac{x^4(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\,(b^2-4ac)^{3/2}} + \frac{\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \frac{x^9}{(ax+bx^3+cx^5)^2}\,dx = \frac{\frac{2(-2a^2c+b^3x^2+ab(b-3cx^2))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{2b(b^2-6ac)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[In] Integrate[x^9/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x^2 + a*b*(b - 3*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2
 + 4*a*c)^(3/2) + Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.36

method	result	size
default	$\frac{\frac{b\left(3ac-b^2\right)x^2}{c^2\left(4ac-b^2\right)}+\frac{a\left(2ac-b^2\right)}{\left(4ac-b^2\right)c^2}}{2c\,x^4+2b\,x^2+2a}+\frac{\frac{\left(4ac-b^2\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(-ab-\frac{\left(4ac-b^2\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c(4ac-b^2)}$	179
risch	Expression too large to display	1017

```
[In] int(x^9/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x^2+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^
4+b*x^2+a)+1/2/c/(4*a*c-b^2)*(1/2*(4*a*c-b^2)/c*ln(c*x^4+b*x^2+a)+2*(-a*b-1
/2*(4*a*c-b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(120) = 240.

Time = 0.30 (sec) , antiderivative size = 663, normalized size of antiderivative = 5.02

$$\int \frac{x^9}{(ax+bx^3+cx^5)^2}\,dx = \left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+2\,(b^5-7\,ab^3c+12\,a^2bc^2)x^2+((b^3c-6\,abc^2)x^4+ab^3-6\,a^2bc+(b^4-6\,ab^2c}{4\,(ab^4c^2-8\,a^2b^2c^3}\right.$$

```
[In] integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c
^2)*x^2 + ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*
x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2
+ b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4
 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a
^2*b*c^4)*x^2), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3
*c + 12*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (
b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 +
4*a*c)/(b^2 - 4*a*c)) + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(c*x^4 +
 b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^9}{(ax+bx^3+cx^5)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**9/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^9}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^9}{(cx^5+bx^3+ax)^2}\,dx$$

```
[In] integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^2 - 2*a^2*c + (b^3 - 3*a*b*c)*x^2)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c
^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) - integrate(-((b^2 - 4*a*c)*
x^3 + a*b*x)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.15

$$\int \frac{x^9}{(ax+bx^3+cx^5)^2}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}} - \frac{b^2cx^4-4\,ac^2x^4-b^3x^2+2\,abcx^2-ab^2}{4\,(cx^4+bx^2+a)(b^2c^2-4\,ac^3)} + \frac{\log\left(cx^4+bx^2+a\right)}{4\,c^2}$$

```
[In] integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4
*a*c^3)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 - b^3*x^2 + 2*a*
b*c*x^2 - a*b^2)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/4*log(c*x^4
+ b*x^2 + a)/c^2
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 1336, normalized size of antiderivative = 10.12

$$\int \frac{x^9}{(ax+bx^3+cx^5)^2}\,dx$$

$$= \frac{\frac{a\,(2\,a\,c-b^2)}{2\,c^2\,(4\,a\,c-b^2)}+\frac{b\,x^2\,(3\,a\,c-b^2)}{2\,c^2\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a} - \frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(-128\,a^3\,c^3+96\,a^2\,b^2\,c^2-24\,a\,b^4\,c+2\,b^6\right)}{2\,\left(256\,a^3\,c^5-192\,a^2\,b^2\,c^4+48\,a\,b^4\,c^3-4\,b^6\,c^2\right)}$$

$$+ \frac{b\,\mathrm{atan}\left(\frac{\left(8\,a\,c^3\,(4\,a\,c-b^2)^3-2\,b^2\,c^2\,(4\,a\,c-b^2)^3\right)\left(x^2\left(\frac{\frac{b\left(\frac{6\,b^3\,c^2-28\,a\,b\,c^3}{4\,a\,c^3-b^2\,c^2}+\frac{\left(8\,b^3\,c^4-32\,a\,b\,c^5\right)\left(-128\,a^3\,c^3+96\,a^2\,b^2\,c^2-24\,a\,b^4\,c+2\,b^6\right)}{2\left(4\,a\,c^3-b^2\,c^2\right)\left(256\,a^3\,c^5-192\,a^2\,b^2\,c^4+48\,a\,b^4\,c^3-4\,b^6\,c^2\right)}\right)\left(6\,a\,c-\cdots\right.}{8\,c^2\,(4\,a\,c-b^2)^{3/2}}}{a\,\left(4\,a\,c-b\cdots\right.}\cdots\right.\right.}{\cdots}\right)}{\cdots}$$

```
[In] int(x^9/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] ((a*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)) + (b*x^2*(3*a*c - b^2))/(2*c^2*(4*
a*c - b^2)))/(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a^3
```

```
*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4
*c^3 - 192*a^2*b^2*c^4)) + (b*atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4
*a*c - b^2)^3)*(x^2*(((b*((6*b^3*c^2 - 28*a*b*c^3)/(4*a*c^3 - b^2*c^2) + ((
8*b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)
)/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*
b^2*c^4))))*(6*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(3/2)) + (b*(8*b^3*c^4 - 32*
a*b*c^5)*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))
/(16*c^2*(4*a*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 +
 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) - (b*((b^3 - 5*a*b*c)/
(4*a*c^3 - b^2*c^2) + (((6*b^3*c^2 - 28*a*b*c^3)/(4*a*c^3 - b^2*c^2) + ((8*
b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/
(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^
2*c^4)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c
^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (b^2*((b^3*c^4)/2 - 2*a
*b*c^5)*(6*a*c - b^2)^2)/(c^4*(4*a*c - b^2)^3*(4*a*c^3 - b^2*c^2))))/(2*a*(
4*a*c - b^2)^(3/2))) - ((b*(6*a*c - b^2)*(8*a + (8*a*c^2*(2*b^6 - 128*a^3*c
^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3
- 192*a^2*b^2*c^4)))/(8*c^2*(4*a*c - b^2)^(3/2)) + (a*b*(6*a*c - b^2)*(2*b^
6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/((4*a*c - b^2)^(3/2)*(256*a
^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) +
(b*(a/c^2 + ((8*a + (8*a*c^2*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b
^4*c))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(2*b^6 -
 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c^5 - 4*b^6*c^2 +
48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (a*b^2*(6*a*c - b^2)^2)/(c^2*(4*a*c - b^
2)^3)))/(2*a*(4*a*c - b^2)^(3/2))))/(b^6 + 36*a^2*b^2*c^2 - 12*a*b^4*c))*(6
*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(3/2))
```

3.92 $\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 271

$$\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx = -\frac{bx}{2c(b^2-4ac)}+\frac{x^3(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(b^2-6ac-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2-6ac+\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/2*b*x/c/(-4*a*c+b^2)+1/2*x^3*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/
4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-6*a*c-b*(-8*a
*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-
6*a*c+b*(-8*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-
4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

= {1599, 1134, 1293, 1180, 211}

$$\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx = \frac{\left(-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x^3(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{bx}{2c(b^2-4ac)}$$

```
[In] Int[x^8/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -1/2*(b*x)/(c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) + ((b^2 - 6*a*c - (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2
- 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
```

```
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^6}{(a+bx^2+cx^4)^2}\,dx \\
&= \frac{x^3(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{x^2(6a+bx^2)}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)} \\
&= -\frac{bx}{2c(b^2-4ac)} + \frac{x^3(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\int \frac{ab+(b^2-6ac)x^2}{a+bx^2+cx^4}\,dx}{2c(b^2-4ac)} \\
&= -\frac{bx}{2c(b^2-4ac)} + \frac{x^3(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\
&\quad + \frac{\left(b^2-6ac-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4c(b^2-4ac)} \\
&\quad + \frac{\left(b^2-6ac+\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4c(b^2-4ac)} \\
&= -\frac{bx}{2c(b^2-4ac)} + \frac{x^3(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\
&\quad + \frac{\left(b^2-6ac-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad + \frac{\left(b^2-6ac+\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.04

$$\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx$$
$$= \frac{-\frac{2\sqrt{c}x(b^2x^2+a(b-2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-b^3+8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^3-8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4\right.}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{}}}}{4c^{3/2}}$$

```
[In] Integrate[x^8/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((-2*Sqrt[c]*x*(b^2*x^2 + a*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (Sqrt[2]*(-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*
a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3 - 8*a*b*c + b^2*Sqr
t[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/
(4*c^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.56

method	result
risch	$\frac{-\frac{(2ac-b^2)x^3}{2c(4ac-b^2)}+\frac{abx}{2c(4ac-b^2)}}{cx^4+bx^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(\frac{(6ac-b^2)\_R^2}{4ac-b^2}-\frac{ab}{4ac-b^2}\right)\ln(x-\_R)}{2c\_R^3+\_Rb}}{4c}$
default	$\frac{-\frac{(2ac-b^2)x^3}{2c(4ac-b^2)}+\frac{abx}{2c(4ac-b^2)}}{cx^4+bx^2+a}+\frac{\frac{\left(6ac\sqrt{-4ac+b^2}-b^2\sqrt{-4ac+b^2}+8abc-b^3\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{4c\sqrt{-4ac+b^2}\sqrt{(b+\sqrt{-4ac+b^2})c}}-\frac{\left(6ac\sqrt{-4ac+b^2}-b^2\sqrt{-4ac+b^2}\right.}{4c\sqrt{-4a}}}{4ac-b^2}$

```
[In] int(x^8/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*(2*a*c-b^2)/c/(4*a*c-b^2)*x^3+1/2*a*b/c/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a
)+1/4/c*sum(((6*a*c-b^2)/(4*a*c-b^2)*_R^2-a*b/(4*a*c-b^2))/(2*_R^3*c+_R*b)*
ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2257 vs. 2(227) = 454.

Time = 0.34 (sec) , antiderivative size = 2257, normalized size of antiderivative = 8.33

$$\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(b^2 - 2*a*c)*x^3 + 2*a*b*x - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 +
a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60
*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b
^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*
a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 8
8*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 -
 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c
+ 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqr
t((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) +
sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c
^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 +
 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6
 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 +
48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x
- 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c
^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((
b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 6
4*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c
^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a
*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*
a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^
4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*
a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a
*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88
*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 -
640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^
6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c +
 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 -
 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) + s
```

```
qrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^
2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 +
48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6
- 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 4
8*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x -
 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8*c^
3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b
^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4
 + 48*a^2*b^2*c^5 - 64*a^3*c^6))))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a
^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**8/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^8}{(cx^5+bx^3+ax)^2}\,dx$$

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2 - 2*a*c)*x^3 + a*b*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2
*c^2 + (b^3*c - 4*a*b*c^2)*x^2) - 1/2*integrate(-((b^2 - 6*a*c)*x^2 + a*b)/
(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2736 vs. 2(227) = 454.

Time = 0.94 (sec) , antiderivative size = 2736, normalized size of antiderivative = 10.10

$$\int \frac{x^8}{(ax + bx^3 + cx^5)^2} dx = \text{Too large to display}$$

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*x^3 - 2*a*c*x^3 + a*b*x)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
- 1/16*(2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24
*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6 - (2*b^4*c^2 - 20*a
*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
- 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*
c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2 - 2*(sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*
c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a
^2*b*c^4)*abs(b^2*c - 4*a*c^2))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^
2 + sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3
)))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a
^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2
*b^2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c)) - 1/16*(2*b^8*c^4 - 32*
a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
```

```
 - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^
5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6 - (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*
a*c^3)*(b^2*c - 4*a*c^2)^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a
^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b
*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(b^2*c - 4
*a*c^2))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b
*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)
))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3
*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*
abs(b^2*c - 4*a*c^2)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 6293, normalized size of antiderivative = 23.22

$$\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

```
[In] int(x^8/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] - ((x^3*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b*x)/(2*c*(4*a*c - b^2)))/(
a + b*x^2 + c*x^4) - atan(((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c
^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^
3)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^
8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16
```

*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 1
6*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^1
0*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^
2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 614
4*a^5*b^2*c^8)))^(1/2)*1i - (((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*
c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c
^3)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2
*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*
c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b
^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(1
6*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c +
16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a
^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9
*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^
10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b
^2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2)
- 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 -
27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 61
44*a^5*b^2*c^8)))^(1/2)*1i)/((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5
*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*
c^3)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^
2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a
c - b^2)^9)^(1/2))/(32(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*
b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(
16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c +
16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*
a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^
9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b
^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*
b^2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*
(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2)
- 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 -
27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3
- 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6
144*a^5*b^2*c^8)))^(1/2) + (((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c
^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^
3)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*

b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (5*a^2*b^4 + 216*a^4*c^2 - 66*a^3*b^2*c)/(4*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*2i - atan(((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*1i - (((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^

4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c
^2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(
4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3
 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(409
6*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 +
 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*1i)/((((16*a*b^7*c^2 - 1024*a
^4*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a
*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3
840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*
a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24
*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*
a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2
*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c -
 b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840
*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c
^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a
^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*
c^2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-
(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^
3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(40
96*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6
 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (((16*a*b^7*c^2 - 1024*a^4
*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b
^4*c^2 + 48*a^2*b^2*c^3)) + (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 384
0*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*
b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a
*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^
5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b
^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c - b
^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a
^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9
 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4
*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^
2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4
*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3
+ 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096
*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 +
3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (5*a^2*b^4 + 216*a^4*c^2 - 6
6*a^3*b^2*c)/(4*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))))*(-(
b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280
*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*2i

3.93 $\int \frac{x^7}{(ax+bx^3+cx^5)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 78

$$\int \frac{x^7}{(ax+bx^3+cx^5)^2} dx = \frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{2a\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*x^2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*a*arctanh((2*c*x^2+b)/(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1599, 1128, 736, 632, 212}

$$\int \frac{x^7}{(ax+bx^3+cx^5)^2} dx = \frac{2a\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} + \frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

```
[In] Int[x^7/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*ArcTanh[(b
 + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c))), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^5}{\left(a+bx^2+cx^4\right)^2}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)\\
&= \frac{x^2(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{a\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{b^2-4ac}\\
&= \frac{x^2(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{(2a)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}\\
&= \frac{x^2(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

$$\int \frac{x^7}{(ax+bx^3+cx^5)^2}\,dx = \frac{b^2x^2+a(b-2cx^2)}{2c(-b^2+4ac)(a+bx^2+cx^4)} + \frac{2a\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

```
[In] Integrate[x^7/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (b^2*x^2 + a*(b - 2*c*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a
*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.33

method	result
default	$\frac{-\frac{(2ac-b^2)x^2}{c(4ac-b^2)}+\frac{ab}{c(4ac-b^2)}}{2cx^4+2bx^2+2a}+\frac{2a\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{(2ac-b^2)x^2}{2c(4ac-b^2)}+\frac{ab}{2c(4ac-b^2)}}{cx^4+bx^2+a}+\frac{a\ln\left(\left((-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)x^2+8ca^2-2b^2a\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{a\ln\left(\left((-4ac+b^2)^{\frac{3}{2}}-4abc+b^3\right)x^2-8ca^2+2b\right.}{(-4ac+b^2)^{\frac{3}{2}}}$

```
[In] int(x^7/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-(2*a*c-b^2)/c/(4*a*c-b^2)*x^2+a*b/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2*a/
(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(72) = 144.

Time = 0.30 (sec) , antiderivative size = 407, normalized size of antiderivative = 5.22

$$\int \frac{x^7}{(ax+bx^3+cx^5)^2}\,dx$$

$$= \left[-\frac{ab^3-4a^2bc+(b^4-6ab^2c+8a^2c^2)x^2+2(ac^2x^4+abcx^2+a^2c)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac-(2}{cx^4+bx^2+a}\right.}{2(ab^4c-8a^2b^2c^2+16a^3c^3+(b^4c^2-8ab^2c^3+16a^2c^4)x^4+(b^5c-8ab^3c^2+16a^2bc^3)x^2}\right.$$

$$-\frac{ab^3-4a^2bc+(b^4-6ab^2c+8a^2c^2)x^2-4(ac^2x^4+abcx^2+a^2c)\sqrt{-b^2+4ac}\arctan\left(-\frac{(2cx^2+b)\sqrt{-b^2+4}}{b^2-4ac}\right.}{2(ab^4c-8a^2b^2c^2+16a^3c^3+(b^4c^2-8ab^2c^3+16a^2c^4)x^4+(b^5c-8ab^3c^2+16a^2bc^3)x^2)}$$

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x^2 + 2*(a*c^2*x^4
 + a*b*c*x^2 + a^2*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4*c - 8
*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a
*b^2*c + 8*a^2*c^2)*x^2 - 4*(a*c^2*x^4 + a*b*c*x^2 + a^2*c)*sqrt(-b^2 + 4*a
*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a*b^4*c - 8*a
^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(70) = 140$.

Time = 0.81 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.62

$$\int \frac{x^7}{(ax+bx^3+cx^5)^2}\,dx = -a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{-16a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}-ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab}{2ac}\right) + a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{16a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab}{2ac}\right) + \frac{ab+x^2(-2ac+b^2)}{8a^2c^2-2ab^2c+x^4\cdot(8ac^3-2b^2c^2)+x^2\cdot(8abc^2-2b^3c)}$$

```
[In] integrate(x**7/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**3*c**2*sqrt(-1/(4*a*c - b*
*2)**3) + 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - a*b**4*sqrt(-1/(4*a*c
- b**2)**3) + a*b)/(2*a*c)) + a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a
**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)*
*3) + a*b**4*sqrt(-1/(4*a*c - b**2)**3) + a*b)/(2*a*c)) + (a*b + x**2*(-2*a
*c + b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**
2*(8*a*b*c**2 - 2*b**3*c))
```

Maxima [F]

$$\int \frac{x^7}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^7}{(cx^5+bx^3+ax)^2}\,dx$$

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*a*integrate(x/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c) - 1/2*((b^2 - 2*a*c)
*x^2 + a*b)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b
*c^2)*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23

$$\int \frac{x^7}{(ax+bx^3+cx^5)^2}\,dx = -\frac{2\,a\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{b^2x^2-2\,acx^2+ab}{2\,(cx^4+bx^2+a)(b^2c-4\,ac^2)}$$

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -2*a*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*
a*c)) - 1/2*(b^2*x^2 - 2*a*c*x^2 + a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c
^2))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.40

$$\int \frac{x^7}{(ax+bx^3+cx^5)^2}\,dx = -\frac{\frac{x^2\,(2\,a\,c-b^2)}{2\,c\,(4\,a\,c-b^2)}-\frac{a\,b}{2\,c\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a} - \frac{2\,a\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{(4\,a\,c-b^2)^{3/2}}-\frac{x^2\left(\frac{4\,a\,c^2}{(4\,a\,c-b^2)^{7/2}}+\frac{4\,a\,(b^3\,c^2-4\,a\,b\,c^3)\,(b^3-4\,a\,b\,c)}{(4\,a\,c-b^2)^{13/2}}\right)(4\,a\,c-b^2)^4}{8\,a^2\,c^2}\right)}{(4\,a\,c-b^2)^{3/2}}$$

```
[In] int(x^7/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] - ((x^2*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b)/(2*c*(4*a*c - b^2)))/(a
+ b*x^2 + c*x^4) - (2*a*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*((4
*a*c^2)/(4*a*c - b^2)^(7/2) + (4*a*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c))/(
4*a*c - b^2)^(13/2))*(4*a*c - b^2)^4)/(8*a^2*c^2)))/(4*a*c - b^2)^(3/2)
```

3.94 $\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 237

$$\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx=\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\left(b-\frac{b^2+4ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)
/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c-b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(3/2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1599, 1134, 1180, 211}

$$\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx=\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

```
[In] Int[x^6/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b - (b^2 + 4*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 +
 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{x(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx}{2\,(b^2-4ac)}$$

$$= \frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4(b^2-4ac)^{3/2}} + \frac{\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4(b^2-4ac)^{3/2}}$$

$$= \frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.99

$$\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx = \frac{1}{4}\left(\frac{2(2ax+bx^3)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)$$

```
[In] Integrate[x^6/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sq
rt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]]))/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.52

method	result
risch	$\frac{-\frac{b x^3}{2(4ac-b^2)}-\frac{ax}{4ac-b^2}}{c x^4+b x^2+a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(-\frac{b\_R^2}{4ac-b^2}+\frac{2a}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_R b}\right)}{4}$
default	$\frac{-\frac{b x^3}{2(4ac-b^2)}-\frac{ax}{4ac-b^2}}{c x^4+b x^2+a}+\frac{2c\left(\frac{\left(-b\sqrt{-4ac+b^2}-4ac-b^2\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-b\sqrt{-4ac+b^2}+4ac+b^2\right)\sqrt{2}\operatorname{arctanh}\left(\frac{cx}{\sqrt{\left(-b+\sqrt{}\right.}}\right.}{8\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right.}{4ac-b^2}$

[In] int(x^6/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)

[Out] (-1/2*b/(4*a*c-b^2)*x^3-a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4*sum((-b/(4*a*c-b^2)*_R^2+2*a/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1668 vs. $2(193) = 386$.

Time = 0.28 (sec) , antiderivative size = 1668, normalized size of antiderivative = 7.04

$$\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] 1/4*(2*b*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^

```
2*c^3 - 64*a^3*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a
^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2
+ 4*a*c)*x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - 2*(b^7*c - 12*a*b^5*
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^
2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3
 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a
*b^2*c + 16*a^2*c^2 - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c
^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3
+ 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 +
48*a^2*b^2*c^3 - 64*a^3*c^4))) + 4*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*
a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [A] (verification not implemented)

Time = 7.36 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.25

$$\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx = \frac{-2ax-bx^3}{8a^2c-2ab^2+x^4\cdot(8ac^2-2b^2c)+x^2\cdot(8abc-2b^3)}$$

$$+\operatorname{RootSum}\left(t^4\cdot\left(1048576a^6c^7-1572864a^5b^2c^6+983040a^4b^4c^5-327680a^3b^6c^4+61440a^2b^8c^3-6144ab^{10}\right.\right.$$

```
[In] integrate(x**6/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-2*a*x - b*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*
(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**6*c**7 - 1572864*a**5*b**2*
c**6 + 983040*a**4*b**4*c**5 - 327680*a**3*b**6*c**4 + 61440*a**2*b**8*c**3
 - 6144*a*b**10*c**2 + 256*b**12*c) + _t**2*(-12288*a**4*b*c**4 + 8192*a**3
*b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**3*c**2 + 24*a**2*b**2*c
 + 9*a*b**4, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4 - 12288*_t**3*a
**2*b**3*c**3 + 3072*_t**3*a*b**5*c**2 - 256*_t**3*b**7*c + 64*_t*a**2*c**2
 - 128*_t*a*b**2*c - 4*_t*b**4)/(4*a*c + 3*b**2))))
```

Maxima [F]

$$\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^6}{(cx^5+bx^3+ax)^2}\,dx$$

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^3 + 2*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
*c)*x^2) + 1/2*integrate((b*x^2 - 2*a)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2132 vs. 2(193) = 386.

Time = 0.88 (sec) , antiderivative size = 2132, normalized size of antiderivative = 9.00

$$\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^3 + 2*a*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) + 1/16*(2*b^7*c^2 -
 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4 - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)
*b*c^2)*(b^2 - 4*a*c)^2 - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*
c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^
3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2
 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*abs(b^2 - 4*a*c))*arctan(2*s
qrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*
```

```
c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a
*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^
3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*b^
7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4 -
 (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 -
 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2 - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b^4*c^2 + 16*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2
*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 -
 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*abs(b^2 - 4*a*c))*arc
tan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 -
 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^
2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4
- 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 4973, normalized size of antiderivative = 20.98

$$\int \frac{x^6}{(ax + bx^3 + cx^5)^2} \, dx = \text{Too large to display}$$

```
[In] int(x^6/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] - atan(((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4
)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(((-(4*a*c - b^
2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(
b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
+ 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3
- 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((
(-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b
^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280
```

*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (x*(b^4*c + 8
*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b
^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*
(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*1i - (((2048*a^4*c^5 - 32*a
*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^
2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*
c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b
^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*
b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3
c^4))/(2(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9
 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6
*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^
5 - 6144*a^5*b^2*c^6)))^(1/2) + (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b
*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*
b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5
*b^2*c^6)))^(1/2)*1i)/((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1
536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*
(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3
*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 12
80*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 -
 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 -
8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*
c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2
*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)
- (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))
*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^
3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1
280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (4*a^2*b*c
^2 + 3*a*b^3*c)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((2
048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2)
- b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 409
6*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b
^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b
*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b
^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*
(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) + (x*(b^4*c + 8*a^2*c^3 + 2
*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2)
 - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 40
96*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*
b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 76
8*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7

- 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6
144*a^5*b^2*c^6)))^(1/2)*2i - atan(((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^
2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*
b^4*c)) - (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5
*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^
2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)
*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^
4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^1
0*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^
2*c^6)))^(1/2) - (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*
b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240
*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1
/2)*1i - (((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^
4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(b^9 + (-(4*
a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32
*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^
4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^
3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*
(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^
3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1
280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) + (x*(b^4*c
+ 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-
(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/
(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6
*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*1i)/((((2048*a^4*c^5 -
32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 4
8*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a
^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 2
4*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144
*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^
2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)
^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 40
96*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*
b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))
/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 76
8*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7
- 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6
144*a^5*b^2*c^6)))^(1/2) + (((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3
 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) +
 (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 51
2*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3
 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*
c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c

```
^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^
2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 2
40*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^
(1/2) + (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^
2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 +
 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*
c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (4*
a^2*b*c^2 + 3*a*b^3*c)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512
*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3
- 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*2i - ((a*
x)/(4*a*c - b^2) + (b*x^3)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

3.95 $\int \frac{x^5}{(ax+bx^3+cx^5)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 75

$$\int \frac{x^5}{(ax+bx^3+cx^5)^2} dx = \frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{b\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*arctanh((2*c*x^2+b)/(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1599, 1128, 652, 632, 212}

$$\int \frac{x^5}{(ax+bx^3+cx^5)^2} dx = \frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{b\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[In] Int[x^5/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)\\
&= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}\\
&= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}\\
&= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{x^5}{(ax+bx^3+cx^5)^2}\,dx = \frac{2a+bx^2}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{b\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

```
[In] Integrate[x^5/(a*x + b*x^3 + c*x^5)^2,x]
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTan[(b + 2*c*x^
2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

method	result
default	$\frac{-b\,x^2-2a}{2(4ac-b^2)(c\,x^4+b\,x^2+a)}-\frac{b\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{b\,x^2}{2\left(4ac-b^2\right)}-\frac{a}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{b\ln\left(\left(-(-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)x^2+8c\,a^2-2b^2a\right)}{2(-4ac+b^2)^{\frac{3}{2}}}-\frac{b\ln\left(\left(-(-4ac+b^2)^{\frac{3}{2}}-4abc+b^3\right)x^2-8c\,a^2+2b^2a\right)}{2(-4ac+b^2)^{\frac{3}{2}}}$

```
[In] int(x^5/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)-b/(4*a*c-b^2)^(3/2)*arctan((2*
c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(69) = 138.

Time = 0.27 (sec) , antiderivative size = 360, normalized size of antiderivative = 4.80

$$\int \frac{x^5}{(ax+bx^3+cx^5)^2}\,dx = \left[\frac{2\,ab^2-8\,a^2c+(b^3-4\,abc)x^2-(bcx^4+b^2x^2+ab)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\,(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^4+(b^5-8\,ab^3c+16\,a^2bc^2)x^2)}, 2\right.$$

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
[Out] [1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - (b*c*x^4 + b^2*x^2 + a*b)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 +
```

```
 (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*
x^2), 1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - 2*(b*c*x^4 + b^2*x^2 +
 a*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*
a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^
3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(63) = 126.

Time = 0.69 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.59

$$\int \frac{x^5}{(ax+bx^3+cx^5)^2}\,dx = \frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)}{2} - \frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)}{2} + \frac{-2a-bx^2}{8a^2c-2ab^2+x^4\cdot(8ac^2-2b^2c)+x^2\cdot(8abc-2b^3)}$$

```
[In] integrate(x**5/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b*
*2)**3) + b**2)/(2*b*c))/2 - b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a*
*2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3
) + b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c))/2 + (-2*a - b*x**2)/(8
*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3))
```

Maxima [F]

$$\int \frac{x^5}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^5}{(cx^5+bx^3+ax)^2}\,dx$$

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] b*integrate(x/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c) + 1/2*(b*x^2 + 2*a)/((b
^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{(ax+bx^3+cx^5)^2}\,dx = \frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} + \frac{bx^2+2\,a}{2\,(cx^4+bx^2+a)(b^2-4\,ac)}$$

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c
)) + 1/2*(b*x^2 + 2*a)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.37

$$\int \frac{x^5}{(ax+bx^3+cx^5)^2}\,dx = \frac{b\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{(4\,a\,c-b^2)^{3/2}} - \frac{x^2\,(4\,a\,c-b^2)^4\left(\frac{b^2\,c^2}{a\,\left(4\,a\,c-b^2\right)^{7/2}} + \frac{b^2\,\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)\,\left(b^3-4\,a\,b\,c\right)}{2\,a\,\left(4\,a\,c-b^2\right)^{13/2}}\right)}{2\,b^2\,c^2}\right)}{(4\,a\,c-b^2)^{3/2}} - \frac{\frac{a}{4\,a\,c-b^2} + \frac{b\,x^2}{2\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a}$$

```
[In] int(x^5/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] (b*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((b^2*c^
2)/(a*(4*a*c - b^2)^(7/2)) + (b^2*(2*b^3*c^2 - 8*a*b*c^3)*(b^3 - 4*a*b*c))/
(2*a*(4*a*c - b^2)^(13/2))))/(2*b^2*c^2)))/(4*a*c - b^2)^(3/2) - (a/(4*a*c
- b^2) + (b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

3.96 $\int \frac{x^4}{(ax+bx^3+cx^5)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 221

$$\int \frac{x^4}{(ax+bx^3+cx^5)^2} dx = -\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}(2b-\sqrt{b^2-4ac})\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}(2b+\sqrt{b^2-4ac})\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/2*x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*arctan(x*2^(1/2)*c^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{1599, 1133, 1180, 211\}$

$$\int \frac{x^4}{(ax+bx^3+cx^5)^2}\,dx = \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{x(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[In] Int[x^4/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -1/2*(x*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(2*b
- Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*
b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1133

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
 1)*(b^2 - 4*a*c))), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx \\
&= -\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)} \\
&= -\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(c\left(1+\frac{2b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2(b^2-4ac)} \\
&\quad + \frac{\left(c\left(2b-\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2(b^2-4ac)^{3/2}} \\
&= -\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad - \frac{\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int \frac{x^4}{(ax+bx^3+cx^5)^2}\,dx &= \frac{-bx-2cx^3}{2(b^2-4ac)(a+bx^2+cx^4)} \\
&\quad - \frac{\sqrt{c}\left(-2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad - \frac{\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

```
[In] Integrate[x^4/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (-(b*x) - 2*c*x^3)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(-2*b +
 Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*b
 + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.55

method	result
risch	$\frac{\frac{c x^3}{4ac-b^2}+\frac{bx}{8ac-2b^2}}{c x^4+b x^2+a}+\frac{\left(\sum_{\_R=\text{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(\frac{2c\_R^2}{4ac-b^2}-\frac{b}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{4}$
default	$16c^2\left(\frac{\frac{\sqrt{-4ac+b^2}\,x}{8c\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}-\frac{\left(-b+\frac{\sqrt{-4ac+b^2}}{2}\right)\sqrt{2}\,\text{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}}{4c(4ac-b^2)\sqrt{-4ac+b^2}}+\frac{\frac{\sqrt{-4ac+b^2}\,x}{8c\left(x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}\right)}+\frac{\left(b+\frac{\sqrt{-4ac+b^2}}{2}\right)\cdots}{4\cdots}}{4c(4ac-b^2)\sqrt{-4a\cdots}}\right.$

```
[In] int(x^4/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (c/(4*a*c-b^2)*x^3+1/2*b/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4*sum((2*c/(4*a*c
-b^2)*_R^2-b/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+
a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1680 vs. 2(180) = 360.

Time = 0.30 (sec) , antiderivative size = 1680, normalized size of antiderivative = 7.60

$$\int \frac{x^4}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c
^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
)/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*
c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6
*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3
*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) - sqrt(1/2)*
((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3
+ 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*
```

```
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c +
48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x - 1/2*sqrt(1/2)*(b^
5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256
*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(
-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqr
t(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^
4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a
^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^
3))*log((3*b^2*c + 4*a*c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c
^2 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2
- 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(
a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2
)*x - 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + (a*b^8 - 8*a^2*b^6*c
+ 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^
2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + 2*b*x)/((b^2*
c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(ax+bx^3+cx^5)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**4/(c*x**5+b*x**3+a*x)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^4}{(cx^5+bx^3+ax)^2}\,dx$$

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^3 + b*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*x^2) - 1/2*integrate((2*c*x^2 - b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*
c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1970 vs. 2(180) = 360.

Time = 0.88 (sec) , antiderivative size = 1970, normalized size of antiderivative = 8.91

$$\int \frac{x^4}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*c*x^3 + b*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) + 1/8*(4*b^6*c^2 -
 32*a*b^4*c^3 + 64*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 - 4*(b^2 - 4*a*c)*b^4*c^2 + 16*(b^2 - 4*a*c)*a*b^2*c^3 - (2*b^2*c^2 - 8*a*
c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)
^2 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c
^2)*abs(b^2 - 4*a*c))*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3
- 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/
((a*b^6 - 12*a^2*b^4*c - 2*a*b^5*c + 48*a^3*b^2*c^2 + 16*a^2*b^3*c^2 + a*b^
```

```
4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^3 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*abs(b^2 - 4
*a*c)*abs(c)) - 1/8*(4*b^6*c^2 - 32*a*b^4*c^3 + 64*a^2*b^2*c^4 - 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 16*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*(b^2 - 4*a*c)*b^4*c^2 + 16*(b^2 - 4*a*
c)*a*b^2*c^3 - (2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*
(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)^2 - (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*
c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*abs(b^2 - 4*a*c))*arctan(2*sqrt(1/2)*x/
sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c -
 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6 - 12*a^2*b^4*c - 2*a*b^5*c + 48*a^3
*b^2*c^2 + 16*a^2*b^3*c^2 + a*b^4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^3 - 8*a^2*b
^2*c^3 + 16*a^3*c^4)*abs(b^2 - 4*a*c)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 4854, normalized size of antiderivative = 21.96

$$\int \frac{x^4}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

```
[In] int(x^4/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] atan(((((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^
6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1
/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 +
 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a
^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*
b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)
^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*
b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 +
3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b
^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*
c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2
```

*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*
b^2*c^5)))^(1/2)*1i - (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2
*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(((-(4*
a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^
3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*
b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^
5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(
((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*
b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 128
0*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (x*(4*a*c^4
- 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) -
b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*
a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4
*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*1i)/((4*a*c^4 + 3*b^2*c^3)/(2*(b^6 - 64*a^
3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a
^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^
2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b
^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8
*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c
^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*
b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240
*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1
/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c
- b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(
32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*
c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (((8*b^7*c^2 - 96*a*b^
5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)) - (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 +
96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*
c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^
5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b
^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*
c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2
*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*
b^2*c^5)))^(1/2) + (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
)*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a
^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 -
1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)))*(((-(4*a*
c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)
/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^
6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*2i + atan(((((8*b^7*c^
2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*
a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 -

24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 614
4*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*
b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2)
 - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7
*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^
4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2
 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b
^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*
a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/
2)*1i - (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(-(b^9 + (-(4*a*c - b
^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^1
2 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 384
0*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a
^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*
a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32
*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^
3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (x*(4*a*c^4 - 5*b^2*c^3)
)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a
^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 2
4*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144
*a^6*b^2*c^5)))^(1/2)*1i)/((4*a*c^4 + 3*b^2*c^3)/(2*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c)) + (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 +
384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*
(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^
3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1
280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 -
 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b
^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 +
 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*
c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*
(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c -
b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^
12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 38
40*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (((8*b^7*c^2 - 96*a*b^5*c^3 -
512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c)) - (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*
b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240
*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1
/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16
*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 -
96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*
c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^
5)))^(1/2) + (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b
^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^

```
3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*
a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)))*(-(b^9 + (-(4*
a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32
*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^
3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*2i + ((b*x)/(2*(4*a*c - b^
2)) + (c*x^3)/(4*a*c - b^2))/(a + b*x^2 + c*x^4)
```

3.97 $\int \frac{x^3}{(ax+bx^3+cx^5)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 74

$$\int \frac{x^3}{(ax+bx^3+cx^5)^2}\,dx=-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] 1/2*(-2*c*x^2-b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*c*arctanh((2*c*x^2+b)/(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1599, 1121, 628, 632, 212}

$$\int \frac{x^3}{(ax+bx^3+cx^5)^2}\,dx=\frac{2c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[In] Int[x^3/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -1/2*(b + 2*c*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*c*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x}{(a+bx^2+cx^4)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}\,dx, x, x^2\right) \\
&= -\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{c\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{b^2-4ac} \\
&= -\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{(2c)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac} \\
&= -\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{(ax+bx^3+cx^5)^2}\,dx = -\frac{\frac{b+2cx^2}{a+bx^2+cx^4}+\frac{4c\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2\left(b^2-4ac\right)}$$

```
[In] Integrate[x^3/(a*x + b*x^3 + c*x^5)^2,x]
[Out] -1/2*((b + 2*c*x^2)/(a + b*x^2 + c*x^4) + (4*c*ArcTan[(b + 2*c*x^2)/Sqrt[-b
^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

method	result	si
default	$\frac{2c\,x^2+b}{2(4ac-b^2)(c\,x^4+b\,x^2+a)}+\frac{2c\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	7
risch	$\frac{\frac{c\,x^2}{4ac-b^2}+\frac{b}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{c\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)x^2+8c\,a^2-2b^2a\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{c\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)x^2-8c\,a^2+2b^2a\right)}{(-4ac+b^2)^{\frac{3}{2}}}$	1

```
[In] int(x^3/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+2*c/(4*a*c-b^2)^(3/2)*arctan((2
*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(68) = 136.

Time = 0.26 (sec) , antiderivative size = 361, normalized size of antiderivative = 4.88

$$\int \frac{x^3}{(ax+bx^3+cx^5)^2}\,dx$$
$$= \left[-\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)x^2+2\,(c^2x^4+bcx^2+ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right.}{2\,(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^4+(b^5-8\,ab^3c+16\,a^2bc^2)x^2)}\right.$$
$$\left.-\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)x^2-4\,(c^2x^4+bcx^2+ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{2\,(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^4+(b^5-8\,ab^3c+16\,a^2bc^2)x^2)}\right]$$

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + 2*(c^2*x^4 + b*c*x^2 + a*c
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b
)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^
2)*x^2), -1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - 4*(c^2*x^4 + b*c*x
^2 + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
- 4*a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(66) = 132$.

Time = 0.68 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.61

$$\int \frac{x^3}{(ax+bx^3+cx^5)^2}\,dx =$$

$$-c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{-16a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+bc}{2c^2}\right)$$
$$+c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{16a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+bc}{2c^2}\right)$$
$$+\frac{b+2cx^2}{8a^2c-2ab^2+x^4\cdot(8ac^2-2b^2c)+x^2\cdot(8abc-2b^3)}$$

```
[In] integrate(x**3/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*c**3*sqrt(-1/(4*a*c - b*
*2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b**4*c*sqrt(-1/(4*a*c
- b**2)**3) + b*c)/(2*c**2)) + c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*
a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)
**3) + b**4*c*sqrt(-1/(4*a*c - b**2)**3) + b*c)/(2*c**2)) + (b + 2*c*x**2)/
(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3)
)
```

Maxima [F]

$$\int \frac{x^3}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{x^3}{\left(cx^5+bx^3+ax\right)^2}\,dx$$

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*c*integrate(x/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c) - 1/2*(2*c*x^2 + b)/
((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\left(ax+bx^3+cx^5\right)^2}\,dx = -\frac{2\,c\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{2\,cx^2+b}{2\,(cx^4+bx^2+a)(b^2-4\,ac)}$$

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -2*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*
a*c)) - 1/2*(2*c*x^2 + b)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.32

$$\begin{aligned}
&\int \frac{x^3}{\left(ax+bx^3+cx^5\right)^2}\,dx \\
&= \frac{\frac{b}{2\,(4\,a\,c-b^2)}+\frac{c\,x^2}{4\,a\,c-b^2}}{c\,x^4+b\,x^2+a} \\
&- \frac{2\,c\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{(4\,a\,c-b^2)^{3/2}}-\frac{x^2\,(4\,a\,c-b^2)^4\left(\frac{4\,c^4}{a\,(4\,a\,c-b^2)^{7/2}}+\frac{4\,c^2\,(b^3\,c^2-4\,a\,b\,c^3)\,(b^3-4\,a\,b\,c)}{a\,(4\,a\,c-b^2)^{13/2}}\right)}{8\,c^4}\right)}{(4\,a\,c-b^2)^{3/2}}
\end{aligned}$$

```
[In] int(x^3/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] (b/(2*(4*a*c - b^2)) + (c*x^2)/(4*a*c - b^2))/(a + b*x^2 + c*x^4) - (2*c*at
an((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((4*c^4)/(a*(
4*a*c - b^2)^(7/2)) + (4*c^2*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c))/(a*(4*a
*c - b^2)^(13/2))))/(8*c^4)))/(4*a*c - b^2)^(3/2)
```

3.98 $\int \frac{x^2}{(ax+bx^3+cx^5)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 252

$$\int \frac{x^2}{(ax+bx^3+cx^5)^2}\,dx = \frac{x(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/
2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)
^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arcta
n(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b*(-4
*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

= {1599, 1106, 1180, 211}

$$\int \frac{x^2}{(ax+bx^3+cx^5)^2}\,dx = \frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x(-2ac+b^2+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[In] Int[x^2/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx\\
&= \frac{x(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{b^2-2ac-2\left(b^2-4ac\right)-bcx^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}\\
&= \frac{x(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(c\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)^{3/2}}\\
&\quad+\frac{\left(c\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)^{3/2}}\\
&= \frac{x(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\\
&\quad-\frac{\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.96

$$
\int \frac{x^2}{\left(ax+bx^3+cx^5\right)^2}\,dx
= \frac{\frac{2x(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a}
$$

```
[In] Integrate[x^2/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.60

method	result
risch	$\frac{-\frac{b c x^3}{2a(4ac-b^2)}+\frac{(2ac-b^2)x}{2a(4ac-b^2)}}{c x^4+b x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2 b+a)}\frac{\left(-\frac{bc\_R^2}{4ac-b^2}+\frac{6ac-b^2}{4ac-b^2}\right)\ln(x-\_R)}{2c\_R^3+\_Rb}}{4a}$
default	$16c^2\left(-\frac{-\frac{\left(-b\sqrt{-4ac+b^2}+4ac-b^2\right)x}{16a c^2\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}-\frac{\left(b^2-12ac+b\sqrt{-4ac+b^2}\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{16ac\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}}{4\sqrt{-4ac+b^2}\,(4ac-b^2)}-\frac{\left(b\sqrt{-4ac+b^2}+4ac-b^2\right)x}{16a c^2\left(x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}\right)}\right.$

```
[In] int(x^2/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/a*b*c/(4*a*c-b^2)*x^3+1/2*(2*a*c-b^2)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a
)+1/4/a*sum((-b*c/(4*a*c-b^2)*_R^2+(6*a*c-b^2)/(4*a*c-b^2))/(2*_R^3*c+_R*b)
*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2309 vs. 2(206) = 412.

Time = 0.36 (sec) , antiderivative size = 2309, normalized size of antiderivative = 9.16

$$\int \frac{x^2}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c +
 (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*
a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6
- 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^
3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 67
2*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 4
48*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c
+ 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqr
t((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
```

```
- 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) -
sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c +
 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*
x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 8
64*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 +
512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 +
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2
*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(
a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b
^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)
*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3
*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sq
rt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*
b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x
^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c +
 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(
(5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*
c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^
2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*
sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2 - 64*a^6*c^3))) + 2*(b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax+bx^3+cx^5)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(c*x**5+b*x**3+a*x)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^2}{(cx^5+bx^3+ax)^2}\,dx$$

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
[Out] 1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^
3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c*x
^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2682 vs. $2(206) = 412$.

Time = 0.99 (sec) , antiderivative size = 2682, normalized size of antiderivative = 10.64

$$\int \frac{x^2}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
[Out] 1/2*(b*c*x^3 + b^2*x - 2*a*c*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1
/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*
```

b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4 + (2*b^
3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c
)*b*c^2)*(a*b^2 - 4*a^2*c)^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b
^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c
)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))*arctan(2*sq
rt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2
 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*
a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 6
4*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)
abs(c)) + 1/16(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5
*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7
+ 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 -
 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b
*c^4 + (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*
(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
2 - 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 4
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c + 20*
(b^2 - 4*a*c)*a^2*b^2*c^2 - 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))

```
*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2
- 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a
^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3
*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^
2 - 4*a^2*c)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 6404, normalized size of antiderivative = 25.41

$$\int \frac{x^2}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

```
[In] int(x^2/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] ((x*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^3)/(2*a*(4*a*c - b^2)))/(a
+ b*x^2 + c*x^4) + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 +
 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b
^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^
2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4
*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
 - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 38
40*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840
*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*
b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
- b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 384
0*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
+ 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*
a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6 + 16*a*b^8*c^2
 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2
)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4
*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 409
6*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b
^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a
^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-
(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1
504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 128
0*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c
^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(
```

```
b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280
*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((((6144*a^
5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^
5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11
+ b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^
3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*
b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16
*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^
(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 24
0*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(
1/2) + (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4
- 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^
8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(10
24*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2
*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 -
24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 614
4*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*
b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27
*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 2
4*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144
*a^8*b^2*c^5)))^(1/2) + (5*b^3*c^4 - 36*a*b*c^5)/(4*(a^2*b^6 - 64*a^5*c^3 -
 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 -
24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 614
4*a^8*b^2*c^5)))^(1/2)*2i + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*
b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2)
- 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 -
```

27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2

```
*c^5)))^(1/2) + (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3
*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a
^4*b^2*c^2)) + (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^
(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4)
)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9
)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^
3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a
^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*
c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))
/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)
^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3
*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^
9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c
^4 - 6144*a^8*b^2*c^5)))^(1/2) + (5*b^3*c^4 - 36*a*b*c^5)/(4*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11 - b^2*(-(4*a*c - b^2)^9
)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^
3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a
^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*
c^4 - 6144*a^8*b^2*c^5)))^(1/2)*2i
```

3.99 $\int \frac{x}{(ax+bx^3+cx^5)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 122

$$\int \frac{x}{(ax+bx^3+cx^5)^2}\,dx=\frac{b^2-2ac+bcx^2}{2a(b^2-4ac)(a+bx^2+cx^4)}+\frac{b(b^2-6ac)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}}+\frac{\log(x)}{a^2}-\frac{\log(a+bx^2+cx^4)}{4a^2}$$

```
[Out] 1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*b*(-6*a*c+b^2)*a
rctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1/4
*ln(c*x^4+b*x^2+a)/a^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1599, 1128, 754, 814, 648, 632, 212, 642}

$$\int \frac{x}{(ax+bx^3+cx^5)^2}\,dx=\frac{b(b^2-6ac)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}}-\frac{\log(a+bx^2+cx^4)}{4a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx^2}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

```
[In] Int[x/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (b*(b^2 -
 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2
)) + Log[x]/a^2 - Log[a + b*x^2 + c*x^4]/(4*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\text{Subst}\left(\int \frac{-b^2+4ac-bcx}{x(a+bx+cx^2)}\,dx,x,x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\text{Subst}\left(\int \left(\frac{-b^2+4ac}{ax}+\frac{b(b^2-5ac)+c(b^2-4ac)x}{a(a+bx+cx^2)}\right)dx,x,x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log(x)}{a^2}-\frac{\text{Subst}\left(\int \frac{b(b^2-5ac)+c(b^2-4ac)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log(x)}{a^2}-\frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^2} - \frac{\left(b\left(b^2-6ac\right)\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2} + \frac{\left(b\left(b^2-6ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.70

$$\int \frac{x}{(ax+bx^3+cx^5)^2}\,dx$$
$$= \frac{\frac{2a(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)} + 4\log(x) - \frac{\left(b^3-6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{\left(b^3-6abc-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-\cdots}\right.}{(b^2-4ac)\cdots}}{4a^2}$$

```
[In] Integrate[x/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*Log[
x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log
[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c -
b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.52

method	result
default	$\frac{\ln(x)}{a^2} - \frac{\frac{\frac{abc\,x^2}{4ac-b^2} - \frac{a\left(2ac-b^2\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a} + \frac{\frac{\left(4a\,c^2-b^2c\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c} + \frac{2\left(5abc-b^3-\frac{\left(4a\,c^2-b^2c\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^2}$
risch	$\frac{-\frac{bc\,x^2}{2a\left(4ac-b^2\right)} + \frac{2ac-b^2}{2a\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a} + \frac{\ln(x)}{a^2} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^5c^3-48a^4b^2c^2+12a^3b^4c-b^6a^2\right)\_Z^2+\left(64c^3a^3-48a^2b^2c^2+12a\,b^4c-b^6\right)\_Z+16\cdots\right.}\cdots\right.}{\cdots}$

```
[In] int(x/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/a^2-1/2/a^2*((a*b*c/(4*a*c-b^2)*x^2-a*(2*a*c-b^2)/(4*a*c-b^2))/(c*x^4
+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(4*a*c^2-b^2*c)/c*ln(c*x^4+b*x^2+a)+2*(5*a*b*c
-b^3-1/2*(4*a*c^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b
^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(112) = 224$.

Time = 0.32 (sec) , antiderivative size = 813, normalized size of antiderivative = 6.66

$$\int \frac{x}{(ax+bx^3+cx^5)^2}\,dx = \left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+2\,(ab^3c-4\,a^2bc^2)x^2+((b^3c-6\,abc^2)x^4+ab^3-6\,a^2bc+(b^4-6\,ab^2c)x^2)\sqrt{}}{}\right.$$

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 +
 ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt
(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt
(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^
2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c -
8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(x
))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^
4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2*a*b^4 - 12
*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b
*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*a
rctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c
 + 16*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*
a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)
]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(ax+bx^3+cx^5)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x}{(cx^5+bx^3+ax)^2}\,dx$$

[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] 1/2*(b*c*x^2 + b^2 - 2*a*c)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + integrate(-((b^2*c - 4*a*c^2)*x^3 + (b^3 - 5*a *b*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c) + log(x)/a^2

Giac [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.36

$$\int \frac{x}{(ax+bx^3+cx^5)^2}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^2b^2-4\,a^3c)\sqrt{-b^2+4\,ac}} + \frac{b^2cx^4-4\,ac^2x^4+b^3x^2-2\,abcx^2+3\,ab^2-8\,a^2c}{4\,(cx^4+bx^2+a)(a^2b^2-4\,a^3c)} - \frac{\log\left(cx^4+bx^2+a\right)}{4\,a^2} + \frac{\log\left(x^2\right)}{2\,a^2}$$

[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4 *a^3*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 + b^3*x^2 - 2*a* b*c*x^2 + 3*a*b^2 - 8*a^2*c)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c)) - 1/ 4*log(c*x^4 + b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2

Mupad [B] (verification not implemented)

Time = 10.96 (sec) , antiderivative size = 5048, normalized size of antiderivative = 41.38

$$\int \frac{x}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

[In] int(x/(a*x + b*x^3 + c*x^5)^2,x)

[Out] log(x)/a^2 + ((2*a*c - b^2)/(2*a*(4*a*c - b^2)) - (b*c*x^2)/(2*a*(4*a*c - b ^2)))/(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (b*atan((x^2*((((((b*((320*a^5*b*c^6 - 2*a^2*b^7*c^3 +

36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*
a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*
b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*
b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2
*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*
c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056
*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*
a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*
b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4
c))/(2(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (b*((
6*a*b^5*c^4 + 80*a^3*b*c^6 - 44*a^2*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + 36*a^3*b^5*c^
4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)
 - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 1
2*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*
(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5
*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*
c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2
c^2)))(6*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2)) + (b^3*(6*a*c - b^2)^3*(
2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688
*a^6*b^3*c^5))/(64*a^6*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c)
)/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 7
2*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*((((6*a*b^5*c^4 + 80*a^3*
b*c^6 - 44*a^2*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2) + (((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/
(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*
c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a
^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c +
 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2
*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*
a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^
3*b^4*c + 192*a^4*b^2*c^2)) - (b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2) - (b*(6*a*c - b^2)*((b*((320*a^5*b*c^6 - 2*a^2*b^7*c^3
+ 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c +
48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(256
0*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^
6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^
2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a
^2*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^
2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 10
56*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 6
4*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^

3*b^4*c + 192*a^4*b^2*c^2))))/(4*a^2*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b
^2)^2*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 +
12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(
32*a^4*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^
2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))))/(8*a^3*c^2
*(4*a*c - b^2)^3*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)))*(16
*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b
^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2)))/(b^6*c^2 -
12*a*b^4*c^3 + 36*a^2*b^2*c^4) + (((b*((4*a*b^4*c^3 - 17*a^2*b^2*c^4)/(a^3
*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) - (((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 + 80*a
^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5
*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4
c))/(2(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*
a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a
*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6
*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2)) - (((b*((4*a^2*b^6*c^2 - 36*a^3*b^
4*c^3 + 80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*
c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^
2 - 24*a*b^4*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a
^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b
^2)^(3/2)) + (b*(6*a*c - b^2)*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*
c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(8*a^2*(4*a*c - b
^2)^(3/2)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 4
8*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24
*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) +
(b^3*(6*a*c - b^2)^3*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4))/(6
4*a^6*(4*a*c - b^2)^(9/2)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))*(16*a^6*b^
6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4
*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*b^6 - 40*a^3*c^
3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(b^6*c^2 -
12*a*b^4*c^3 + 36*a^2*b^2*c^4)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72
*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*(16*a^6*b^6*(4*a*c - b^2)^
(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2
) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*((((4*a*b^4*c^3 - 17*a^2*b^2*c^4)/
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) - (((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 +
80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32
*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a
*b^4*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 -
48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 -
24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))
)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*
a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (b^2*c^4)/(a^3*b^4 + 16*a^5*c^
2 - 8*a^4*b^2*c) + (b*(6*a*c - b^2)*((b*((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 +
80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32
*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a

```
*b^4*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 -
 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2
)) + (b*(6*a*c - b^2)*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*
b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(8*a^2*(4*a*c - b^2)^(3/2
)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^
4*c + 192*a^4*b^2*c^2))))/(4*a^2*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b^2)^
2*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 +
96*a^2*b^2*c^2 - 24*a*b^4*c))/(32*a^4*(4*a*c - b^2)^3*(a^3*b^4 + 16*a^5*c^2
 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))
))/(8*a^3*c^2*(4*a*c - b^2)^3*(b^6*c^2 - 12*a*b^4*c^3 + 36*a^2*b^2*c^4)*(6*
b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)))*(6*a*c - b^2))/(2*a^2*(
4*a*c - b^2)^(3/2))
```

3.100 $\int \frac{1}{(ax+bx^3+cx^5)^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 308

$$\begin{aligned}
&\int \frac{1}{(ax+bx^3+cx^5)^2} \, dx \\
&= -\frac{3b^2-10ac}{2a^2(b^2-4ac)x} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x(a+bx^2+cx^4)} \\
&\quad - \frac{\sqrt{c}\left(3b^3-16abc+(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad + \frac{\sqrt{c}\left(3b^3-16abc-(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

```
[Out] 1/2*(10*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2
)/x/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*c^(1/2)*(3*b^3-16*a*b*c+(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c
+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c-(-10*a*c+3*b^2)*(-
4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1608, 1135, 1295, 1180, 211}

$$\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx$$
$$= -\frac{\sqrt{c}\left((3b^2-10ac)\sqrt{b^2-4ac}-16abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\left(-(3b^2-10ac)\sqrt{b^2-4ac}-16abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$-\frac{3b^2-10ac}{2a^2x(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{2ax(b^2-4ac)(a+bx^2+cx^4)}$$

```
[In] Int[(a*x + b*x^3 + c*x^5)^(-2),x]
```

```
[Out] -1/2*(3*b^2 - 10*a*c)/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*
(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2
- 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*
c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-3b^2+10ac-3bcx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{\int \frac{-b\left(3b^2-13ac\right)-c\left(3b^2-10ac\right)x^2}{a+bx^2+cx^4}\,dx}{2a^2\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{16abc}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a^2\left(b^2-4ac\right)} - \frac{\left(c\left(3b^2-10ac-\frac{3b^3}{\sqrt{b^2-4ac}}+\frac{16abc}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a^2\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\sqrt{c}\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{16abc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(3b^2-10ac-\frac{3b^3}{\sqrt{b^2-4ac}}+\frac{16abc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.98

$$\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx$$

$$= \frac{-\frac{4}{x}-\frac{2x(b^3-3abc+b^2cx^2-2ac^2x^2)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(3b^3-16abc-\right.}{}}{4a^2}$$

[In] Integrate[(a*x + b*x^3 + c*x^5)^(-2),x]

[Out] (-4/x - (2*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.95

method	result
default	$-\frac{1}{a^2x}-\frac{\frac{\frac{c(2ac-b^2)x^3}{8ac-2b^2}+\frac{b(3ac-b^2)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(10ac\sqrt{-4ac+b^2}-3b^2\sqrt{-4ac+b^2}-16abc+3b^3\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(10ac\sqrt{-4ac\right.}{}\right.}{4ac-b^2}}{a^2}$
risch	$\frac{-\frac{c(10ac-3b^2)x^4}{2a^2(4ac-b^2)}-\frac{b(11ac-3b^2)x^2}{2(4ac-b^2)a^2}-\frac{1}{a}}{x(c\,x^4+b\,x^2+a)}+\left(\left(\_R=\mathrm{RootOf}\left(\left(4096a^{11}c^6-6144a^{10}b^2c^5+3840a^9b^4c^4-1280a^8b^6c^3+240a^7b^8c^2-24a^6b^{10}c+a^5b^{12}\right.\right.\right.\right.$

[In] int(1/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)

[Out] -1/a^2/x-1/a^2*((1/2*c*(2*a*c-b^2)/(4*a*c-b^2)*x^3+1/2*b*(3*a*c-b^2)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/8*(10*a*c*(-4*a*c+b^2)^(1/2)-3*b^2*(-4*a*c+b^2)^(1/2)-16*a*b*c+3*b^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(10*a*c*(-4*a*c+b^2)^(1/2)-3*b^2*(-4*a*c+b^2)^(1/2)+16*a*b*c-3*b^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2912 vs. $2(260) = 520$.

Time = 0.45 (sec) , antiderivative size = 2912, normalized size of antiderivative = 9.45

$$\int \frac{1}{(ax + bx^3 + cx^5)^2} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*x^4 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c
)*x^2 - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3
+ (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420
*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((8
1*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^
6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4
+ 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c
 + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^
5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*
a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2
- 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c
^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*
b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8
 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b
^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4
*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^
5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a
*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7
*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550
*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6
4*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-
(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sq
rt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 144
08*a^4*b^3*c^4 - 5200*a^5*b*c^5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*
c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 9
18*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*
c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*
c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*
b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - sqrt(1/
2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*
a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a
^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b
```

```
^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^
7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*
c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7
*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5 + (3*a^5*b^10
 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1
280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^
6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 -
4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^
2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a
^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 -
1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sqrt(1/2)*(27*b^1
1 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4
- 5200*a^5*b*c^5 + (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*
b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 30
51*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3
*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^
3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a
^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b
^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))))/((a^2*b^2*c - 4*a^3*c^2
)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{1}{(cx^5+bx^3+ax)^2}\,dx$$

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*b^2*c - 10*a*c^2)*x^4 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*x^2
)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a
^4*c)*x) + 1/2*integrate(-(3*b^3 - 13*a*b*c + (3*b^2*c - 10*a*c^2)*x^2)/(c*
x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3087 vs. $2(260) = 520$.

Time = 0.69 (sec) , antiderivative size = 3087, normalized size of antiderivative = 10.02

$$\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^2*c*x^4 - 10*a*c^2*x^4 + 3*b^3*x^2 - 11*a*b*c*x^2 + 2*a*b^2 - 8*a
^2*c)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*(6*a^4*b^8*c^2 - 8
0*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 5
6*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4 + (6*b^4*c^2 -
44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^
2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 -
3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*
a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2 + 2*(3*sqrt(2)
```

*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 74*a^3*b^5*c^2 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + 416*a^5*b*c^4 + 6*(b^2 - 4*a*c)*a^2*b^5*c - 50*(b^2 - 4*a*c)*a^3*b^3*c^2 + 104*(b^2 - 4*a*c)*a^4*b*c^3)*abs(a^2*b^2 - 4*a^3*c))*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^3 - 4*a^3*b*c + sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6 - 12*a^6*b^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2 - 64*a^8*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*abs(a^2*b^2 - 4*a^3*c)*abs(c)) + 1/16*(6*a^4*b^8*c^2 - 80*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4 + (6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2 - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 - 37*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 74*a^3*b^5*c^2 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 + 304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 416*a^5*b*c^4 - 6*(b^2 - 4*a*c)*a^2*b^5*c + 50*(b^2 - 4*a*c)*a^3*b^3*c^2 - 104*(b^2 - 4*a*c)*a^4*b*c^3)*abs(a^2*b^2 - 4*a^3*c))*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^3 - 4*a^3*b*c - sqrt((a^2*b^3 - 4*a

```
^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*
a^3*c^2)))/((a^5*b^6 - 12*a^6*b^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6
*b^3*c^2 + a^5*b^4*c^2 - 64*a^8*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7
*c^4)*abs(a^2*b^2 - 4*a^3*c)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 7555, normalized size of antiderivative = 24.53

$$\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] - atan((((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 207
7*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -
 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c -
 b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8
*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(85
1968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4
 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(
9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^
2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*
(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/
2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*
a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b
*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680
*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) + x*(204800*a^
12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*
a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4
*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*
b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2
)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b
^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 +
3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i - ((-(9*b^13 - 9*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^
3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 +
4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a
^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c
^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*
a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 - x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9
)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^
4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a
*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^
```

6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) - x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i)/(((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) + x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) + ((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 - x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) - x*(204800*a^12*c^9 + 144*a^6*b^12*c^

3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^1
0*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2
) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*
c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*
c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24
*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*
a^10*b^2*c^5)))^(1/2) + 128000*a^10*c^9 + 504*a^6*b^8*c^5 - 8112*a^7*b^6*c^
6 + 48704*a^8*b^4*c^7 - 129280*a^9*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b
^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^
11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*
c^4 - 6144*a^10*b^2*c^5)))^(1/2)*2i - atan((((-(9*b^13 + 9*b^4*(-(4*a*c - b
^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^
11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*
c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 46
72*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^
5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2)
 + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c
^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c
 - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*
a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a
^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^
11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1
572864*a^15*b^3*c^7)) + x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^
10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458
752*a^11*b^2*c^8))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b
*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5
*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 2
40*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))
^(1/2)*1i - ((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 +
 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c
^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a
c - b^2)^9)^(1/2))/(32(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7
*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)
*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9
*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 - x
*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^
9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*
c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1
280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^

16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 32
7680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) - x*(20480
0*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143
360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 + 9
b^4(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*
a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c -
 b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a
^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^
3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i)/(((-(9*b^13 + 9*b^4*(
-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^
7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^
9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^1
2 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 38
40*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^
13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778
240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^
2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 3024
0*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 2
13*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^1
1*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c
^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6
144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*
b^5*c^6 - 1572864*a^15*b^3*c^7)) + x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 -
3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^
4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) +
26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4
- 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c -
51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6
*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10
*b^2*c^5)))^(1/2) + ((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6
*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a
^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*
c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c +
 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)
))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*
a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3
c^7 - x(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 207
7*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 +
 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c -
 b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8
*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(10
48576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*
c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) -
x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c
^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*

b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) + 128000*a^10*c^9 + 504*a^6*b^8*c^5 - 8112*a^7*b^6*c^6 + 48704*a^8*b^4*c^7 - 129280*a^9*b^2*c^8))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*2i - (1/a + (b*x^2*(11*a*c - 3*b^2))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(10*a*c - 3*b^2))/(2*a^2*(4*a*c - b^2)))/(a*x + b*x^3 + c*x^5)

3.101 $\int \frac{1}{x(ax+bx^3+cx^5)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 162

$$\int \frac{1}{x(ax+bx^3+cx^5)^2}\,dx = -\frac{b^2-3ac}{a^2(b^2-4ac)x^2}+\frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^2(a+bx^2+cx^4)} -\frac{(b^4-6ab^2c+6a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} -\frac{2b\log(x)}{a^3}+\frac{b\log(a+bx^2+cx^4)}{2a^3}$$

```
[Out] (3*a*c-b^2)/a^2/(-4*a*c+b^2)/x^2+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2
/(c*x^4+b*x^2+a)-(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)
^(1/2))/a^3/(-4*a*c+b^2)^(3/2)-2*b*ln(x)/a^3+1/2*b*ln(c*x^4+b*x^2+a)/a^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1599, 1128, 754, 814, 648, 632, 212, 642}

$$\int \frac{1}{x(ax+bx^3+cx^5)^2}\,dx = \frac{b\log(a+bx^2+cx^4)}{2a^3}-\frac{2b\log(x)}{a^3}-\frac{b^2-3ac}{a^2x^2(b^2-4ac)} -\frac{(6a^2c^2-6ab^2c+b^4)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} +\frac{-2ac+b^2+bcx^2}{2ax^2(b^2-4ac)(a+bx^2+cx^4)}$$

```
[In] Int[1/(x*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*x^2)) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^
2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan
h[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x]
)/a^3 + (b*Log[a + b*x^2 + c*x^4])/(2*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^3\left(a+bx^2+cx^4\right)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right) \\
&= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)}-\frac{\text{Subst}\left(\int \frac{-2\left(b^2-3ac\right)-2bcx}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^2\right)}{2a\left(b^2-4ac\right)} \\
&= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} \\
&\quad -\frac{\text{Subst}\left(\int \left(\frac{2\left(-b^2+3ac\right)}{ax^2}-\frac{2b\left(-b^2+4ac\right)}{a^2x}+\frac{2\left(-b^4+5ab^2c-3a^2c^2-bc\left(b^2-4ac\right)x\right)}{a^2\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)}{2a\left(b^2-4ac\right)} \\
&= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} \\
&\quad -\frac{2b\log(x)}{a^3}-\frac{\text{Subst}\left(\int \frac{-b^4+5ab^2c-3a^2c^2-bc\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx,x,x^2\right)}{a^3\left(b^2-4ac\right)} \\
&= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)}-\frac{2b\log(x)}{a^3} \\
&\quad +\frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^3}+\frac{\left(b^4-6ab^2c+6a^2c^2\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^3\left(b^2-4ac\right)} \\
&= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)}-\frac{2b\log(x)}{a^3} \\
&\quad +\frac{b\log\left(a+bx^2+cx^4\right)}{2a^3}-\frac{\left(b^4-6ab^2c+6a^2c^2\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{a^3\left(b^2-4ac\right)}
\end{aligned}$$

$$= -\frac{b^2-3ac}{a^2(b^2-4ac)x^2} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^2(a+bx^2+cx^4)} - \frac{(b^4-6ab^2c+6a^2c^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} - \frac{2b\log(x)}{a^3} + \frac{b\log(a+bx^2+cx^4)}{2a^3}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.53

$$\int \frac{1}{x(ax+bx^3+cx^5)^2}\,dx = \frac{-\frac{a}{x^2} - \frac{a(b^3-3abc+b^2cx^2-2ac^2x^2)}{(b^2-4ac)(a+bx^2+cx^4)} - 4b\log(x) + \frac{\left(b^4-6ab^2c+6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{(-b^4+6a}{}}{2a^3}$$

```
[In] Integrate[1/(x*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] (-(a/x^2) - (a*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) - 4*b*Log[x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[
b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(b^2 - 4*a*c)^(3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 -
4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(
b^2 - 4*a*c)^(3/2))/(2*a^3)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.31

method	result
default	$-\frac{1}{2a^2x^2} - \frac{2b\ln(x)}{a^3} - \frac{\frac{\frac{ac(2ac-b^2)x^2}{4ac-b^2}+\frac{ab(3ac-b^2)}{4ac-b^2}}{cx^4+bx^2+a} + \frac{\frac{(-4abc^2+b^3c)\ln(cx^4+bx^2+a)}{c}+\frac{4\left(3a^2c^2-5ab^2c+b^4-\frac{(-4abc^2+b^3c)b}{2c}\right)\arctan(}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^3}$
risch	$\frac{-\frac{c(3ac-b^2)x^4}{a^2(4ac-b^2)}-\frac{b(7ac-2b^2)x^2}{2(4ac-b^2)a^2}-\frac{1}{2a}}{x^2(cx^4+bx^2+a)} - \frac{2b\ln(x)}{a^3} + \left(\sum_{\_R=\text{RootOf}((64a^6c^3-48b^2a^5c^2+12a^4b^4c-b^6a^3)\_Z^2+(-64bc^3a^3+48b^3c^2a^2-}\right.$

```
[In] int(1/x/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2/x^2-2*b*ln(x)/a^3-1/2/a^3*((a*c*(2*a*c-b^2)/(4*a*c-b^2)*x^2+a*b*(3
*a*c-b^2)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*(1/2*(-4*a*b*c^2+b^3*c
)/c*ln(c*x^4+b*x^2+a)+2*(3*a^2*c^2-5*a*b^2*c+b^4-1/2*(-4*a*b*c^2+b^3*c)*b/c
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(154) = 308$.

Time = 0.37 (sec) , antiderivative size = 1007, normalized size of antiderivative = 6.22

$$\int \frac{1}{x(ax+bx^3+cx^5)^2}\,dx$$

$$= \left[-\frac{a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + 2\,(ab^4c - 7\,a^2b^2c^2 + 12\,a^3c^3)x^4 + (2\,ab^5 - 15\,a^2b^3c + 28\,a^3bc^2)x^2 + ((b^4c - 6\right.$$

$$-\frac{a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + 2\,(ab^4c - 7\,a^2b^2c^2 + 12\,a^3c^3)x^4 + (2\,ab^5 - 15\,a^2b^3c + 28\,a^3bc^2)x^2 + 2\,((b^4c -$$

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12
*a^3*c^3)*x^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + ((b^4*c - 6*a
*b^2*c^2 + 6*a^2*c^3)*x^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 -
6*a^2*b^2*c + 6*a^3*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2
+ b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)
*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4
*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3*b^4*c -
8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^
4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2), -1/2*(a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^4 + (2*a*b^5 - 15
*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^6 +
 (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x^
2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c
)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*
b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2
+ a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*
a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3*
b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b
*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/x/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)^2}\,dx=\int \frac{1}{\left(cx^5+bx^3+ax\right)^2x}\,dx$$

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b^2*c - 3*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (2*b^3 - 7*a*b*c)*x^2)/((
a^2*b^2*c - 4*a^3*c^2)*x^6 + (a^2*b^3 - 4*a^3*b*c)*x^4 + (a^3*b^2 - 4*a^4*c
)*x^2) - 2*integrate(-((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 5*a*b^2*c + 3*a^2*c
^2)*x)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c) - 2*b*log(x)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.60 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)^2}\,dx=\frac{\left(b^4-6\,ab^2c+6\,a^2c^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^3b^2-4\,a^4c\right)\sqrt{-b^2+4\,ac}}-\frac{2\,b^2cx^4-6\,ac^2x^4+2\,b^3x^2-7\,abcx^2+ab^2-4\,a^2c}{2\,\left(cx^6+bx^4+ax^2\right)\left(a^2b^2-4\,a^3c\right)}+\frac{b\log\left(cx^4+bx^2+a\right)}{2\,a^3}-\frac{b\log\left(x^2\right)}{a^3}$$

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^
3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^2*c*x^4 - 6*a*c^2*x^4 + 2*b
^3*x^2 - 7*a*b*c*x^2 + a*b^2 - 4*a^2*c)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^2 -
 4*a^3*c)) + 1/2*b*log(c*x^4 + b*x^2 + a)/a^3 - b*log(x^2)/a^3
```

Mupad [B] (verification not implemented)

Time = 11.26 (sec) , antiderivative size = 5491, normalized size of antiderivative = 33.90

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(x*(a*x + b*x^3 + c*x^5)^2),x)
```

```
[Out] (log(a + b*x^2 + c*x^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))
/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) - (1/(2*a) - (x
^2*(2*b^3 - 7*a*b*c))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(3*a*c - b^2))/(a^2*(4
*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) - (2*b*log(x))/a^3 + (atan(((2*a^9*b^
6*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(4
*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*b^6 - 3*a^3*c^3
 + 36*a^2*b^2*c^2 - 21*a*b^4*c)*((4*(2*b^5*c^4 - 12*a*b^3*c^5 + 18*a^2*b*c^
6))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) + (((4*(9*a^5*c^6 - 4*a^2*b^6*c^3
+ 29*a^3*b^4*c^4 - 54*a^4*b^2*c^5))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) -
(((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c^3 - 46*a^6*b^3*c^4))/(a^6
*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9
*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((a^6*b^4 + 1
6*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64
*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^
3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2)) + (((((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c^3 - 46*a^6*b^3*c
^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3
 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((a^
6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2))
- ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^2 - 6*a*b^
2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(a^3*(4*a*c - b^2)
^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*
b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^
2)^(3/2)) - ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^
2 - 6*a*b^2*c)^2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*a^6
*(4*a*c - b^2)^3*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4
- 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)*(36*a^4*c^6 + b^8
*c^2 - 12*a*b^6*c^3 + 48*a^2*b^4*c^4 - 72*a^3*b^2*c^5)) - (x^2*((((4*(54*a^
3*c^8 - 2*b^6*c^5 + 18*a*b^4*c^6 - 54*a^2*b^2*c^7))/(a^6*b^6 - 64*a^9*c^3 -
 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(276*a^5*b*c^7 - 6*a^2*b^7*c^4 + 65*
a^3*b^5*c^5 - 233*a^4*b^3*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2) - (((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*
c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c
```

^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6
 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a
^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3
- 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 1
2*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^
7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2)) - (((((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^
5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a
^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a
*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4
- 672*a^9*b^3*c^5))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)
*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 + 6*a^2*c^2
- 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*
(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*
b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/(a^3*(4*a*c
- b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^
6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^
2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7 -
 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^
2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/(2*a^6*(4*a*c - b^
2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a
^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c
^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b
^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*((((((4*(480*a^8*c^7 - a^4*b^8
*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9
*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3
*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a
^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a
^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 +
 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 -
6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c
^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/(
a^3*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^7 - 64*a^3*
b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4
*c + 48*a^5*b^2*c^2)) - (((4*(276*a^5*b*c^7 - 6*a^2*b^7*c^4 + 65*a^3*b^5*c^
5 - 233*a^4*b^3*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
) - (((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*
a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(
b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b
^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a^3*b^6 - 6
4*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b
^4*c + 48*a^8*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)
)/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*
c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2

```
*c)^3*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 -
672*a^9*b^3*c^5))/(2*a^9*(4*a*c - b^2)^(9/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2)))*(3*b^6 - 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*
c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 3
82*a^3*b^2*c^3 + 72*a*b^6*c)))*(2*a^9*b^6*(4*a*c - b^2)^(9/2) - 128*a^12*c^
3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2
*(4*a*c - b^2)^(9/2)))/(36*a^4*c^6 + b^8*c^2 - 12*a*b^6*c^3 + 48*a^2*b^4*c^
4 - 72*a^3*b^2*c^5) + (b*((((((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5
*c^3 - 46*a^6*b^3*c^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*
c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2
- 12*a*b^5*c))/((a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3
- 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4
*a*c - b^2)^(3/2)) - ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 +
 6*a^2*c^2 - 6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))
/(a^3*(4*a*c - b^2)^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 6
4*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b
^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2)) - (((4*(9*a^5*c^6 - 4*a^2*b^6*c^3 + 29*a^3*b^4*c^4 - 54*a^4*b^2*c^5))
/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2
+ 18*a^5*b^5*c^3 - 46*a^6*b^3*c^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) -
(2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*
a^2*b^3*c^2 - 12*a*b^5*c))/((a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 -
 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2
*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) + ((a^7
*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3)
/(2*a^9*(4*a*c - b^2)^(9/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))*(2*a^9*b
^6*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(
4*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*b^6 - 49*a^3*c
^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^
4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)*(36*a^4*c^6 + b
^8*c^2 - 12*a*b^6*c^3 + 48*a^2*b^4*c^4 - 72*a^3*b^2*c^5)))*(b^4 + 6*a^2*c^2
 - 6*a*b^2*c))/(a^3*(4*a*c - b^2)^(3/2))
```

3.102 $\int \frac{1}{x^2(ax+bx^3+cx^5)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 361

$$\begin{aligned}\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)^2}\,dx &= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)x^3}+\frac{b(5b^2-19ac)}{2a^3\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} \\ &\quad+\frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+b(5b^2-19ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\ &\quad-\frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2-b(5b^2-19ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

```
[Out] 1/6*(14*a*c-5*b^2)/a^2/(-4*a*c+b^2)/x^3+1/2*b*(-19*a*c+5*b^2)/a^3/(-4*a*c+b
^2)/x+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^3/(c*x^4+b*x^2+a)+1/4*arctan
(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+
28*a^2*c^2+b*(-19*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)*2^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2-b*(-19*a*c+5*b^2)*(-
4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2)
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1599, 1135, 1295, 1180, 211}

$$\begin{aligned}
&\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)^2}\,dx\\
&=\frac{b(5b^2-19ac)}{2a^3x\left(b^2-4ac\right)}-\frac{5b^2-14ac}{6a^2x^3\left(b^2-4ac\right)}\\
&\quad+\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b(5b^2-19ac)\sqrt{b^2-4ac}+5b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\\
&\quad-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b(5b^2-19ac)\sqrt{b^2-4ac}+5b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}\\
&\quad+\frac{-2ac+b^2+bcx^2}{2ax^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}
\end{aligned}$$

```
[In] Int[1/(x^2*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] -1/6*(5*b^2 - 14*a*c)/(a^2*(b^2 - 4*a*c)*x^3) + (b*(5*b^2 - 19*a*c))/(2*a^3
*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x^3*(a + b*x
^2 + c*x^4)) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*
c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt
[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(
b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^4 \left(a+bx^2+cx^4\right)^2}\, dx$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-5b^2+14ac-5bcx^2}{x^4\left(a+bx^2+cx^4\right)}\, dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)x^3} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} + \frac{\int \frac{-3b\left(5b^2-19ac\right)-3c\left(5b^2-14ac\right)x^2}{x^2\left(a+bx^2+cx^4\right)}\, dx}{6a^2\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)x^3} + \frac{b(5b^2-19ac)}{2a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-3\left(5b^4-24ab^2c+14a^2c^2\right)-3bc\left(5b^2-19ac\right)x^2}{a+bx^2+cx^4}\, dx}{6a^3\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)x^3} + \frac{b(5b^2-19ac)}{2a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(5b^4-29ab^2c+28a^2c^2-b(5b^2-19ac)\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\, dx}{4a^3\left(b^2-4ac\right)^{3/2}} + \frac{\left(c\left(5b^4-29ab^2c+28a^2c^2+b(5b^2-19ac)\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\, dx}{4a^3\left(b^2-4ac\right)^{3/2}}$$

$$= -\frac{5b^2-14ac}{6a^2(b^2-4ac)x^3} + \frac{b(5b^2-19ac)}{2a^3(b^2-4ac)x} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^3(a+bx^2+cx^4)}$$
$$+ \frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+b(5b^2-19ac)\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$- \frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2-b(5b^2-19ac)\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2(ax+bx^3+cx^5)^2}\,dx$$
$$= \frac{-\frac{4a}{x^3} + \frac{24b}{x} + \frac{6x(b^4-4ab^2c+2a^2c^2+b^3cx^2-3abc^2x^2)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{3\sqrt{2}\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+5b^3\sqrt{b^2-4ac}-19abc\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{12a^3}$$

```
[In] Integrate[1/(x^2*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] ((-4*a)/x^3 + (24*b)/x + (6*x*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 - 3*
a*b*c^2*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(5*b
^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 -
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29
*a*b^2*c - 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^
(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.93

method	result
default	$-\frac{1}{3a^2x^3} + \frac{2b}{a^3x} - \frac{\frac{-\frac{bc(3ac-b^2)x^3}{2(4ac-b^2)} + \frac{(2a^2c^2-4ab^2c+b^4)x}{8ac-2b^2}}{cx^4+bx^2+a} + \frac{2c\left(\frac{\left(-19\sqrt{-4ac+b^2}\,abc+5\sqrt{-4ac+b^2}\,b^3-28a^2c^2+29ab^2c-5b^4\right)\sqrt{2}\,\arctan\left(\right.}{8\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right.}{}}{}$
risch	$\frac{\frac{cb(19ac-5b^2)x^6}{2(4ac-b^2)a^3} - \frac{(14a^2c^2-62ab^2c+15b^4)x^4}{6a^3(4ac-b^2)} + \frac{5bx^2}{3a^2} - \frac{1}{3a}}{x^3(cx^4+bx^2+a)} + \frac{\left(\_R=\text{RootOf}\left(\left(4096a^{13}c^6-6144a^{12}b^2c^5+3840a^{11}b^4c^4-1280a^{10}b^6c^3+240a^9b\right.\right.\right.}{}$

```
[In] int(1/x^2/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^2/x^3+2/a^3*b/x-1/a^3*((-1/2*b*c*(3*a*c-b^2)/(4*a*c-b^2)*x^3+1/2*(2*
a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/8*
(-19*(-4*a*c+b^2)^(1/2)*a*b*c+5*(-4*a*c+b^2)^(1/2)*b^3-28*a^2*c^2+29*a*b^2*
c-5*b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(5*b^4-29*a*b^2*c+28*a^2
*c^2+5*(-4*a*c+b^2)^(1/2)*b^3-19*(-4*a*c+b^2)^(1/2)*a*b*c)/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3435 vs. 2(311) = 622.

Time = 0.70 (sec) , antiderivative size = 3435, normalized size of antiderivative = 9.52

$$\int \frac{1}{x^2 (ax + bx^3 + cx^5)^2} dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(5*b^3*c - 19*a*b*c^2)*x^6 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*x
^4 - 4*a^2*b^2 + 16*a^3*c + 20*(a*b^3 - 4*a^2*b*c)*x^2 + 3*sqrt(1/2)*((a^3*
b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^
3)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260
*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((
625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^
4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 4
8*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 -
64*a^10*c^3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 504
21*a^3*b^2*c^7 + 9604*a^4*c^8)*x + 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^12*c
+ 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*
b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7 - (5*a^7*b^11 - 94*a^8*b^9*c +
700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*
sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76
686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4
*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^
2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 4
8*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8
*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6
*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6
 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) - 3*sqrt(1/2)*((a^3*b^2*c
 - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sq
rt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*
b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b
```

^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4
*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^1
6*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^
10*c^3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^
3*b^2*c^7 + 9604*a^4*c^8)*x - 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^12*c + 189
40*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c
^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7 - (5*a^7*b^11 - 94*a^8*b^9*c + 700*a
^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*sqrt(
(625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a
^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c +
48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5
*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9
*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2
- 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)
/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12
*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*
a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(
25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4
 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 -
 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4
- 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2
*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^
3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2
*c^7 + 9604*a^4*c^8)*x + 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^12*c + 18940*a^
2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 +
79408*a^6*b^2*c^6 - 10976*a^7*c^7 + (5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^
7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*sqrt((625*
b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^
4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^
16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2
- 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*
c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 836
30*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^1
4*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*
b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c
^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(25*b^
9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a
^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250
*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 241
08*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2
- 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*l
og((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7
+ 9604*a^4*c^8)*x - 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^12*c + 18940*a^2*b^1
0*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408
*a^6*b^2*c^6 - 10976*a^7*c^7 + (5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2

```
- 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*sqrt((625*b^12
- 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4
 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^
2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 241
5*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 -
 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^
3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6
 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c
 + 48*a^9*b^2*c^2 - 64*a^10*c^3))))/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3
 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (ax + bx^3 + cx^5)^2} \, dx = \text{Timed out}$$

```
[In] integrate(1/x**2/(c*x**5+b*x**3+a*x)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x^2 (ax + bx^3 + cx^5)^2} \, dx = \int \frac{1}{(cx^5 + bx^3 + ax)^2 x^2} \, dx$$

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
[Out] 1/6*(3*(5*b^3*c - 19*a*b*c^2)*x^6 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*x^4
- 2*a^2*b^2 + 8*a^3*c + 10*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2*c - 4*a^4*c^2
)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) - 1/2*integrat
e(-(5*b^4 - 24*a*b^2*c + 14*a^2*c^2 + (5*b^3*c - 19*a*b*c^2)*x^2)/(c*x^4 +
b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3651 vs. $2(311) = 622$.

Time = 0.98 (sec) , antiderivative size = 3651, normalized size of antiderivative = 10.11

$$\int \frac{1}{x^2 (ax + bx^3 + cx^5)^2} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c*x^3 - 3*a*b*c^2*x^3 + b^4*x - 4*a*b^2*c*x + 2*a^2*c^2*x)/((a^3*b
^2 - 4*a^4*c)*(c*x^4 + b*x^2 + a)) + 1/16*(10*a^6*b^9*c^2 - 138*a^7*b^7*c^3
 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2 - 4*a*c)*a^6*b^7
*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^8*b^3*c^4 + 224*(
b^2 - 4*a*c)*a^9*b*c^5 + (10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 39*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 -
4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2 + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c -
10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c - 10*a^3*b^8*c + 286*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b^6*c^2 + 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 -
44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 - 572*a^5*b^4*c^3 +
224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^5*b^2*c^4 + 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^6*c^5 - 448*a^7*c^5 + 10*(b^2 - 4*a*c)*a^3*b^6*c - 88*(b^2 - 4*a*c)*a
^4*b^4*c^2 + 220*(b^2 - 4*a*c)*a^5*b^2*c^3 - 112*(b^2 - 4*a*c)*a^6*c^4)*abs
(a^3*b^2 - 4*a^4*c))*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^3 - 4*a^4*b*c + sqrt(
(a^3*b^3 - 4*a^4*b*c)^2 - 4*(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2)))/(
a^3*b^2*c - 4*a^4*c^2)))/((a^7*b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a^9*b^
2*c^2 + 16*a^8*b^3*c^2 + a^7*b^4*c^2 - 64*a^10*c^3 - 32*a^9*b*c^3 - 8*a^8*b
^2*c^3 + 16*a^9*c^4)*abs(a^3*b^2 - 4*a^4*c)*abs(c)) + 1/16*(10*a^6*b^9*c^2
- 138*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqr
```

```
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2
 - 4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^
8*b^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5 + (10*b^5*c^2 - 78*a*b^3*c^3 + 152*
a^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5
 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 1
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3
*c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2 + 2*(5*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^4*b^6*c - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c + 10*
a^3*b^8*c + 286*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^6*c^2 - 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^5*b^3*c^3 - 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 + 5
72*a^5*b^4*c^3 + 224*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^4 - 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^6*c^5 + 448*a^7*c^5 - 10*(b^2 - 4*a*c)*a^3*b^6*c + 8
8*(b^2 - 4*a*c)*a^4*b^4*c^2 - 220*(b^2 - 4*a*c)*a^5*b^2*c^3 + 112*(b^2 - 4*
a*c)*a^6*c^4)*abs(a^3*b^2 - 4*a^4*c))*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^3 -
4*a^4*b*c - sqrt((a^3*b^3 - 4*a^4*b*c)^2 - 4*(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c
 - 4*a^4*c^2)))/(a^3*b^2*c - 4*a^4*c^2)))/((a^7*b^6 - 12*a^8*b^4*c - 2*a^7*
b^5*c + 48*a^9*b^2*c^2 + 16*a^8*b^3*c^2 + a^7*b^4*c^2 - 64*a^10*c^3 - 32*a^
9*b*c^3 - 8*a^8*b^2*c^3 + 16*a^9*c^4)*abs(a^3*b^2 - 4*a^4*c)*abs(c)) + 1/3*
(6*b*x^2 - a)/(a^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 10.32 (sec) , antiderivative size = 8739, normalized size of antiderivative = 24.21

$$\int \frac{1}{x^2 (ax + bx^3 + cx^5)^2} \, dx = \text{Too large to display}$$

```
[In] int(1/(x^2*(a*x + b*x^3 + c*x^5)^2),x)
[Out] atan((((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 636
6*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^
5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
 - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9
)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 -
1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(320*a^1
2*b^14*c^2 - 917504*a^19*c^9 - 7936*a^13*b^12*c^3 + 82816*a^14*b^10*c^4 - 4
68480*a^15*b^8*c^5 + 1536000*a^16*b^6*c^6 - 2867200*a^17*b^4*c^7 + 2719744*
a^18*b^2*c^8 + x*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b
*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*
a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 61
5*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*
c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*
b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2
)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17
*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7
)) - x*(401408*a^16*c^10 - 400*a^9*b^14*c^3 + 9440*a^10*b^12*c^4 - 92816*a^
11*b^10*c^5 + 488096*a^12*b^8*c^6 - 1458688*a^13*b^6*c^7 + 2401280*a^14*b^4
*c^8 - 1871872*a^15*b^2*c^9))*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2)
- 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*
c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^
9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*
b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10
*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^
2*c^5)))^(1/2)*1i + ((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a
^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219
744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2)
- 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(
4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*
a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^
(1/2)*(917504*a^19*c^9 - 320*a^12*b^14*c^2 + 7936*a^13*b^12*c^3 - 82816*a^1
4*b^10*c^4 + 468480*a^15*b^8*c^5 - 1536000*a^16*b^6*c^6 + 2867200*a^17*b^4*
c^7 - 2719744*a^18*b^2*c^8 + x*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2)
 - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7
*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)
^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a
```

```
*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^1
0*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b
^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^
3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 157286
4*a^20*b^3*c^7)) - x*(401408*a^16*c^10 - 400*a^9*b^14*c^3 + 9440*a^10*b^12*
c^4 - 92816*a^11*b^10*c^5 + 488096*a^12*b^8*c^6 - 1458688*a^13*b^6*c^7 + 24
01280*a^14*b^4*c^8 - 1871872*a^15*b^2*c^9))*(-(25*b^15 - 25*b^6*(-(4*a*c -
b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 1
16928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-
(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^
(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6
 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4
- 6144*a^12*b^2*c^5)))^(1/2)*1i)/(((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(
1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4
*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c -
b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 1
65*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8
*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^
12*b^2*c^5)))^(1/2)*(320*a^12*b^14*c^2 - 917504*a^19*c^9 - 7936*a^13*b^12*c
^3 + 82816*a^14*b^10*c^4 - 468480*a^15*b^8*c^5 + 1536000*a^16*b^6*c^6 - 286
7200*a^17*b^4*c^7 + 2719744*a^18*b^2*c^8 + x*(-(25*b^15 - 25*b^6*(-(4*a*c -
 b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 +
116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(
-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)
^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^
6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4
 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 614
4*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^
5*c^6 - 1572864*a^20*b^3*c^7)) - x*(401408*a^16*c^10 - 400*a^9*b^14*c^3 + 9
440*a^10*b^12*c^4 - 92816*a^11*b^10*c^5 + 488096*a^12*b^8*c^6 - 1458688*a^1
3*b^6*c^7 + 2401280*a^14*b^4*c^8 - 1871872*a^15*b^2*c^9))*(-(25*b^15 - 25*b
^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a
^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 +
 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*
a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 +
 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840
*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) - ((-(25*b^15 - 25*b^6*(-(4*a*c
- b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 +
 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*
(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9
)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c
^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^
4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*a^19*c^9 - 320*a^12*b^14*c^2 + 7936*
a^13*b^12*c^3 - 82816*a^14*b^10*c^4 + 468480*a^15*b^8*c^5 - 1536000*a^16*b^
6*c^6 + 2867200*a^17*b^4*c^7 - 2719744*a^18*b^2*c^8 + x*(-(25*b^15 - 25*b^6
```

*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3
*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 4
9*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*
c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4
096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a
^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^1
3*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983
040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7)) - x*(401408*a^16*c^10 - 400*a^9*b
^14*c^3 + 9440*a^10*b^12*c^4 - 92816*a^11*b^10*c^5 + 488096*a^12*b^8*c^6 -
1458688*a^13*b^6*c^7 + 2401280*a^14*b^4*c^8 - 1871872*a^15*b^2*c^9))*(-(25*
b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^
2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^
6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^
2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*
(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6
*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) + 476672*a^13*b*c^10
+ 1800*a^9*b^9*c^6 - 29080*a^10*b^7*c^7 + 176032*a^11*b^5*c^8 - 473216*a^12
*b^3*c^9))*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 +
6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^
5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^
13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^
2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*2i -
(1/(3*a) - (5*b*x^2)/(3*a^2) + (x^4*(15*b^4 + 14*a^2*c^2 - 62*a*b^2*c))/(6
a^3(4*a*c - b^2)) + (c*x^6*(5*b^3 - 19*a*b*c))/(2*a^3*(4*a*c - b^2)))/(a*
x^3 + b*x^5 + c*x^7) + atan((((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2)
- 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*
c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^
9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*
b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10
*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^
2*c^5)))^(1/2)*(320*a^12*b^14*c^2 - 917504*a^19*c^9 - 7936*a^13*b^12*c^3 +
82816*a^14*b^10*c^4 - 468480*a^15*b^8*c^5 + 1536000*a^16*b^6*c^6 - 2867200*
a^17*b^4*c^7 + 2719744*a^18*b^2*c^8 + x*(-(25*b^15 + 25*b^6*(-(4*a*c - b^2)
^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 11692
8*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a
*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2
) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 2
4*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 61
44*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^1
6*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6
- 1572864*a^20*b^3*c^7)) - x*(401408*a^16*c^10 - 400*a^9*b^14*c^3 + 9440*a
^10*b^12*c^4 - 92816*a^11*b^10*c^5 + 488096*a^12*b^8*c^6 - 1458688*a^13*b^6
*c^7 + 2401280*a^14*b^4*c^8 - 1871872*a^15*b^2*c^9))*(-(25*b^15 + 25*b^6*(-
(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^

9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a
^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c -
 b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096
*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11
*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i + ((-(25*b^15 + 25*b^6*(-(4*a*c -
b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 1
16928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-
(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^
(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6
 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4
- 6144*a^12*b^2*c^5)))^(1/2)*(917504*a^19*c^9 - 320*a^12*b^14*c^2 + 7936*a^
13*b^12*c^3 - 82816*a^14*b^10*c^4 + 468480*a^15*b^8*c^5 - 1536000*a^16*b^6*
c^6 + 2867200*a^17*b^4*c^7 - 2719744*a^18*b^2*c^8 + x*(-(25*b^15 + 25*b^6*(
-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b
^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*
a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c
 - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 409
6*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^1
1*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*
c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 98304
0*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7)) - x*(401408*a^16*c^10 - 400*a^9*b^1
4*c^3 + 9440*a^10*b^12*c^4 - 92816*a^11*b^10*c^5 + 488096*a^12*b^8*c^6 - 14
58688*a^13*b^6*c^7 + 2401280*a^14*b^4*c^8 - 1871872*a^15*b^2*c^9))*(-(25*b^
15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2
 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*
b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*
c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a
^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c
^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i)/(((-(25*b^15 + 25*b
^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a
^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 -
 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*
a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 +
 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840
*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(320*a^12*b^14*c^2 - 917504*a^19
*c^9 - 7936*a^13*b^12*c^3 + 82816*a^14*b^10*c^4 - 468480*a^15*b^8*c^5 + 153
6000*a^16*b^6*c^6 - 2867200*a^17*b^4*c^7 + 2719744*a^18*b^2*c^8 + x*(-(25*b
^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2
 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6
*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2
c^2(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*
c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 +
256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b
^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7)) - x*(401408*a^16*c^10

- 400*a^9*b^14*c^3 + 9440*a^10*b^12*c^4 - 92816*a^11*b^10*c^5 + 488096*a^12*b^8*c^6 - 1458688*a^13*b^6*c^7 + 2401280*a^14*b^4*c^8 - 1871872*a^15*b^2*c^9))*(-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) - ((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*a^19*c^9 - 320*a^12*b^14*c^2 + 7936*a^13*b^12*c^3 - 82816*a^14*b^10*c^4 + 468480*a^15*b^8*c^5 - 1536000*a^16*b^6*c^6 + 2867200*a^17*b^4*c^7 - 2719744*a^18*b^2*c^8 + x*(-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7)) - x*(401408*a^16*c^10 - 400*a^9*b^14*c^3 + 9440*a^10*b^12*c^4 - 92816*a^11*b^10*c^5 + 488096*a^12*b^8*c^6 - 1458688*a^13*b^6*c^7 + 2401280*a^14*b^4*c^8 - 1871872*a^15*b^2*c^9))*(-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) + 476672*a^13*b*c^10 + 1800*a^9*b^9*c^6 - 29080*a^10*b^7*c^7 + 176032*a^11*b^5*c^8 - 473216*a^12*b^3*c^9))*(-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*2i

3.103 $\int \frac{1}{x^3(ax+bx^3+cx^5)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 219

$$\begin{aligned}\int \frac{1}{x^3\left(ax+bx^3+cx^5\right)^2}\,dx = &-\frac{3b^2-8ac}{4a^2\left(b^2-4ac\right)x^4}+\frac{b(3b^2-11ac)}{2a^3\left(b^2-4ac\right)x^2}\\&+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)}\\&+\frac{b(3b^4-20ab^2c+30a^2c^2)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{3/2}}\\&+\frac{(3b^2-2ac)\log(x)}{a^4}-\frac{(3b^2-2ac)\log\left(a+bx^2+cx^4\right)}{4a^4}\end{aligned}$$

```
[Out] 1/4*(8*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x^4+1/2*b*(-11*a*c+3*b^2)/a^3/(-4*a*c+b^
2)/x^2+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^4/(c*x^4+b*x^2+a)+1/2*b*(30
*a^2*c^2-20*a*b^2*c+3*b^4)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*
a*c+b^2)^(3/2)+(-2*a*c+3*b^2)*ln(x)/a^4-1/4*(-2*a*c+3*b^2)*ln(c*x^4+b*x^2+a
)/a^4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

$= \{1599, 1128, 754, 814, 648, 632, 212, 642\}$

$$\int \frac{1}{x^3 (ax + bx^3 + cx^5)^2} \, dx = -\frac{(3b^2 - 2ac) \log (a + bx^2 + cx^4)}{4a^4} + \frac{\log(x) (3b^2 - 2ac)}{a^4} + \frac{b(3b^2 - 11ac)}{2a^3x^2 (b^2 - 4ac)} - \frac{3b^2 - 8ac}{4a^2x^4 (b^2 - 4ac)} + \frac{b(30a^2c^2 - 20ab^2c + 3b^4) \operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4 (b^2 - 4ac)^{3/2}} + \frac{-2ac + b^2 + bcx^2}{2ax^4 (b^2 - 4ac) (a + bx^2 + cx^4)}$$

```
[In] Int[1/(x^3*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] -1/4*(3*b^2 - 8*a*c)/(a^2*(b^2 - 4*a*c)*x^4) + (b*(3*b^2 - 11*a*c))/(2*a^3*
(b^2 - 4*a*c)*x^2) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x^4*(a + b*
x^2 + c*x^4)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/
a^4 - ((3*b^2 - 2*a*c)*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^5 (a+bx^2+cx^4)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3 (a+bx+cx^2)^2}\,dx, x, x^2\right) \\
&= \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)\,x^4 (a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{-3b^2+8ac-3bcx}{x^3(a+bx+cx^2)}\,dx, x, x^2\right)}{2a(b^2-4ac)} \\
&= \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)\,x^4 (a+bx^2+cx^4)} \\
&\quad - \frac{\text{Subst}\left(\int \left(\frac{-3b^2+8ac}{ax^3} + \frac{3b^3-11abc}{a^2x^2} + \frac{(b^2-4ac)(-3b^2+2ac)}{a^3x} + \frac{b(3b^4-17ab^2c+19a^2c^2)+c(b^2-4ac)(3b^2-2ac)x}{a^3(a+bx+cx^2)}\right) dx, x,\right.}{2a(b^2-4ac)}
\end{aligned}
$$

$$= -\frac{3b^2-8ac}{4a^2\left(b^2-4ac\right)x^4}+\frac{b(3b^2-11ac)}{2a^3\left(b^2-4ac\right)x^2}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)} + \frac{(3b^2-2ac)\log(x)}{a^4}-\frac{\text{Subst}\left(\int \frac{b(3b^4-17ab^2c+19a^2c^2)+c(b^2-4ac)(3b^2-2ac)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^4\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-8ac}{4a^2\left(b^2-4ac\right)x^4}+\frac{b(3b^2-11ac)}{2a^3\left(b^2-4ac\right)x^2}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)} + \frac{(3b^2-2ac)\log(x)}{a^4}-\frac{(3b^2-2ac)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^4} - \frac{(b(3b^4-20ab^2c+30a^2c^2))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^4\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-8ac}{4a^2\left(b^2-4ac\right)x^4}+\frac{b(3b^2-11ac)}{2a^3\left(b^2-4ac\right)x^2}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)} + \frac{(3b^2-2ac)\log(x)}{a^4}-\frac{(3b^2-2ac)\log\left(a+bx^2+cx^4\right)}{4a^4} + \frac{(b(3b^4-20ab^2c+30a^2c^2))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2a^4\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-8ac}{4a^2\left(b^2-4ac\right)x^4}+\frac{b(3b^2-11ac)}{2a^3\left(b^2-4ac\right)x^2}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)} + \frac{b(3b^4-20ab^2c+30a^2c^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{3/2}} + \frac{(3b^2-2ac)\log(x)}{a^4}-\frac{(3b^2-2ac)\log\left(a+bx^2+cx^4\right)}{4a^4}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.50

$$\int \frac{1}{x^3\left(ax+bx^3+cx^5\right)^2}\,dx$$

$$= \frac{-\frac{a^2}{x^4}+\frac{4ab}{x^2}+\frac{2a(b^4-4ab^2c+2a^2c^2+b^3cx^2-3abc^2x^2)}{(b^2-4ac)(a+bx^2+cx^4)}+4(3b^2-2ac)\log(x)-\frac{\left(3b^5-20ab^3c+30a^2bc^2+3b^4\sqrt{b^2-4ac}-14ab^2c\sqrt{b^2-\right.}{(b^2-4ac)}}{}$$

```
[In] Integrate[1/(x^3*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] (-(a^2/x^4) + (4*a*b)/x^2 + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 -
 3*a*b*c^2*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*(3*b^2 - 2*a*c)*Lo
g[x] - ((3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2 + 3*b^4*Sqrt[b^2 - 4*a*c] - 14*a
*b^2*c*Sqrt[b^2 - 4*a*c] + 8*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 -
4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((3*b^5 - 20*a*b^3*c + 30*a^2*b*c^
2 - 3*b^4*Sqrt[b^2 - 4*a*c] + 14*a*b^2*c*Sqrt[b^2 - 4*a*c] - 8*a^2*c^2*Sqrt
[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(
4*a^4)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.20

method	result
default	$-\frac{1}{4a^2x^4}+\frac{(-2ac+3b^2)\ln(x)}{a^4}+\frac{b}{a^3x^2}+\frac{\frac{\frac{acb(3ac-b^2)x^2}{4ac-b^2}-\frac{a(2a^2c^2-4a\,b^2c+b^4)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\frac{(8a^2c^3-14b^2a\,c^2+3b^4c)\ln(c\,x^4+b\,x^2+a)}{2c}+\frac{2\left(19a^2b\right.}{}}{}}{2a^4}$
risch	$\frac{\frac{bc(11ac-3b^2)x^6}{2a^3(4ac-b^2)}-\frac{(8a^2c^2-25a\,b^2c+6b^4)x^4}{4a^3(4ac-b^2)}+\frac{3b\,x^2}{4a^2}-\frac{1}{4a}}{x^4(c\,x^4+b\,x^2+a)}-\frac{2\ln(x)c}{a^3}+\frac{3b^2\ln(x)}{a^4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((64a^7c^3-48a^6b^2c^2+12a^5b^4c-b^6a^4)\_\right.}\right.}{}$

[In] int(1/x^3/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)

[Out] -1/4/a^2/x^4+(-2*a*c+3*b^2)*ln(x)/a^4+1/a^3*b/x^2+1/2/a^4*((a*c*b*(3*a*c-b^2)/(4*a*c-b^2)*x^2-a*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)/c*ln(c*x^4+b*x^2+a)+2*(19*a^2*b*c^2-17*a*b^3*c+3*b^5-1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. 2(205) = 410.

Time = 0.45 (sec) , antiderivative size = 1242, normalized size of antiderivative = 5.67

$$\int \frac{1}{x^3\left(ax+bx^3+cx^5\right)^2}\,dx = \text{Too large to display}$$

[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] [-1/4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^6 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3)*x^4 - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^8 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^6 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^4)*log(c*x^4 + b*x^2 + a) - 4*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^4)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^8 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^6 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^4), -1/4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 2

```
3*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^6 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c
^2 - 32*a^4*c^3)*x^4 - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2 - 2*((3
*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^8 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^
2*c^2)*x^6 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^4)*sqrt(-b^2 + 4*a*c
)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((3*b^6*c - 26*
a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c + 64*a^2
*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 3
2*a^4*c^3)*x^4)*log(c*x^4 + b*x^2 + a) - 4*((3*b^6*c - 26*a*b^4*c^2 + 64*a^
2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3
*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^4)*l
og(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^8 + (a^4*b^5 - 8*a^5*b^3
*c + 16*a^6*b*c^2)*x^6 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (ax + bx^3 + cx^5)^2} \, dx = \text{Timed out}$$

```
[In] integrate(1/x**3/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x^3 (ax + bx^3 + cx^5)^2} \, dx = \int \frac{1}{(cx^5 + bx^3 + ax)^2 x^3} \, dx$$

```
[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(3*b^3*c - 11*a*b*c^2)*x^6 + (6*b^4 - 25*a*b^2*c + 8*a^2*c^2)*x^4 -
a^2*b^2 + 4*a^3*c + 3*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2*c - 4*a^4*c^2)*x^8
 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^5*c)*x^4) - integrate(((3*b^4
*c - 14*a*b^2*c^2 + 8*a^2*c^3)*x^3 + (3*b^5 - 17*a*b^3*c + 19*a^2*b*c^2)*x)
/(c*x^4 + b*x^2 + a), x)/(a^4*b^2 - 4*a^5*c) + (3*b^2 - 2*a*c)*log(x)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^3\left(ax+bx^3+cx^5\right)^2}\,dx = -\frac{(3\,b^5-20\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^4b^2-4\,a^5c)\sqrt{-b^2+4\,ac}} + \frac{3\,b^4cx^4-14\,ab^2c^2x^4+8\,a^2c^3x^4+3\,b^5x^2-12\,ab^3cx^2+2\,a^2bc^2x^2+5\,ab^4-22\,a^2b^2c+12\,a^3c^2}{4\,(a^4b^2-4\,a^5c)(cx^4+bx^2+a)} - \frac{(3\,b^2-2\,ac)\log\left(cx^4+bx^2+a\right)}{4\,a^4} + \frac{(3\,b^2-2\,ac)\log\left(x^2\right)}{2\,a^4} - \frac{9\,b^2x^4-6\,acx^4-4\,abx^2+a^2}{4\,a^4x^4}$$

```
[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4
*a*c))/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(3*b^4*c*x^4 - 14*a*b
^2*c^2*x^4 + 8*a^2*c^3*x^4 + 3*b^5*x^2 - 12*a*b^3*c*x^2 + 2*a^2*b*c^2*x^2 +
 5*a*b^4 - 22*a^2*b^2*c + 12*a^3*c^2)/((a^4*b^2 - 4*a^5*c)*(c*x^4 + b*x^2 +
 a)) - 1/4*(3*b^2 - 2*a*c)*log(c*x^4 + b*x^2 + a)/a^4 + 1/2*(3*b^2 - 2*a*c)
*log(x^2)/a^4 - 1/4*(9*b^2*x^4 - 6*a*c*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^4)
```

Mupad [B] (verification not implemented)

Time = 11.78 (sec) , antiderivative size = 5999, normalized size of antiderivative = 27.39

$$\int \frac{1}{x^3\left(ax+bx^3+cx^5\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^3*(a*x + b*x^3 + c*x^5)^2),x)
```

```
[Out] (b*atan((x^2*((((((b*((2240*a^10*b*c^7 - 6*a^6*b^9*c^3 + 40*a^7*b^7*c^4 + 1
08*a^8*b^5*c^5 - 1248*a^9*b^3*c^6)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c +
 48*a^11*b^2*c^2) - ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 +
 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*
c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^
4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^
2*c^2)))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*(4*a*c - b^2)^(3/2)) - (
b*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)*(2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184
*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^4*c^4
 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(8*a^4*(4*a*c - b^2)^(3
/2)*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 -
256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*
a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 -
48*a^5*b^4*c + 192*a^6*b^2*c^2)) + (b*((1760*a^7*b*c^8 + 54*a^3*b^9*c^4 - 6
```

57*a^4*b^7*c^5 + 2775*a^5*b^5*c^6 - 4484*a^6*b^3*c^7)/(a^9*b^6 - 64*a^12*c^
3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) + (((2240*a^10*b*c^7 - 6*a^6*b^9*c^3 +
 40*a^7*b^7*c^4 + 108*a^8*b^5*c^5 - 1248*a^9*b^3*c^6)/(a^9*b^6 - 64*a^12*c^
3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) - ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 -
 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^4
*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(a^9*b^6 - 64*a^
12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5
*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^
3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a
^6*b^2*c^2)))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*(4*a*c - b^2)^(3/2)
) + (b^3*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)^3*(2560*a^13*b*c^6 + 12*a^9*b^9*
c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5))/(64*a^12*(
4*a*c - b^2)^(9/2)*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2
)))*(9*b^8 + 80*a^4*c^4 + 270*a^2*b^4*c^2 - 285*a^3*b^2*c^3 - 87*a*b^6*c))/
(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(54*b^10 - 1600*a^5*c^5 + 3480*a^2*b^6*c^2 -
 7200*a^3*b^4*c^3 + 5775*a^4*b^2*c^4 - 720*a*b^8*c)) + (3*b*((27*b^9*c^5 -
297*a*b^7*c^6 + 1089*a^2*b^5*c^7 - 1331*a^3*b^3*c^8)/(a^9*b^6 - 64*a^12*c^3
 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) - (((1760*a^7*b*c^8 + 54*a^3*b^9*c^4 -
657*a^4*b^7*c^5 + 2775*a^5*b^5*c^6 - 4484*a^6*b^3*c^7)/(a^9*b^6 - 64*a^12*c
^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) + (((2240*a^10*b*c^7 - 6*a^6*b^9*c^3
+ 40*a^7*b^7*c^4 + 108*a^8*b^5*c^5 - 1248*a^9*b^3*c^6)/(a^9*b^6 - 64*a^12*c
^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) - ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2
- 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^
4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(a^9*b^6 - 64*a
^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^
5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a
^3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*
a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 7
6*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2))
+ (b*((b*((2240*a^10*b*c^7 - 6*a^6*b^9*c^3 + 40*a^7*b^7*c^4 + 108*a^8*b^5*c
^5 - 1248*a^9*b^3*c^6)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2
*c^2) - ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b
^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^
3*b^2*c^3 - 76*a*b^6*c))/(2*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^1
1*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*
b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*(4*a*c - b^2)^(3/2)) - (b*(3*b^4 + 3
0*a^2*c^2 - 20*a*b^2*c)*(2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^
3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b
^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(8*a^4*(4*a*c - b^2)^(3/2)*(a^9*b^6
 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3
- 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a
^4*(4*a*c - b^2)^(3/2)) - (b^2*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)^2*(2560*a^
13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^1
2*b^3*c^5)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*
b^6*c))/(32*a^8*(4*a*c - b^2)^3*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48

```
*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))
*(3*b^6 - 25*a^3*c^3 + 50*a^2*b^2*c^2 - 23*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^
2)^3*(54*b^10 - 1600*a^5*c^5 + 3480*a^2*b^6*c^2 - 7200*a^3*b^4*c^3 + 5775*a
^4*b^2*c^4 - 720*a*b^8*c)))*(16*a^12*b^6*(4*a*c - b^2)^(9/2) - 1024*a^15*c^
3*(4*a*c - b^2)^(9/2) - 192*a^13*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^14*b^2*c
^2*(4*a*c - b^2)^(9/2)))/(9*b^10*c^2 - 120*a*b^8*c^3 + 580*a^2*b^6*c^4 - 12
00*a^3*b^4*c^5 + 900*a^4*b^2*c^6) + (((b*((36*a^3*b^8*c^3 - 309*a^4*b^6*c^4
 + 778*a^5*b^4*c^5 - 473*a^6*b^2*c^6)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c
) - (((12*a^6*b^8*c^2 - 116*a^7*b^6*c^3 + 348*a^8*b^4*c^4 - 304*a^9*b^2*c^5
)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c) + ((4*a^10*b^6*c^2 - 32*a^11*b^4*c
^3 + 64*a^12*b^2*c^4)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*
c^3 - 76*a*b^6*c))/(2*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a^4*b^6 - 2
56*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*a
^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 4
8*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*
(4*a*c - b^2)^(3/2)) - (((b*((12*a^6*b^8*c^2 - 116*a^7*b^6*c^3 + 348*a^8*b^
4*c^4 - 304*a^9*b^2*c^5)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c) + ((4*a^10*
b^6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*b^2*c^4)*(6*b^8 + 256*a^4*c^4 + 336*a^2
*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(a^9*b^4 + 16*a^11*c^2 - 8*a^1
0*b^2*c)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^
4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*(4*a*c - b^2)^(3/2)) + (b*(3*b^4 + 30*
a^2*c^2 - 20*a*b^2*c)*(4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*b^2*c^4)*
(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(8*
a^4*(4*a*c - b^2)^(3/2)*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a^4*b^6 -
 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336
*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 -
 48*a^5*b^4*c + 192*a^6*b^2*c^2)) + (b^3*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)^
3*(4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*b^2*c^4))/(64*a^12*(4*a*c - b
^2)^(9/2)*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)))*(16*a^12*b^6*(4*a*c - b^
2)^(9/2) - 1024*a^15*c^3*(4*a*c - b^2)^(9/2) - 192*a^13*b^4*c*(4*a*c - b^2)
^(9/2) + 768*a^14*b^2*c^2*(4*a*c - b^2)^(9/2))*(9*b^8 + 80*a^4*c^4 + 270*a^
2*b^4*c^2 - 285*a^3*b^2*c^3 - 87*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(
9*b^10*c^2 - 120*a*b^8*c^3 + 580*a^2*b^6*c^4 - 1200*a^3*b^4*c^5 + 900*a^4*b
^2*c^6)*(54*b^10 - 1600*a^5*c^5 + 3480*a^2*b^6*c^2 - 7200*a^3*b^4*c^3 + 577
5*a^4*b^2*c^4 - 720*a*b^8*c)) - (3*b*(16*a^12*b^6*(4*a*c - b^2)^(9/2) - 102
4*a^15*c^3*(4*a*c - b^2)^(9/2) - 192*a^13*b^4*c*(4*a*c - b^2)^(9/2) + 768*a
^14*b^2*c^2*(4*a*c - b^2)^(9/2))*((((36*a^3*b^8*c^3 - 309*a^4*b^6*c^4 + 778
*a^5*b^4*c^5 - 473*a^6*b^2*c^6)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c) - ((
(12*a^6*b^8*c^2 - 116*a^7*b^6*c^3 + 348*a^8*b^4*c^4 - 304*a^9*b^2*c^5)/(a^9
*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c) + ((4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 6
4*a^12*b^2*c^4)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 -
76*a*b^6*c))/(2*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a^4*b^6 - 256*a^7
*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4
*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*
b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3
```

```
*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^
6*b^2*c^2)) - (27*b^8*c^4 - 216*a*b^6*c^5 + 495*a^2*b^4*c^6 - 242*a^3*b^2*c
^7)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c) + (b*((b*((12*a^6*b^8*c^2 - 116*
a^7*b^6*c^3 + 348*a^8*b^4*c^4 - 304*a^9*b^2*c^5)/(a^9*b^4 + 16*a^11*c^2 - 8
*a^10*b^2*c) + ((4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*b^2*c^4)*(6*b^8
 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(a^9*b
^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c +
192*a^6*b^2*c^2)))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*(4*a*c - b^2)^
(3/2)) + (b*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)*(4*a^10*b^6*c^2 - 32*a^11*b^4
*c^3 + 64*a^12*b^2*c^4)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^
2*c^3 - 76*a*b^6*c))/(8*a^4*(4*a*c - b^2)^(3/2)*(a^9*b^4 + 16*a^11*c^2 - 8*
a^10*b^2*c)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3
*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*(4*a*c - b^2)^(3/2)) + (b^2*(3*b^4
+ 30*a^2*c^2 - 20*a*b^2*c)^2*(4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*b^
2*c^4)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*
c))/(32*a^8*(4*a*c - b^2)^3*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a^4*b
^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^6 - 25*a^3*c^3 +
50*a^2*b^2*c^2 - 23*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*b^10*c^2 - 120*
a*b^8*c^3 + 580*a^2*b^6*c^4 - 1200*a^3*b^4*c^5 + 900*a^4*b^2*c^6)*(54*b^10
- 1600*a^5*c^5 + 3480*a^2*b^6*c^2 - 7200*a^3*b^4*c^3 + 5775*a^4*b^2*c^4 - 7
20*a*b^8*c)))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(2*a^4*(4*a*c - b^2)^(3/2)
) - (log(x)*(2*a*c - 3*b^2))/a^4 - (log(a + b*x^2 + c*x^4)*(6*b^8 + 256*a^4
*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256
*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)) - (1/(4*a) - (3*b*x^2)/(4*a^2)
+ (x^4*(6*b^4 + 8*a^2*c^2 - 25*a*b^2*c))/(4*a^3*(4*a*c - b^2)) - (b*c*x^6*(
11*a*c - 3*b^2))/(2*a^3*(4*a*c - b^2)))/(a*x^4 + b*x^6 + c*x^8)
```

3.104 $\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{2x^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}$$

```
[Out] 2/3*x^2*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1944, 1155, 524}

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{2x^2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}$$

```
[In] Int[x/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a*x + b*x^3 + c*x^5]
)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1944

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Dist[(a*x^q + b*x^n + c*x^(2*n - q))^p/(x^(p*q)*(a + b*x^(n
 - q) + c*x^(2*(n - q)))^p), Int[x^(m + p*q)*(a + b*x^(n - q) + c*x^(2*(n -
 q)))^p, x], x] /; FreeQ[{a, b, c, m, n, p, q}, x] && EqQ[r, 2*n - q] &&  !
IntegerQ[p] && PosQ[n - q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\left(\sqrt{x}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{x}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{ax+bx^3+cx^5}} \\
&= \frac{2x^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 11.07 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.20

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{2x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{3\sqrt{x\left(a+bx^2+cx^4\right)}}$$

```
[In] Integrate[x/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1
/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])])/(3*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [F]

$$\int \frac{x}{\sqrt{c\,x^{5}+b\,x^{3}+ax}}dx$$

```
[In] int(x/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] int(x/(c*x^5+b*x^3+a*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)/(c*x^4 + b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{x}{\sqrt{x\,(a+bx^2+cx^4)}}\,dx$$

```
[In] integrate(x/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx=\int \frac{x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx=\int \frac{x}{\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

```
[In] int(x/(a*x + b*x^3 + c*x^5)^(1/2),x)
[Out] int(x/(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.105 $\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 380

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx=-\frac{2(b^2-3ac)\,x^{3/2}(a+bx^2+cx^4)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$
$$+\frac{\sqrt{x}(b+3cx^2)\sqrt{ax+bx^3+cx^5}}{15c}$$
$$+\frac{2\sqrt[4]{a}(b^2-3ac)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{ax+bx^3+cx^5}}$$
$$-\frac{\sqrt[4]{a}\left(2b^2+\sqrt{a}b\sqrt{c}-6ac\right)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] -2/15*(-3*a*c+b^2)*x^(3/2)*(c*x^4+b*x^2+a)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))/(c
*x^5+b*x^3+a*x)^(1/2)+1/15*(3*c*x^2+b)*x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2)/c+2/
15*a^(1/4)*(-3*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-
b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a
^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^5+b*x^3+a*x)^(1/2)-1/30*a^(1/4)*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))
*(a^(1/2)+x^2*c^(1/2))*(2*b^2-6*a*c+b*a^(1/2)*c^(1/2))*x^(1/2)*((c*x^4+b*x^
2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1933, 1967, 1211, 1117, 1209}

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx =$$

$$-\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{ax+bx^3+cx^5}}$$
$$+\frac{2\sqrt[4]{a}\sqrt{x}(b^2-3ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{ax+bx^3+cx^5}}$$
$$-\frac{2x^{3/2}(b^2-3ac)(a+bx^2+cx^4)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{x}(b+3cx^2)\sqrt{ax+bx^3+cx^5}}{15c}$$

```
[In] Int[x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (-2*(b^2 - 3*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(15*c^(3/2)*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*(b + 3*c*x^2)*Sqrt[a*x + b*x^
3 + c*x^5])/(15*c) + (2*a^(1/4)*(b^2 - 3*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(7/4)*Sqrt[a*x + b
*x^3 + c*x^5]) - (a^(1/4)*(2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*Sqrt[x]*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(7
/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1933

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Dist[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), Int[x^(m - (n - 2*q
))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1967

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)
*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{x}(b+3cx^2)\sqrt{ax+bx^3+cx^5}}{15c}+\frac{\int \frac{\sqrt{x}(-ab-2(b^2-3ac)x^2)}{\sqrt{ax+bx^3+cx^5}}\,dx}{15c}\\
&= \frac{\sqrt{x}(b+3cx^2)\sqrt{ax+bx^3+cx^5}}{15c}+\frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{-ab-2(b^2-3ac)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{15c\sqrt{ax+bx^3+cx^5}}\\
&= \frac{\sqrt{x}(b+3cx^2)\sqrt{ax+bx^3+cx^5}}{15c}\\
&\quad+\frac{\left(2\sqrt{a}(b^2-3ac)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{15c^{3/2}\sqrt{ax+bx^3+cx^5}}\\
&\quad+\frac{\left(\sqrt{a}\left(-\sqrt{a}b\sqrt{c}-2(b^2-3ac)\right)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{15c^{3/2}\sqrt{ax+bx^3+cx^5}}
\end{aligned}$$

$$= -\frac{2(b^2-3ac)\,x^{3/2}(a+bx^2+cx^4)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} + \frac{\sqrt{x}(b+3cx^2)\sqrt{ax+bx^3+cx^5}}{15c}$$
$$+ \frac{2\sqrt[4]{a}(b^2-3ac)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{ax+bx^3+cx^5}}$$
$$- \frac{\sqrt[4]{a}(2b^2+\sqrt{a}b\sqrt{c}-6ac)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{ax+bx^3+cx^5}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.28

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx = \frac{\sqrt{x}\left(2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(b+3cx^2)\left(a+bx^2+cx^4\right)-i(b^2-3ac)\left(-b+\sqrt{b^2-4ac}\right.\right.}{}$$

```
[In] Integrate[x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b + 3*c*x^2)*(a + b*x^2 +
c*x^4) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*
c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(
-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b
^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]
*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
- 4*a*c])]))/(30*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*
x^4)])
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.20

method	result
risch	$\frac{x^{\frac{3}{2}}(3cx^2+b)(cx^4+bx^2+a)}{15c\sqrt{x(cx^4+bx^2+a)}} - \frac{\left(\frac{ab\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}, \frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac\cdots}\right)}{ac}}}{2}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}\right.}{}$
default	Expression too large to display

```
[In] int(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*x^(3/2)*(3*c*x^2+b)*(c*x^4+b*x^2+a)/c/(x*(c*x^4+b*x^2+a))^(1/2)-1/15/c
*(1/4*a*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*
b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2*(6*a*c-2*b^2)*a*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(
-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2)
)*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))*(c*x^4+b*x
^2+a)^(1/2)*x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.97

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx =$$

$$-\frac{2\sqrt{\frac{1}{2}}\left((b^2c-3\,ac^2)x^2\sqrt{\frac{b^2-4\,ac}{c^2}}-(b^3-3\,abc)x^2\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+b\cdots}{2\,ac}}{}$$

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/30*(2*sqrt(1/2)*((b^2*c - 3*a*c^2)*x^2*sqrt((b^2 - 4*a*c)/c^2) - (b^3 -
3*a*b*c)*x^2)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(ar
csin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b
^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((2*b^2*c - (6*a + b)*c^
2)*x^2*sqrt((b^2 - 4*a*c)/c^2) - (2*b^3 - (6*a*b - b^2)*c)*x^2)*sqrt(c)*sqr
t((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sq
rt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 -
```

```
2*a*c)/(a*c)) - 2*(3*c^3*x^4 + b*c^2*x^2 - 2*b^2*c + 6*a*c^2)*sqrt(c*x^5 +
b*x^3 + a*x)*sqrt(x))/(c^3*x^2)
```

Sympy [F]

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx=\int x^{\frac{3}{2}}\sqrt{x\,(a+bx^2+cx^4)}\,dx$$

```
[In] integrate(x**(3/2)*(c*x**5+b*x**3+a*x)**(1/2),x)
[Out] Integral(x**(3/2)*sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx=\int \sqrt{cx^5+bx^3+ax}x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)
```

Giac [F]

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx=\int \sqrt{cx^5+bx^3+ax}x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx=\int x^{3/2}\,\sqrt{c\,x^5+b\,x^3+a\,x}\,dx$$

```
[In] int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(1/2),x)
[Out] int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.106 $\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 129

$$\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx=\frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{(b^2-4ac)\sqrt{x}\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] -1/16*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x
^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(c*x^5+b*x^3+a*x)^(1/2)+1/8*(2*c*x^2+b
)*(c*x^5+b*x^3+a*x)^(1/2)/c/x^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1932, 1928, 1121, 635, 212}

$$\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx=\frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\sqrt{x}(b^2-4ac)\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}\sqrt{ax+bx^3+cx^5}}$$

```
[In] Int[Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(8*c*Sqrt[x]) - ((b^2 - 4*a*c)*Sq
rt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x
^2 + c*x^4])])/(16*c^(3/2)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1932

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1))), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}} - \frac{(b^2-4ac)\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx}{8c} \\
&= \frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}} - \frac{\left((b^2-4ac)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx}{8c\sqrt{ax+bx^3+cx^5}} \\
&= \frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}} - \frac{\left((b^2-4ac)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{16c\sqrt{ax+bx^3+cx^5}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}} \\
&\quad - \frac{\left((b^2-4ac)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8c\sqrt{ax+bx^3+cx^5}} \\
&= \frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}} - \frac{(b^2-4ac)\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}\sqrt{ax+bx^3+cx^5}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$
\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx = \frac{\sqrt{x\,(a+bx^2+cx^4)}\left(\sqrt{c}(b+2cx^2)+\frac{(b^2-4ac)\mathrm{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}-\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}\sqrt{x}}
$$

```
[In] Integrate[Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x*(a + b*x^2 + c*x^4)]*(Sqrt[c]*(b + 2*c*x^2) + ((b^2 - 4*a*c)*ArcTan
h[(Sqrt[c]*x^2)/(Sqrt[a] - Sqrt[a + b*x^2 + c*x^4])])/Sqrt[a + b*x^2 + c*x^
4]))/(8*c^(3/2)*Sqrt[x])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95

method	result
risch	$\frac{(2c\,x^2+b)\,(c\,x^4+b\,x^2+a)\sqrt{x}}{8c\sqrt{x(c\,x^4+b\,x^2+a)}}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{x}}{16c^{\frac{3}{2}}\sqrt{x(c\,x^4+b\,x^2+a)}}$
default	$\frac{\sqrt{x(c\,x^4+b\,x^2+a)}\left(4c^{\frac{3}{2}}x^2\sqrt{c\,x^4+b\,x^2+a}+4\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}+b}{2\sqrt{c}}\right)ac-\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}+b}{2\sqrt{c}}\right)b^2+2b\sqrt{c\,x^4+b\,x^2+a}\right.}{16c^{\frac{3}{2}}\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}}$

```
[In] int(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2*c*x^2+b)*(c*x^4+b*x^2+a)/c*x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)+1/16*(4
*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*(c*x^4+b*
x^2+a)^(1/2)*x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.80

$$\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx$$
$$= \left[-\frac{(b^2-4\,ac)\sqrt{c}x\log\left(-\frac{8\,c^2x^5+8\,bcx^3+4\,\sqrt{cx^5+bx^3+ax}(2\,cx^2+b)\sqrt{c}\sqrt{x}+(b^2+4\,ac)x}{x}\right)-4\,\sqrt{cx^5+bx^3+ax}(2\,c^2x^2+b}{32\,c^2x}\right.$$

```
[In] integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5
+ b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x) - 4*sqrt
(c*x^5 + b*x^3 + a*x)*(2*c^2*x^2 + b*c)*sqrt(x))/(c^2*x), 1/16*((b^2 - 4*a*
c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*s
qrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c^2*x^
2 + b*c)*sqrt(x))/(c^2*x)]
```

Sympy [F]

$$\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx = \int \sqrt{x}\sqrt{x\left(a+bx^2+cx^4\right)}\,dx$$

```
[In] integrate(x**(1/2)*(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx = \int \sqrt{cx^5+bx^3+ax}\sqrt{x}\,dx$$

```
[In] integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.94

$$\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx = \frac{1}{8}\sqrt{cx^4+bx^2+a}\left(2x^2+\frac{b}{c}\right) + \frac{(b^2-4ac)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{16c^{\frac{3}{2}}} - \frac{b^2\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)-4ac\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+2\sqrt{a}b\sqrt{c}}{16c^{\frac{3}{2}}}$$

```
[In] integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + 1/16*(b^2 - 4*a*c)*log(abs(2*(s
qrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(3/2) - 1/16*(b^2*log
(abs(b - 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 2*sq
rt(a)*b*sqrt(c))/c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx = \int \sqrt{x}\,\sqrt{c\,x^5+b\,x^3+a\,x}\,dx$$

```
[In] int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2),x)
[Out] int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.107 $\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}} dx$

Optimal result

Integrand size = 24, antiderivative size = 347

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}} dx = \frac{bx^{3/2}(a+bx^2+cx^4)}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5}$$
$$-\frac{\sqrt[4]{a}b\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{3/4}\sqrt{ax+bx^3+cx^5}}$$
$$+\frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] 1/3*b*x^(3/2)*(c*x^4+b*x^2+a)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))/(c*x^5+b*x^3+a*
x)^(1/2)+1/3*x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2)-1/3*a^(1/4)*b*(cos(2*arctan(c^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2
*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c
^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*
x^5+b*x^3+a*x)^(1/2)+1/6*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))
),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(b+2*a^(1/2)*c^(1/
2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^5+
b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1935, 1967, 1211, 1117, 1209}

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx$$
$$= \frac{\sqrt[4]{a}\sqrt{x}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{ax+bx^3+cx^5}}$$
$$-\frac{\sqrt[4]{a}b\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{3/4}\sqrt{ax+bx^3+cx^5}}$$
$$+\frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5}+\frac{bx^{3/2}(a+bx^2+cx^4)}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$

```
[In] Int[Sqrt[a*x + b*x^3 + c*x^5]/Sqrt[x],x]
```

```
[Out] (b*x^(3/2)*(a + b*x^2 + c*x^4))/(3*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x
 + b*x^3 + c*x^5]) + (Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5])/3 - (a^(1/4)*b*Sqr
t[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
)/(3*c^(3/4)*Sqrt[a*x + b*x^3 + c*x^5]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*
Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/(6*c^(3/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1935

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*(2
*n - q) + 1)), x] + Dist[(n - q)*(p/(m + p*(2*n - q) + 1)), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1967

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)
*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5}+\frac{1}{3}\int\frac{\sqrt{x}(2a+bx^2)}{\sqrt{ax+bx^3+cx^5}}\,dx\\
&= \frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5}+\frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3\sqrt{ax+bx^3+cx^5}}\\
&= \frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5}+\frac{\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int\frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{3\sqrt{ax+bx^3+cx^5}}\\
&\quad-\frac{\left(\sqrt{a}b\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3\sqrt{c}\sqrt{ax+bx^3+cx^5}}
\end{aligned}$$

$$= \frac{bx^{3/2}(a+bx^2+cx^4)}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} + \frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5} - \frac{\sqrt[4]{a}b\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{3/4}\sqrt{ax+bx^3+cx^5}} + \frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{ax+bx^3+cx^5}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx$$

$$= \frac{\sqrt{x}\left(4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(a+bx^2+cx^4)+ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\text{arcsinh}\right.\right.}{}$$

```
[In] Integrate[Sqrt[a*x + b*x^3 + c*x^5]/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) + I*b*(
-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b +
Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2
- 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*S
qrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Elliptic
F[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])]))/(12*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
x*(a + b*x^2 + c*x^4)])
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.24

method	result
risch	$\frac{x^{\frac{3}{2}}\left(c\,x^{4}+b\,x^{2}+a\right)}{3\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}}+\frac{\left(\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{6\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}\right.}{}$
default	$\frac{\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}\left(\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{-4ac+b^{2}}\,c\,x^{5}+\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,bc\,x^{5}+\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{-4ac+b^{2}}\,b\,x^{3}+\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\right.}{}$

```
[In] int((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(3/2)*(c*x^4+b*x^2+a)/(x*(c*x^4+b*x^2+a))^(1/2)+(1/6*a*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*
(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))
/a/c)^(1/2))-1/6*b*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4
*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x
^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Ell
ipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a
*c+b^2)^(1/2))/a/c)^(1/2))))*(c*x^4+b*x^2+a)^(1/2)*x^(1/2)/(x*(c*x^4+b*x^2+
a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx$$
$$=\frac{\sqrt{\frac{1}{2}}\left(bcx^{2}\sqrt{\frac{b^{2}-4\,ac}{c^{2}}}-b^{2}x^{2}\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^{2}-4\,ac}{c^{2}}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^{2}-4\,ac}{c^{2}}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^{2}-4\,ac}{c^{2}}}+b^{2}-2\,ac}{2\,ac})-\sqrt{\frac{1}{2}}\left((bc-\right.}{}$$

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(1/2)*(b*c*x^2*sqrt((b^2 - 4*a*c)/c^2) - b^2*x^2)*sqrt(c)*sqrt((c*
sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b
^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c
)/(a*c)) - sqrt(1/2)*((b*c - 2*c^2)*x^2*sqrt((b^2 - 4*a*c)/c^2) - (b^2 + 2*
```

```
b*c)*x^2)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin
(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 -
 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*sqrt(c*x^5 + b*x^3 + a*x)*(c^2*x^2 +
 b*c)*sqrt(x))/(c^2*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{ax + bx^3 + cx^5}}{\sqrt{x}}\, dx = \int \frac{\sqrt{x\left(a + bx^2 + cx^4\right)}}{\sqrt{x}}\, dx$$

```
[In] integrate((c*x**5+b*x**3+a*x)**(1/2)/x**(1/2),x)
[Out] Integral(sqrt(x*(a + b*x**2 + c*x**4))/sqrt(x), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax + bx^3 + cx^5}}{\sqrt{x}}\, dx = \int \frac{\sqrt{cx^5 + bx^3 + ax}}{\sqrt{x}}\, dx$$

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)
```

Giac [F]

$$\int \frac{\sqrt{ax + bx^3 + cx^5}}{\sqrt{x}}\, dx = \int \frac{\sqrt{cx^5 + bx^3 + ax}}{\sqrt{x}}\, dx$$

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax + bx^3 + cx^5}}{\sqrt{x}}\, dx = \int \frac{\sqrt{c\,x^5 + b\,x^3 + a\,x}}{\sqrt{x}}\, dx$$

```
[In] int((a*x + b*x^3 + c*x^5)^(1/2)/x^(1/2),x)
[Out] int((a*x + b*x^3 + c*x^5)^(1/2)/x^(1/2), x)
```

3.108 $\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 194

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx = \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} - \frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}} + \frac{b\sqrt{x}\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(1/2)*x^(1/2)
*(c*x^4+b*x^2+a)^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)+1/4*b*arctanh(1/2*(2*c*x^2+b
)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(c*x
^5+b*x^3+a*x)^(1/2)+1/2*(c*x^5+b*x^3+a*x)^(1/2)/x^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1935, 1967, 1265, 857, 635, 212, 738}

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx = -\frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}} + \frac{b\sqrt{x}\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\sqrt{ax+bx^3+cx^5}} + \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}$$

```
[In] Int[Sqrt[a*x + b*x^3 + c*x^5]/x^(3/2),x]
```

```
[Out] Sqrt[a*x + b*x^3 + c*x^5]/(2*Sqrt[x]) - (Sqrt[a]*Sqrt[x]*Sqrt[a + b*x^2 + c
*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a
*x + b*x^3 + c*x^5]) + (b*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*
x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*Sqrt[a*x + b*x^3 + c*
x^5])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1935

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*(2
*n - q) + 1)), x] + Dist[(n - q)*(p/(m + p*(2*n - q) + 1)), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
```

```
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1967

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)
*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} + \frac{1}{2}\int \frac{2a+bx^2}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx \\
&= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} + \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{2a+bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx}{2\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} + \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{2a+bx}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} + \frac{\left(a\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2\sqrt{ax+bx^3+cx^5}} \\
&\quad + \frac{\left(b\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} - \frac{\left(a\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{ax+bx^3+cx^5}} \\
&\quad + \frac{\left(b\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} - \frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}} \\
&\quad + \frac{b\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\sqrt{ax+bx^3+cx^5}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx = \frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+4\sqrt{a}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)-b\,\text{lo}\right.}{4\sqrt{c}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

```
[In] Integrate[Sqrt[a*x + b*x^3 + c*x^5]/x^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4] + 4*Sqr
t[a]*Sqrt[c]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]] - b*L
og[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]))/(4*Sqrt[c]*Sqrt[x*(a
+ b*x^2 + c*x^4)])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}\,\left(2\sqrt{a}\,\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)\sqrt{c}-b\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{2\sqrt{c}}\right)-2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}\right)}{4\sqrt{x}\,\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}}$	136

```
[In] int((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(x*(c*x^4+b*x^2+a))^(1/2)*(2*a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*
x^2+a)^(1/2))/x^2)*c^(1/2)-b*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2
)+b)/c^(1/2))-2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)^(1/2
)/c^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 666, normalized size of antiderivative = 3.43

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx = \left[\frac{b\sqrt{c}x\log\left(-\frac{8\,c^2x^5+8\,bcx^3+4\,\sqrt{cx^5+bx^3+ax}\,(2\,cx^2+b)\sqrt{c}\sqrt{x}+(b^2+4\,ac)x}{x}\right)+2\,\sqrt{a}cx\log\left(-\frac{(b^2+4}{}\right.}{8\,cx}\right.$$

$$-\frac{b\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\,(2\,cx^2+b)\sqrt{-c}\sqrt{x}}{2\,(c^2x^5+bcx^3+acx)}\right)-\sqrt{a}cx\log\left(-\frac{(b^2+4\,ac)x^5+8\,abx^3+8\,a^2x-4\,\sqrt{cx^5+bx^3+ax}\,(bx^2+2\,a)\sqrt{a}\sqrt{x}}{x^5}\right)-}{4\,cx}$$

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(b*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)
*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x) + 2*sqrt(a)*c*x*log(-(
(b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^
2 + 2*a)*sqrt(a)*sqrt(x))/x^5) + 4*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*
x), -1/4*(b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*s
qrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) - sqrt(a)*c*x*log(-((b^2 + 4*a
*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*s
qrt(a)*sqrt(x))/x^5) - 2*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*x), 1/8*(4
*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*s
qrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x)) + b*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x
^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a
*c)*x)/x) + 4*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*x), 1/4*(2*sqrt(-a)*c
*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c
*x^5 + a*b*x^3 + a^2*x)) - b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x
)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*sqrt(c*x^
5 + b*x^3 + a*x)*c*sqrt(x))/(c*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx = \int \frac{\sqrt{x\,(a+bx^2+cx^4)}}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*x**5+b*x**3+a*x)**(1/2)/x**(3/2),x)
[Out] Integral(sqrt(x*(a + b*x**2 + c*x**4))/x**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx = \int \frac{\sqrt{cx^5+bx^3+ax}}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/x^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax + bx^3 + cx^5}}{x^{3/2}} \, dx = \text{Exception raised: TypeError}$$

[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax + bx^3 + cx^5}}{x^{3/2}} \, dx = \int \frac{\sqrt{c\,x^5 + b\,x^3 + a\,x}}{x^{3/2}} \, dx$$

[In] int((a*x + b*x^3 + c*x^5)^(1/2)/x^(3/2),x)

[Out] int((a*x + b*x^3 + c*x^5)^(1/2)/x^(3/2), x)

3.109 $\int x^{3/2}(ax+bx^3+cx^5)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 244

$$\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}\,dx=\frac{(15b^4-100ab^2c+128a^2c^2)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}(b(5b^2-4ac)+4c(5b^2-16ac)\,x^2)\sqrt{ax+bx^3+cx^5}}{640c^2}+\frac{\sqrt{x}(3b+8cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{80c}-\frac{3b(b^2-4ac)^2\sqrt{x}\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] 1/80*(8*c*x^2+3*b)*(c*x^5+b*x^3+a*x)^(3/2)*x^(1/2)/c-3/512*b*(-4*a*c+b^2)^2
*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x^(1/2)*(c*x^4+b*x^
2+a)^(1/2)/c^(7/2)/(c*x^5+b*x^3+a*x)^(1/2)-1/640*x^(3/2)*(b*(-4*a*c+5*b^2)+
4*c*(-16*a*c+5*b^2)*x^2)*(c*x^5+b*x^3+a*x)^(1/2)/c^2+1/1280*(128*a^2*c^2-10
0*a*b^2*c+15*b^4)*(c*x^5+b*x^3+a*x)^(1/2)/c^3/x^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used

$= \{1933, 1959, 1963, 12, 1928, 1121, 635, 212\}$

$$\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}\,dx = \frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}} - \frac{3b\sqrt{x}\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}\sqrt{ax+bx^3+cx^5}} - \frac{x^{3/2}\left(4cx^2\left(5b^2-16ac\right)+b\left(5b^2-4ac\right)\right)\sqrt{ax+bx^3+cx^5}}{640c^2} + \frac{\sqrt{x}\left(3b+8cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{80c}$$

```
[In] Int[x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a*x + b*x^3 + c*x^5])/(1280*c^3*
Sqrt[x]) - (x^(3/2)*(b*(5*b^2 - 4*a*c) + 4*c*(5*b^2 - 16*a*c)*x^2)*Sqrt[a*x
 + b*x^3 + c*x^5])/(640*c^2) + (Sqrt[x]*(3*b + 8*c*x^2)*(a*x + b*x^3 + c*x^
5)^(3/2))/(80*c) - (3*b*(b^2 - 4*a*c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*Arc
Tanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(7/2)*Sqrt[
a*x + b*x^3 + c*x^5])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
```

```
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1933

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Dist[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), Int[x^(m - (n - 2*q
))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1959

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
```

```
) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{x}(3b+8cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{80c} + \frac{3\int \sqrt{x}\left(-2ab-(5b^2-16ac)\,x^2\right)\sqrt{ax+bx^3+cx^5}\,dx}{80c}$$

$$= -\frac{x^{3/2}\left(b(5b^2-4ac)+4c(5b^2-16ac)\,x^2\right)\sqrt{ax+bx^3+cx^5}}{640c^2} + \frac{\sqrt{x}(3b+8cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{80c} + \frac{\int \frac{x^{3/2}\left(2ab(5b^2-28ac)+(15b^4-100ab^2c+128a^2c^2)x^2\right)}{\sqrt{ax+bx^3+cx^5}}\,dx}{640c^2}$$

$$= \frac{(15b^4-100ab^2c+128a^2c^2)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}} - \frac{x^{3/2}\left(b(5b^2-4ac)+4c(5b^2-16ac)\,x^2\right)\sqrt{ax+bx^3+cx^5}}{640c^2} + \frac{\sqrt{x}(3b+8cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{80c} - \frac{\int \frac{15b(b^2-4ac)^2x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx}{1280c^3}$$

$$= \frac{(15b^4-100ab^2c+128a^2c^2)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}} - \frac{x^{3/2}\left(b(5b^2-4ac)+4c(5b^2-16ac)\,x^2\right)\sqrt{ax+bx^3+cx^5}}{640c^2} + \frac{\sqrt{x}(3b+8cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{80c} - \frac{\left(3b(b^2-4ac)^2\right)\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx}{256c^3}$$

$$= \frac{(15b^4-100ab^2c+128a^2c^2)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}} - \frac{x^{3/2}\left(b(5b^2-4ac)+4c(5b^2-16ac)\,x^2\right)\sqrt{ax+bx^3+cx^5}}{640c^2} + \frac{\sqrt{x}(3b+8cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{80c} - \frac{\left(3b(b^2-4ac)^2\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx}{256c^3\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{(15b^4 - 100ab^2c + 128a^2c^2)\sqrt{ax + bx^3 + cx^5}}{1280c^3\sqrt{x}} - \frac{x^{3/2}(b(5b^2 - 4ac) + 4c(5b^2 - 16ac)x^2)\sqrt{ax + bx^3 + cx^5}}{640c^2} + \frac{\sqrt{x}(3b + 8cx^2)(ax + bx^3 + cx^5)^{3/2}}{80c} - \frac{\left(3b(b^2 - 4ac)^2\sqrt{x}\sqrt{a + bx^2 + cx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}dx, x, x^2\right)}{512c^3\sqrt{ax + bx^3 + cx^5}}$$

$$= \frac{(15b^4 - 100ab^2c + 128a^2c^2)\sqrt{ax + bx^3 + cx^5}}{1280c^3\sqrt{x}} - \frac{x^{3/2}(b(5b^2 - 4ac) + 4c(5b^2 - 16ac)x^2)\sqrt{ax + bx^3 + cx^5}}{640c^2} + \frac{\sqrt{x}(3b + 8cx^2)(ax + bx^3 + cx^5)^{3/2}}{80c} - \frac{\left(3b(b^2 - 4ac)^2\sqrt{x}\sqrt{a + bx^2 + cx^4}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{256c^3\sqrt{ax + bx^3 + cx^5}}$$

$$= \frac{(15b^4 - 100ab^2c + 128a^2c^2)\sqrt{ax + bx^3 + cx^5}}{1280c^3\sqrt{x}} - \frac{x^{3/2}(b(5b^2 - 4ac) + 4c(5b^2 - 16ac)x^2)\sqrt{ax + bx^3 + cx^5}}{640c^2} + \frac{\sqrt{x}(3b + 8cx^2)(ax + bx^3 + cx^5)^{3/2}}{80c} - \frac{3b(b^2 - 4ac)^2\sqrt{x}\sqrt{a + bx^2 + cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}\sqrt{ax + bx^3 + cx^5}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.74

$$\int x^{3/2}(ax + bx^3 + cx^5)^{3/2}\,dx = \frac{\sqrt{x}\sqrt{a + bx^2 + cx^4}\left(2\sqrt{c}\sqrt{a + bx^2 + cx^4}\left(15b^4 - 10b^3cx^2 + 128c^2(a + cx^4)^2 + 4b^2c(-25a + 2\right.\right.}{2560c^{7/2}\sqrt{x\,(a + b}}$$

```
[In] Integrate[x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(15*b^4
 - 10*b^3*c*x^2 + 128*c^2*(a + c*x^4)^2 + 4*b^2*c*(-25*a + 2*c*x^4) + 8*b*c
^2*x^2*(7*a + 22*c*x^4)) + 15*b*(b^2 - 4*a*c)^2*Log[b + 2*c*x^2 - 2*Sqrt[c]
*Sqrt[a + b*x^2 + c*x^4]]))/(2560*c^(7/2)*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.83

method	result
risch	$\frac{\left(128c^{4}x^{8}+176c^{3}x^{6}b+256a\,c^{3}x^{4}+8b^{2}c^{2}x^{4}+56ab\,c^{2}x^{2}-10x^{2}c\,b^{3}+128a^{2}c^{2}-100a\,b^{2}c+15b^{4}\right)\left(c\,x^{4}+b\,x^{2}+a\right)\sqrt{x}}{1280c^{3}\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}}-\frac{3b\left(16a^{2}c^{2}-8a\,b^{2}c+\right.}{}$
default	$-\frac{\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}\left(-256c^{\frac{9}{2}}x^{8}\sqrt{c\,x^{4}+b\,x^{2}+a}-352b\,c^{\frac{7}{2}}x^{6}\sqrt{c\,x^{4}+b\,x^{2}+a}-512a\,c^{\frac{7}{2}}x^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}-16b^{2}c^{\frac{5}{2}}x^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}-112\right.}{}$

```
[In] int(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1280*(128*c^4*x^8+176*b*c^3*x^6+256*a*c^3*x^4+8*b^2*c^2*x^4+56*a*b*c^2*x^
2-10*b^3*c*x^2+128*a^2*c^2-100*a*b^2*c+15*b^4)*(c*x^4+b*x^2+a)/c^3*x^(1/2)/
(x*(c*x^4+b*x^2+a))^(1/2)-3/512*b*(16*a^2*c^2-8*a*b^2*c+b^4)/c^(7/2)*ln((1/
2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*(c*x^4+b*x^2+a)^(1/2)*x^(1/2)/(x*
(c*x^4+b*x^2+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.62

$$\int x^{3/2}\left(ax+bx^{3}+cx^{5}\right)^{3/2}\,dx=\left[\frac{15\left(b^{5}-8\,ab^{3}c+16\,a^{2}bc^{2}\right)\sqrt{c}x\log\left(-\frac{8\,c^{2}x^{5}+8\,bcx^{3}-4\,\sqrt{cx^{5}+bx^{3}+ax}\left(2\,cx^{2}+b\right)\sqrt{c}\sqrt{x}+\left(b^{2}+4\,ac\right)x}{x}\right)+4\,(12}{}\right.$$

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*
b*c*x^3 - 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2
+ 4*a*c)*x)/x) + 4*(128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2
+ 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2
)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^4*x), 1/2560*(15*(b^5 - 8*a*b^3*c +
 16*a^2*b*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b
)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*(128*c^5*x^8 + 176*b*c^
4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^4
 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^4*
x)]
```

Sympy [F(-1)]

Timed out.

$$\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate(x**(3/2)*(c*x**5+b*x**3+a*x)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}\,dx=\int \left(cx^5+bx^3+ax\right)^{\frac{3}{2}}x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(210) = 420.

Time = 0.45 (sec) , antiderivative size = 640, normalized size of antiderivative = 2.62

$$\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}\,dx=\frac{1}{96}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4x^2+\frac{b}{c}\right)x^2-\frac{3b^2-8ac}{c^2}\right)-\frac{3\left(b^3-4abc\right)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx\right.\right.\right.}{c^{\frac{5}{2}}}\right.$$
$$+\frac{1}{768}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c-12ac^2}{c^3}\right)x^2+\frac{15b^3-52abc}{c^3}\right)+\frac{3\left(5b^4-24ab^2c+\right.}{}\right.$$
$$+\frac{1}{7680}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6\left(8x^2+\frac{b}{c}\right)x^2-\frac{7b^2c^2-16ac^3}{c^4}\right)x^2+\frac{35b^3c-116abc^2}{c^4}\right)x^2-\frac{105b^4-}{}\right.\right.$$

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] 1/96*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2)
 - 3*(b^3 - 4*a*b*c)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt
(c) + b))/c^(5/2) + (3*b^3*log(abs(b - 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(a
bs(b - 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))/c^
(5/2))*a + 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5*b^
2*c - 12*a*c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 3*(5*b^4 - 24*a*b^2*c
 + 16*a^2*c^2)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) +
```

```
b))/c^(7/2) - (15*b^4*log(abs(b - 2*sqrt(a)*sqrt(c))) - 72*a*b^2*c*log(abs(
b - 2*sqrt(a)*sqrt(c))) + 48*a^2*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) + 30*s
qrt(a)*b^3*sqrt(c) - 104*a^(3/2)*b*c^(3/2))/c^(7/2))*b + 1/7680*(2*sqrt(c*x
^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^2*c^2 - 16*a*c^3)/c^4)*x^
2 + (35*b^3*c - 116*a*b*c^2)/c^4)*x^2 - (105*b^4 - 460*a*b^2*c + 256*a^2*c^
2)/c^4) - 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*log(abs(2*(sqrt(c)*x^2 - s
qrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(9/2) + (105*b^5*log(abs(b - 2*sqrt
(a)*sqrt(c))) - 600*a*b^3*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 720*a^2*b*c^2
*log(abs(b - 2*sqrt(a)*sqrt(c))) + 210*sqrt(a)*b^4*sqrt(c) - 920*a^(3/2)*b^
2*c^(3/2) + 512*a^(5/2)*c^(5/2))/c^(9/2))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}\,dx=\int x^{3/2}\,\left(c\,x^5+b\,x^3+a\,x\right)^{3/2}\,dx$$

```
[In] int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x)
[Out] int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2), x)
```

3.110 $\int \sqrt{x}(ax + bx^3 + cx^5)^{3/2}\, dx$

Optimal result

Integrand size = 24, antiderivative size = 487

$$\int \sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}\,dx = \frac{\left(8b^4-57ab^2c+84a^2c^2\right)x^{3/2}(a+bx^2+cx^4)}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$

$$-\frac{\sqrt{x}\left(b(4b^2-9ac)+6c(2b^2-7ac)\,x^2\right)\sqrt{ax+bx^3+cx^5}}{315c^2}+\frac{\left(3b+7cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{63c\sqrt{x}}$$

$$-\frac{\sqrt[4]{a}\left(8b^4-57ab^2c+84a^2c^2\right)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{315c^{11/4}\sqrt{ax+bx^3+cx^5}}$$

$$+\frac{\sqrt[4]{a}\left(8b^4-57ab^2c+84a^2c^2+4\sqrt{a}b\sqrt{c}(b^2-6ac)\right)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{630c^{11/4}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] 1/63*(7*c*x^2+3*b)*(c*x^5+b*x^3+a*x)^(3/2)/c/x^(1/2)+1/315*(84*a^2*c^2-57*a
*b^2*c+8*b^4)*x^(3/2)*(c*x^4+b*x^2+a)/c^(5/2)/(a^(1/2)+x^2*c^(1/2))/(c*x^5+
b*x^3+a*x)^(1/2)-1/315*(b*(-9*a*c+4*b^2)+6*c*(-7*a*c+2*b^2)*x^2)*x^(1/2)*(c
*x^5+b*x^3+a*x)^(1/2)/c^2-1/315*a^(1/4)*(84*a^2*c^2-57*a*b^2*c+8*b^4)*(cos(
2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elli
pticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^
(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/
c^(11/4)/(c*x^5+b*x^3+a*x)^(1/2)+1/630*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1
/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(8*b
^4-57*a*b^2*c+84*a^2*c^2+4*b*(-6*a*c+b^2)*a^(1/2)*c^(1/2))*x^(1/2)*((c*x^4+
b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1933, 1959, 1967, 1211, 1117, 1209}

$$\int \sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}\,dx=\frac{\sqrt[4]{a}\sqrt{x}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}(b^2-6ac)+8b^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\,\text{a}\right.}{630c^{11/4}\sqrt{ax+bx^3+cx^5}}$$

$$-\frac{\sqrt[4]{a}\sqrt{x}(84a^2c^2-57ab^2c+8b^4)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{315c^{11/4}\sqrt{ax+bx^3+cx^5}}$$

$$+\frac{x^{3/2}(84a^2c^2-57ab^2c+8b^4)\left(a+bx^2+cx^4\right)}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$

$$-\frac{\sqrt{x}(6cx^2(2b^2-7ac)+b(4b^2-9ac))\sqrt{ax+bx^3+cx^5}}{315c^2}+\frac{\left(3b+7cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{63c\sqrt{x}}$$

```
[In] Int[Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] ((8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*x^(3/2)*(a + b*x^2 + c*x^4))/(315*c^(5/2
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) - (Sqrt[x]*(b*(4*b^2 -
 9*a*c) + 6*c*(2*b^2 - 7*a*c)*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(315*c^2) + (
(3*b + 7*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(63*c*Sqrt[x]) - (a^(1/4)*(8*b
^4 - 57*a*b^2*c + 84*a^2*c^2)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(11/4)*Sqrt[a*x + b*x^3 + c*x^5])
 + (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2 + 4*Sqrt[a]*b*Sqrt[c]*(b^2 - 6
*a*c))*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sq
rt[c]))/4])/(630*c^(11/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
```

```
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1933

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Dist[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), Int[x^(m - (n - 2*q
))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1959

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1967

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n -
```

```
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)
*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(3b+7cx^2)(ax+bx^3+cx^5)^{3/2}}{63c\sqrt{x}} + \frac{\int \frac{(-ab-2(2b^2-7ac)x^2)\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx}{21c} \\
&= -\frac{\sqrt{x}(b(4b^2-9ac)+6c(2b^2-7ac)x^2)\sqrt{ax+bx^3+cx^5}}{315c^2} \\
&\quad + \frac{(3b+7cx^2)(ax+bx^3+cx^5)^{3/2}}{63c\sqrt{x}} + \frac{\int \frac{\sqrt{x}(4ab(b^2-6ac)+(8b^4-57ab^2c+84a^2c^2)x^2)}{\sqrt{ax+bx^3+cx^5}}\,dx}{315c^2} \\
&= -\frac{\sqrt{x}(b(4b^2-9ac)+6c(2b^2-7ac)x^2)\sqrt{ax+bx^3+cx^5}}{315c^2} \\
&\quad + \frac{(3b+7cx^2)(ax+bx^3+cx^5)^{3/2}}{63c\sqrt{x}} \\
&\quad + \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{4ab(b^2-6ac)+(8b^4-57ab^2c+84a^2c^2)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{315c^2\sqrt{ax+bx^3+cx^5}} \\
&= -\frac{\sqrt{x}(b(4b^2-9ac)+6c(2b^2-7ac)x^2)\sqrt{ax+bx^3+cx^5}}{315c^2} \\
&\quad + \frac{(3b+7cx^2)(ax+bx^3+cx^5)^{3/2}}{63c\sqrt{x}} \\
&\quad - \frac{\left(\sqrt{a}(8b^4-57ab^2c+84a^2c^2)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{315c^{5/2}\sqrt{ax+bx^3+cx^5}} \\
&\quad + \frac{\left(\sqrt{a}\left(8b^4-57ab^2c+84a^2c^2+4\sqrt{a}b\sqrt{c}(b^2-6ac)\right)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{315c^{5/2}\sqrt{ax+bx^3+cx^5}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(8b^4-57ab^2c+84a^2c^2)\,x^{3/2}(a+bx^2+cx^4)}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} \\
&\quad - \frac{\sqrt{x}(b(4b^2-9ac)+6c(2b^2-7ac)\,x^2)\sqrt{ax+bx^3+cx^5}}{315c^2} \\
&\quad + \frac{(3b+7cx^2)\,(ax+bx^3+cx^5)^{3/2}}{63c\sqrt{x}} \\
&\quad - \frac{\sqrt[4]{a}(8b^4-57ab^2c+84a^2c^2)\,\sqrt{x}(\sqrt{a}+\sqrt{c}x^2)\,\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{315c^{11/4}\sqrt{ax+bx^3+cx^5}} \\
&\quad + \frac{\sqrt[4]{a}\left(8b^4-57ab^2c+84a^2c^2+4\sqrt{a}b\sqrt{c}(b^2-6ac)\right)\sqrt{x}(\sqrt{a}+\sqrt{c}x^2)\,\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{630c^{11/4}\sqrt{ax+bx^3+cx^5}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.25

$$
\int \sqrt{x}(ax+bx^3+cx^5)^{3/2}\,dx = \frac{\sqrt{x}\left(4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(-4b^4x^2-b^3cx^4+53b^2c^2x^6+85bc^3x^8+35c^4x^{10}+a^2c(24b+77cx^2)+a\right.}{}
$$

```
[In] Integrate[Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (Sqrt[x]*(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-4*b^4*x^2 - b^3*c*x^4 + 5
3*b^2*c^2*x^6 + 85*b*c^3*x^8 + 35*c^4*x^10 + a^2*c*(24*b + 77*c*x^2) + a*(-
4*b^3 + 27*b^2*c*x^2 + 151*b*c^2*x^4 + 112*c^3*x^6)) + I*(8*b^4 - 57*a*b^2*
c + 84*a^2*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b^5 +
65*a*b^3*c - 132*a^2*b*c^2 + 8*b^4*Sqrt[b^2 - 4*a*c] - 57*a*b^2*c*Sqrt[b^2
- 4*a*c] + 84*a^2*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(1260*c^3*Sq
rt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.34

method	result
risch	$\frac{x^{\frac{3}{2}}\left(35c^{3}x^{6}+50b\,c^{2}x^{4}+77a\,c^{2}x^{2}+3b^{2}c\,x^{2}+24abc-4b^{3}\right)\left(c\,x^{4}+b\,x^{2}+a\right)}{315c^{2}\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}}-\frac{\left(-\frac{a\,b^{3}\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}}{\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}\right.}{}$
default	Expression too large to display

```
[In] int((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/315*x^(3/2)*(35*c^3*x^6+50*b*c^2*x^4+77*a*c^2*x^2+3*b^2*c*x^2+24*a*b*c-4*
b^3)/c^2*(c*x^4+b*x^2+a)/(x*(c*x^4+b*x^2+a))^(1/2)-1/315/c^2*(-a*b^3*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF
(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2
)^(1/2))/a/c)^(1/2))+6*c*b*a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4
-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2*(84*a^2*c^2-5
7*a*b^2*c+8*b^4)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Ellip
ticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c
+b^2)^(1/2))/a/c)^(1/2))))*(c*x^4+b*x^2+a)^(1/2)*x^(1/2)/(x*(c*x^4+b*x^2+a)
)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.00

$$\int \sqrt{x}(ax+bx^3+cx^5)^{3/2}\,dx = \frac{\sqrt{\frac{1}{2}}\left(\left(8\,b^4c-57\,ab^2c^2+84\,a^2c^3\right)x^2\sqrt{\frac{b^2-4\,ac}{c^2}}-\left(8\,b^5-57\,ab^3c+84\,a^2bc^2\right)x^2\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E}{}$$

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/630*(sqrt(1/2)*((8*b^4*c - 57*a*b^2*c^2 + 84*a^2*c^3)*x^2*sqrt((b^2 - 4*a
*c)/c^2) - (8*b^5 - 57*a*b^3*c + 84*a^2*b*c^2)*x^2)*sqrt(c)*sqrt((c*sqrt((b
^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*
a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)
) - sqrt(1/2)*((8*b^4*c + 12*(7*a^2 + 2*a*b)*c^3 - (57*a*b^2 + 4*b^3)*c^2)*
x^2*sqrt((b^2 - 4*a*c)/c^2) - (8*b^5 + 12*(7*a^2*b - 2*a*b^2)*c^2 - (57*a*b
^3 - 4*b^4)*c)*x^2)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*ellipti
c_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*s
qrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*(35*c^5*x^8 + 50*b*c^4*x^6
 + 8*b^4*c - 57*a*b^2*c^2 + 84*a^2*c^3 + (3*b^2*c^3 + 77*a*c^4)*x^4 - 4*(b^
3*c^2 - 6*a*b*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^4*x^2)
```

Sympy [F]

$$\int \sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}\,dx=\int \sqrt{x}\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c*x**5+b*x**3+a*x)**(3/2)*x**(1/2),x)
[Out] Integral(sqrt(x)*(x*(a + b*x**2 + c*x**4))**(3/2), x)
```

Maxima [F]

$$\int \sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}\,dx=\int \left(cx^5+bx^3+ax\right)^{\frac{3}{2}}\sqrt{x}\,dx$$

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="maxima")
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)
```

Giac [F]

$$\int \sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}\,dx=\int \left(cx^5+bx^3+ax\right)^{\frac{3}{2}}\sqrt{x}\,dx$$

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="giac")
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}\,dx=\int \sqrt{x}\,\left(c\,x^5+b\,x^3+a\,x\right)^{3/2}\,dx$$

```
[In] int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(3/2),x)
[Out] int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(3/2), x)
```

3.111 $\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 177

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}}\,dx = -\frac{3(b^2-4ac)(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{(b+2cx^2)(ax+bx^3+cx^5)^{3/2}}{16cx^{3/2}} + \frac{3(b^2-4ac)^2\sqrt{x}\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] 1/16*(2*c*x^2+b)*(c*x^5+b*x^3+a*x)^(3/2)/c/x^(3/2)+3/256*(-4*a*c+b^2)^2*arc
tanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x^(1/2)*(c*x^4+b*x^2+a)
^(1/2)/c^(5/2)/(c*x^5+b*x^3+a*x)^(1/2)-3/128*(-4*a*c+b^2)*(2*c*x^2+b)*(c*x^
5+b*x^3+a*x)^(1/2)/c^2/x^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1932, 1928, 1121, 635, 212}

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}}\,dx = \frac{3\sqrt{x}(b^2-4ac)^2\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}\sqrt{ax+bx^3+cx^5}} - \frac{3(b^2-4ac)(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{(b+2cx^2)(ax+bx^3+cx^5)^{3/2}}{16cx^{3/2}}$$

```
[In] Int[(a*x + b*x^3 + c*x^5)^(3/2)/Sqrt[x],x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(128*c^2*Sqrt[x]
) + ((b + 2*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(16*c*x^(3/2)) + (3*(b^2 -
4*a*c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*S
qrt[a + b*x^2 + c*x^4])])/(256*c^(5/2)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1932

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1))), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rubi steps

$$\text{integral} = \frac{(b+2cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} - \frac{(3(b^2-4ac))\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx}{16c}$$

$$
\begin{aligned}
&= -\frac{3(b^2-4ac)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} \\
&\quad + \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} + \frac{\left(3(b^2-4ac)^2\right)\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx}{128c^2} \\
&= -\frac{3(b^2-4ac)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} \\
&\quad + \frac{\left(3(b^2-4ac)^2\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx}{128c^2\sqrt{ax+bx^3+cx^5}} \\
&= -\frac{3(b^2-4ac)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} \\
&\quad + \frac{\left(3(b^2-4ac)^2\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{256c^2\sqrt{ax+bx^3+cx^5}} \\
&= -\frac{3(b^2-4ac)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} \\
&\quad + \frac{\left(3(b^2-4ac)^2\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{128c^2\sqrt{ax+bx^3+cx^5}} \\
&= -\frac{3(b^2-4ac)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} \\
&\quad + \frac{3(b^2-4ac)^2\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}\sqrt{ax+bx^3+cx^5}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.85

$$
\int \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{\sqrt{x}}\,dx = \frac{\left(x(a+bx^2+cx^4)\right)^{3/2}\left(\frac{\sqrt{c}(b+2cx^2)\left(-3b^2+8bcx^2+4c(5a+2cx^4)\right)}{a+bx^2+cx^4} + \frac{3(b^2-4ac)^2\operatorname{arctanh}\left(\frac{\cdots}{-\sqrt{\cdots}}\right.}{(a+bx^2+cx^4)^{3\cdots}}\right.}{128c^{5/2}x^{3/2}}
$$

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^(3/2)/Sqrt[x],x]
```

```
[Out] ((x*(a + b*x^2 + c*x^4))^(3/2)*((Sqrt[c]*(b + 2*c*x^2)*(-3*b^2 + 8*b*c*x^2
+ 4*c*(5*a + 2*c*x^4)))/(a + b*x^2 + c*x^4) + (3*(b^2 - 4*a*c)^2*ArcTanh[(S
qrt[c]*x^2)/(-Sqrt[a] + Sqrt[a + b*x^2 + c*x^4])])/(a + b*x^2 + c*x^4)^(3/2
)))/(128*c^(5/2)*x^(3/2))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.96

method	result
risch	$\frac{\left(16 c^{3} x^{6}+24 b\,c^{2} x^{4}+40 a\,c^{2} x^{2}+2 b^{2} c\,x^{2}+20 a b c -3 b^{3}\right) \left(c\,x^{4}+b\,x^{2}+a \right) \sqrt{x}}{128 c^{2} \sqrt{x \left(c\,x^{4}+b\,x^{2}+a \right)}}+\frac{3 \left(16 a^{2} c^{2}-8 a\,b^{2} c +b^{4}\right) \ln \left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right) \sqrt{c\,x^{4}+}}{256 c^{\frac{5}{2}} \sqrt{x \left(c\,x^{4}+b\,x^{2}+a \right)}}$
default	$\frac{\sqrt{x \left(c\,x^{4}+b\,x^{2}+a \right)}\, \left(32 c^{\frac{7}{2}} x^{6} \sqrt{c\,x^{4}+b\,x^{2}+a}+48 b\,c^{\frac{5}{2}} x^{4} \sqrt{c\,x^{4}+b\,x^{2}+a}+80 a\,c^{\frac{5}{2}} x^{2} \sqrt{c\,x^{4}+b\,x^{2}+a}+4 b^{2} c^{\frac{3}{2}} x^{2} \sqrt{c\,x^{4}+b\,x^{2}+a}+48 \ln \left(\frac{2 c\,x^{2}+}{}\right.\right.}{25}$

[In] int((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/128*(16*c^3*x^6+24*b*c^2*x^4+40*a*c^2*x^2+2*b^2*c*x^2+20*a*b*c-3*b^3)*(c*x^4+b*x^2+a)/c^2*x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)+3/256*(16*a^2*c^2-8*a*b^2*c+b^4)/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*(c*x^4+b*x^2+a)^(1/2)*x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.88

$$\int \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{\sqrt{x}}\,dx = \left[\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{c}x\log\left(-\frac{8\,c^2x^5+8\,bcx^3+4\,\sqrt{cx^5+bx^3+ax}\left(2\,cx^2+b\right)\sqrt{c}\sqrt{x}+\left(b^2+4\,a\right.}{x}\right.}{}\right.$$

$$-\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\left(2\,cx^2+b\right)\sqrt{-c}\sqrt{x}}{2\left(c^2x^5+bcx^3+acx\right)}\right)-2\left(16\,c^4x^6+24\,bc^3x^4-3\,b^3c+20\,abc^2\right.}{256\,c^3x}$$

[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="fricas")

[Out] [1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x) + 4*(16*c^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^3*x), -1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) - 2*(16*c^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^3*x)]

Sympy [F]

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}}\,dx = \int \frac{(x(a+bx^2+cx^4))^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

```
[In] integrate((c*x**5+b*x**3+a*x)**(3/2)/x**(1/2),x)
[Out] Integral((x*(a + b*x**2 + c*x**4))**(3/2)/sqrt(x), x)
```

Maxima [F]

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}}\,dx = \int \frac{(cx^5+bx^3+ax)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="maxima")
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/sqrt(x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(149) = 298$.

Time = 0.45 (sec) , antiderivative size = 498, normalized size of antiderivative = 2.81

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}}\,dx = \frac{1}{16}\left(2\sqrt{cx^4+bx^2+a}\left(2x^2+\frac{b}{c}\right)+\frac{(b^2-4ac)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+}\right.\right.\right.}{c^{\frac{3}{2}}}$$
$$+\frac{1}{96}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4x^2+\frac{b}{c}\right)x^2-\frac{3b^2-8ac}{c^2}\right)-\frac{3(b^3-4abc)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\right.\right.}{c^{\frac{5}{2}}}\right.$$
$$+\frac{1}{768}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c-12ac^2}{c^3}\right)x^2+\frac{15b^3-52abc}{c^3}\right)+\frac{3(5b^4-24ab^2c+}{}\right.$$

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="giac")
[Out] 1/16*(2*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + (b^2 - 4*a*c)*log(abs(2*(sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(3/2) - (b^2*log(abs(b
 - 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 2*sqrt(a)*
b*sqrt(c))/c^(3/2))*a + 1/96*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^
2 - (3*b^2 - 8*a*c)/c^2) - 3*(b^3 - 4*a*b*c)*log(abs(2*(sqrt(c)*x^2 - sqrt(
c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(5/2) + (3*b^3*log(abs(b - 2*sqrt(a)*sq
rt(c))) - 12*a*b*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^2*sqrt(c)
- 16*a^(3/2)*c^(3/2))/c^(5/2))*b + 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(
```

```
6*x^2 + b/c)*x^2 - (5*b^2*c - 12*a*c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3)
 + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4
+ b*x^2 + a))*sqrt(c) + b))/c^(7/2) - (15*b^4*log(abs(b - 2*sqrt(a)*sqrt(c)
)) - 72*a*b^2*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 48*a^2*c^2*log(abs(b - 2*
sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^3*sqrt(c) - 104*a^(3/2)*b*c^(3/2))/c^(7/2)
)*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax + bx^3 + cx^5)^{3/2}}{\sqrt{x}}\, dx = \int \frac{(c\, x^5 + b\, x^3 + a\, x)^{3/2}}{\sqrt{x}}\, dx$$

```
[In] int((a*x + b*x^3 + c*x^5)^(3/2)/x^(1/2),x)
```

```
[Out] int((a*x + b*x^3 + c*x^5)^(3/2)/x^(1/2), x)
```

3.112 $\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 425

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}}\,dx = -\frac{2b(b^2-8ac)\,x^{3/2}(a+bx^2+cx^4)}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$
$$+\frac{\sqrt{x}(b^2+10ac+3bcx^2)\sqrt{ax+bx^3+cx^5}}{35c}+\frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}}$$
$$+\frac{2\sqrt[4]{a}b(b^2-8ac)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{35c^{7/4}\sqrt{ax+bx^3+cx^5}}$$
$$-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}(b^2-20ac)+2b(b^2-8ac)\right)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{70c^{7/4}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] 1/7*(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2)-2/35*b*(-8*a*c+b^2)*x^(3/2)*(c*x^4+b*x^
2+a)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))/(c*x^5+b*x^3+a*x)^(1/2)+1/35*(3*b*c*x^2+
10*a*c+b^2)*x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2)/c+2/35*a^(1/4)*b*(-8*a*c+b^2)*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))
*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1
/2)/c^(7/4)/(c*x^5+b*x^3+a*x)^(1/2)-1/70*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^
(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2
*b*(-8*a*c+b^2)+(-20*a*c+b^2)*a^(1/2)*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^
(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1935, 1959, 1967, 1211, 1117, 1209}

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}}\,dx =$$
$$-\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}\sqrt{c}(b^2-20ac)+2b(b^2-8ac)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{70c^{7/4}\sqrt{ax+bx^3+cx^5}}$$
$$+\frac{2\sqrt[4]{a}b\sqrt{x}(b^2-8ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{35c^{7/4}\sqrt{ax+bx^3+cx^5}}$$
$$-\frac{2bx^{3/2}(b^2-8ac)\left(a+bx^2+cx^4\right)}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$
$$+\frac{\sqrt{x}(10ac+b^2+3bcx^2)\sqrt{ax+bx^3+cx^5}}{35c}+\frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}}$$

```
[In] Int[(a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2),x]
```

```
[Out] (-2*b*(b^2 - 8*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(35*c^(3/2)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*(b^2 + 10*a*c + 3*b*c*x^2)*
Sqrt[a*x + b*x^3 + c*x^5])/(35*c) + (a*x + b*x^3 + c*x^5)^(3/2)/(7*Sqrt[x])
 + (2*a^(1/4)*b*(b^2 - 8*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(35*c^(7/4)*Sqrt[a*x + b*x^3 + c*x^5]) -
 (a^(1/4)*(Sqrt[a]*Sqrt[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*c))*Sqrt[x]*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(70*c^(
7/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
```

```
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1935

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*(2
*n - q) + 1)), x] + Dist[(n - q)*(p/(m + p*(2*n - q) + 1)), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1959

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1967

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)
*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\text{integral} = \frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}} + \frac{3}{7}\int \frac{(2a+bx^2)\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx$$

$$= \frac{\sqrt{x}(b^2+10ac+3bcx^2)\sqrt{ax+bx^3+cx^5}}{35c} + \frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}} + \frac{\int \frac{\sqrt{x}\left(-a(b^2-20ac)-2b(b^2-8ac)x^2\right)}{\sqrt{ax+bx^3+cx^5}}\,dx}{35c}$$

$$= \frac{\sqrt{x}(b^2+10ac+3bcx^2)\sqrt{ax+bx^3+cx^5}}{35c} + \frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}} + \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{-a(b^2-20ac)-2b(b^2-8ac)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{35c\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{\sqrt{x}(b^2+10ac+3bcx^2)\sqrt{ax+bx^3+cx^5}}{35c} + \frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}} + \frac{\left(2\sqrt{a}b(b^2-8ac)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{35c^{3/2}\sqrt{ax+bx^3+cx^5}} - \frac{\left(\sqrt{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b(b^2-8ac)}{\sqrt{c}}\right)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{35c\sqrt{ax+bx^3+cx^5}}$$

$$= -\frac{2b(b^2-8ac)\,x^{3/2}(a+bx^2+cx^4)}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} + \frac{\sqrt{x}(b^2+10ac+3bcx^2)\sqrt{ax+bx^3+cx^5}}{35c} + \frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}} + \frac{2\sqrt[4]{a}b(b^2-8ac)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{35c^{7/4}\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt[4]{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b(b^2-8ac)}{\sqrt{c}}\right)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right.\right.}{70c^{5/4}\sqrt{ax+bx^3+cx^5}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.27

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}}\,dx = \frac{\sqrt{x}\left(2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(15a^2c+a(b^2+23bcx^2+20c^2x^4)+x^2(b^3+9b^2cx^2+13bc\right.}{}$$

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2),x]
```

```
[Out] (Sqrt[x]*(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(15*a^2*c + a*(b^2 + 23*b*c
*x^2 + 20*c^2*x^4) + x^2*(b^3 + 9*b^2*c*x^2 + 13*b*c^2*x^4 + 5*c^3*x^6)) -
I*b*(b^2 - 8*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/
(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^4 +
9*a*b^2*c - 20*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b
- 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSi
nh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
 Sqrt[b^2 - 4*a*c])]))/(70*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a +
b*x^2 + c*x^4)])
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.46

method	result
risch	$\frac{x^{\frac{3}{2}}(5c^2x^4+8bcx^2+15ac+b^2)(cx^4+bx^2+a)}{35c\sqrt{x(cx^4+bx^2+a)}} + \frac{\left(-\frac{b^2a\sqrt{2}\sqrt{4-\frac{2(-b+\sqrt{-4ac+b^2})x^2}{a}}\sqrt{4+\frac{2(b+\sqrt{-4ac+b^2})x^2}{a}}F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}\right.}{}$
default	Expression too large to display

```
[In] int((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*x^(3/2)*(5*c^2*x^4+8*b*c*x^2+15*a*c+b^2)/c*(c*x^4+b*x^2+a)/(x*(c*x^4+b
*x^2+a))^(1/2)+1/35/c*(-1/4*b^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^
2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+5*c*a^2*2^(1/2
```

```
)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF
(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2
)^(1/2))/a/c)^(1/2))-1/2*(16*a*b*c-2*b^3)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(
1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)
^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))*(c*x^4+b*x^2+a)^(1/2)*x
^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.96

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}}\,dx =$$

$$-\frac{2\sqrt{\frac{1}{2}}\left((b^3c-8\,abc^2)x^2\sqrt{\frac{b^2-4\,ac}{c^2}}-(b^4-8\,ab^2c)x^2\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}}{2\,ac}}{}$$

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] -1/70*(2*sqrt(1/2)*((b^3*c - 8*a*b*c^2)*x^2*sqrt((b^2 - 4*a*c)/c^2) - (b^4
- 8*a*b^2*c)*x^2)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_
e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqr
t((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((2*b^3*c + 20*a*c^3
 - (16*a*b + b^2)*c^2)*x^2*sqrt((b^2 - 4*a*c)/c^2) - (2*b^4 - 20*a*b*c^2 -
(16*a*b^2 - b^3)*c)*x^2)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*el
liptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(
b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 2*(5*c^4*x^6 + 8*b*c^3*
x^4 - 2*b^3*c + 16*a*b*c^2 + (b^2*c^2 + 15*a*c^3)*x^2)*sqrt(c*x^5 + b*x^3 +
 a*x)*sqrt(x))/(c^3*x^2)
```

Sympy [F]

$$\int \frac{(ax + bx^3 + cx^5)^{3/2}}{x^{3/2}}\, dx = \int \frac{(x(a + bx^2 + cx^4))^{\frac{3}{2}}}{x^{\frac{3}{2}}}\, dx$$

```
[In] integrate((c*x**5+b*x**3+a*x)**(3/2)/x**(3/2),x)
[Out] Integral((x*(a + b*x**2 + c*x**4))**(3/2)/x**(3/2), x)
```

Maxima [F]

$$\int \frac{(ax + bx^3 + cx^5)^{3/2}}{x^{3/2}}\, dx = \int \frac{(cx^5 + bx^3 + ax)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\, dx$$

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/x^(3/2), x)
```

Giac [F]

$$\int \frac{(ax + bx^3 + cx^5)^{3/2}}{x^{3/2}}\, dx = \int \frac{(cx^5 + bx^3 + ax)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\, dx$$

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="giac")
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax + bx^3 + cx^5)^{3/2}}{x^{3/2}}\, dx = \int \frac{(c\,x^5 + b\,x^3 + a\,x)^{3/2}}{x^{3/2}}\, dx$$

```
[In] int((a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2),x)
[Out] int((a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2), x)
```

3.113 $\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx=\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\,\mathrm{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] 1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x^(1/2)*(c*x^4+b
*x^2+a)^(1/2)/c^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1928, 1121, 635, 212}

$$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx=\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\,\mathrm{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

```
[In] Int[x^(3/2)/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4])])/(2*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}\\
&= \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2\sqrt{ax+bx^3+cx^5}}\\
&= \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{ax+bx^3+cx^5}}\\
&= \frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{ax+bx^3+cx^5}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$
\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx = -\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{2\sqrt{c}\sqrt{x(a+bx^2+cx^4)}}
$$

```
[In] Integrate[x^(3/2)/Sqrt[a*x + b*x^3 + c*x^5],x]
[Out] -1/2*(Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4]])/(Sqrt[c]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}\,\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{2\sqrt{c}}\right)}{2\sqrt{x}\,\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}}$	72

```
[In] int(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*ln(1/2*(2*c*x^2
+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))/c^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.65

$$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx = \left[\frac{\log\left(-\frac{8\,c^2x^5+8\,bcx^3+4\,\sqrt{cx^5+bx^3+ax}\,(2\,cx^2+b)\sqrt{c}\sqrt{x}+(b^2+4\,ac)x}{x}\right)}{4\,\sqrt{c}},\right.$$
$$\left.-\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\,(2\,cx^2+b)\sqrt{-c}\sqrt{x}}{2\,(c^2x^5+bcx^3+acx)}\right)}{2\,c}\right]$$

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b
)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x)/sqrt(c), -1/2*sqrt(-c)*arctan(1/2*s
qrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3
+ a*c*x))/c]
```

Sympy [F]

$$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

```
[In] integrate(x**(3/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{cx^5+bx^3+ax}}\,dx$$

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(3/2)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.68

$$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx = -\frac{\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{2\sqrt{c}} + \frac{\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)}{2\sqrt{c}}$$

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] -1/2*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/sqrt(c
) + 1/2*log(abs(b - 2*sqrt(a)*sqrt(c)))/sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{x^{3/2}}{\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

```
[In] int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(1/2),x)
[Out] int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.114 $\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 121

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}} \, dx = \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] 1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(
1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^
2)^(1/2)/a^(1/4)/c^(1/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1928, 1117}

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}} \, dx = \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}$$

```
[In] Int[Sqrt[x]/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.08 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx = -\frac{i\sqrt{x}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

```
[In] Integrate[Sqrt[x]/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] ((-I)*Sqrt[x]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*
Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x
^4)])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.46

method	result	si
default	$\frac{\sqrt{x(c\,x^4+b\,x^2+a)}\,\sqrt{-\frac{2\left(\sqrt{-4ac+b^2}\,x^2-b\,x^2-2a\right)}{a}}\,\sqrt{\frac{\sqrt{-4ac+b^2}\,x^2+b\,x^2+2a}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{2}\,\sqrt{\frac{b\sqrt{-4ac+b^2}-2ac+b^2}{ac}}}{2}\right)}{2\sqrt{x}\,(c\,x^4+b\,x^2+a)\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}$	17

```
[In] int(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)/((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2)*(-2*((-4*a*c+b^2)^(1/2)*x^2-b*x^2-2*a)/a)^(1/2)*(1/a*((-4*a*c+
b^2)^(1/2)*x^2+b*x^2+2*a))^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{\sqrt{\frac{1}{2}}\left(c\sqrt{\frac{b^2-4\,ac}{c^2}}+b\right)\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}F(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+b^2-2\,ac}{2\,ac})}{2\,a\sqrt{c}}$$

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*(c*sqrt((b^2 - 4*a*c)/c^2) + b)*sqrt((c*sqrt((b^2 - 4*a*c)/c^
2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)
/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c))/(a*sqrt(c))
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{\sqrt{x}}{\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

```
[In] integrate(x**(1/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx=\int \frac{\sqrt{x}}{\sqrt{cx^5+bx^3+ax}}\,dx$$

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Giac [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx=\int \frac{\sqrt{x}}{\sqrt{cx^5+bx^3+ax}}\,dx$$

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx=\int \frac{\sqrt{x}}{\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

```
[In] int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(1/2),x)
[Out] int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.115 $\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)*x^(1/2)/a^(1/2)/(c*x^5+b*x^3+a*x)^(1/2))/a^(1/
2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1927, 212}

$$\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

```
[In] Int[1/(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]
/Sqrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{\sqrt{x}(2a+bx^2)}{\sqrt{ax+bx^3+cx^5}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx = \frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*
x^4])/Sqrt[a]])/(Sqrt[a]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.41

method	result	size
default	$-\frac{\sqrt{x\left(c\,x^4+b\,x^2+a\right)}\,\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}$	72

```
[In] int(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((2*
a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.69

$$\int \frac{1}{\sqrt{x}\sqrt{ax + bx^3 + cx^5}} \, dx$$
$$= \left[\frac{\log\left(-\frac{(b^2+4\,ac)x^5+8\,abx^3+8\,a^2x-4\,\sqrt{cx^5+bx^3+ax}(bx^2+2\,a)\sqrt{a}\sqrt{x}}{x^5}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}(bx^2+2\,a)\sqrt{-a}\sqrt{x}}{2\,(acx^5+abx^3+a^2x)}\right)}{2\,a} \right]$$

```
[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 +
 a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*
sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3
 + a^2*x))/a]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax + bx^3 + cx^5}} \, dx = \int \frac{1}{\sqrt{x}\sqrt{x\left(a + bx^2 + cx^4\right)}} \, dx$$

```
[In] integrate(1/x**(1/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(x*(a + b*x**2 + c*x**4))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax + bx^3 + cx^5}} \, dx = \int \frac{1}{\sqrt{cx^5 + bx^3 + ax}\sqrt{x}} \, dx$$

```
[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx = \frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

```
[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) - arctan
(sqrt(a)/sqrt(-a))/sqrt(-a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{1}{\sqrt{x}\,\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

```
[In] int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2)),x)
[Out] int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2)), x)
```

3.116 $\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 330

$$\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx = \frac{\sqrt{c}x^{3/2}(a+bx^2+cx^4)}{a\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}} - \frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{ax+bx^3+cx^5}} + \frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] x^(3/2)*(c*x^4+b*x^2+a)*c^(1/2)/a/(a^(1/2)+x^2*c^(1/2))/(c*x^5+b*x^3+a*x)^(
1/2)-(c*x^5+b*x^3+a*x)^(1/2)/a/x^(3/2)-c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1
/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1
/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^5+b*x^3+a*
x)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a
^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1
/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1943, 12, 1928, 1153, 1117, 1209}

$$\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx = \frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{ax+bx^3+cx^5}}$$
$$-\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{ax+bx^3+cx^5}}$$
$$-\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}+\frac{\sqrt{c}x^{3/2}\left(a+bx^2+cx^4\right)}{a\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$

```
[In] Int[1/(x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] (Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x +
 b*x^3 + c*x^5]) - Sqrt[a*x + b*x^3 + c*x^5]/(a*x^(3/2)) - (c^(1/4)*Sqrt[x]
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a
^(3/4)*Sqrt[a*x + b*x^3 + c*x^5]) + (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*Sqrt[a*x + b*x
^3 + c*x^5])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1153

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1943

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - q + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
/(a*(m + p*q + 1))), x] - Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*(b*(m
 + p*q + (n - q)*(p + 1) + 1) + c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ
[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ
[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && LtQ[m + p*q + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}+\frac{\int \frac{cx^{5/2}}{\sqrt{ax+bx^3+cx^5}}\,dx}{a}\\
&= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}+\frac{c\int \frac{x^{5/2}}{\sqrt{ax+bx^3+cx^5}}\,dx}{a}\\
&= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}+\frac{\left(c\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a\sqrt{ax+bx^3+cx^5}}\\
&= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}+\frac{\left(\sqrt{c}\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\sqrt{ax+bx^3+cx^5}}\\
&\quad-\frac{\left(\sqrt{c}\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\sqrt{ax+bx^3+cx^5}}
\end{aligned}
$$

$$= \frac{\sqrt{c}x^{3/2}(a+bx^2+cx^4)}{a\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}} - \frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{ax+bx^3+cx^5}} + \frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{ax+bx^3+cx^5}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.31 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx = \frac{-4(a+bx^2+cx^4) + \frac{i\sqrt{2}\left(-b+\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(E\left(i\text{arcsinh}\left(\right.\right.\right.}{}}{4a\sqrt{x}\sqrt{x\left(a+b\right.}}$$

```
[In] Integrate[1/(x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] (-4*(a + b*x^2 + c*x^4) + (I*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c])*x*Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[x]*S
qrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{cx^4+bx^2+a}{a\sqrt{x}\sqrt{x(cx^4+bx^2+a)}} - \frac{c\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}, \frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4a\right.}}{ac}}}{2}\right.\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\left(b+\sqrt{-4ac+b^2}\right)\sqrt{x(cx^4}}$
default	$\frac{\left(-\sqrt{-4ac+b^2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,cx^4-\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,bcx^4-c\sqrt{-\frac{2\left(\sqrt{-4ac+b^2}\,x^2-bx^2-2a\right)}{a}}\sqrt{\frac{\sqrt{-4ac+b^2}\,x^2+bx^2+2a}{a}}\,axF\left(\frac{x\sqrt{2}\sqrt{}}{}\right.\right.}{}$

```
[In] int(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^4+b*x^2+a)/a/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)-1/2*c*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b
+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))
/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*
(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))*x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/
2)
```

Fricas [F]

$$\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{1}{\sqrt{cx^5+bx^3+ax}\,x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)/(c*x^7 + b*x^5 + a*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

```
[In] integrate(1/x**(3/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(x*(a + b*x**2 + c*x**4))), x)
```

Maxima [F]

$$\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{1}{\sqrt{cx^5+bx^3+ax}\,x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{1}{\sqrt{cx^5+bx^3+ax}x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{1}{x^{3/2}\,\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

```
[In] int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(1/2)),x)
[Out] int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(1/2)), x)
```

3.117 $\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 391

$$\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \frac{x^{3/2}(b^2-2ac+bcx^2)}{a(b^2-4ac)\sqrt{ax+bx^3+cx^5}} - \frac{b\sqrt{c}x^{3/2}(a+bx^2+cx^4)}{a(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} + \frac{b\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}(b^2-4ac)\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{ax+bx^3+cx^5}}$$

```
[Out] x^(3/2)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^5+b*x^3+a*x)^(1/2)-b*x^(3/2
)*(c*x^4+b*x^2+a)*c^(1/2)/a/(-4*a*c+b^2)/(a^(1/2)+x^2*c^(1/2))/(c*x^5+b*x^3
+a*x)^(1/2)+b*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/
a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(
1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(-4*a*c+b^2)/(c*x^5+b*x^3+a*x)^(1/2)-1/2
*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/
a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/
2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1
/2))^2)^(1/2)/a^(3/4)/(b-2*a^(1/2)*c^(1/2))/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1938, 1967, 1211, 1117, 1209}

$$\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \frac{b\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$
$$-\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{ax+bx^3+cx^5}}$$
$$+\frac{x^{3/2}(-2ac+b^2+bcx^2)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{b\sqrt{c}x^{3/2}(a+bx^2+cx^4)}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$

```
[In] Int[x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5
]) - (b*Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(b^2 - 4*a*c)*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (b*c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 - 4*a
*c)*Sqrt[a*x + b*x^3 + c*x^5]) - (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sq
rt[c])*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1938

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n
- q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
+ (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
q] && LtQ[m + p*q + 1, n - q]
```

Rule 1967

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)
*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^{3/2}(b^2-2ac+bcx^2)}{a\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}} - \frac{\int \frac{\sqrt{x}(2ac+bcx^2)}{\sqrt{ax+bx^3+cx^5}}\,dx}{a\,(b^2-4ac)} \\
&= \frac{x^{3/2}(b^2-2ac+bcx^2)}{a\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}} - \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{2ac+bcx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}} \\
&= \frac{x^{3/2}(b^2-2ac+bcx^2)}{a\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}} + \frac{\left(b\sqrt{c}\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}} \\
&\quad - \frac{\left(\left(b+2\sqrt{a}\sqrt{c}\right)\sqrt{c}\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}}
\end{aligned}
$$

$$= \frac{x^{3/2}(b^2-2ac+bcx^2)}{a(b^2-4ac)\sqrt{ax+bx^3+cx^5}} - \frac{b\sqrt{c}x^{3/2}(a+bx^2+cx^4)}{a(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$
$$+ \frac{b\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}(b^2-4ac)\sqrt{ax+bx^3+cx^5}}$$
$$- \frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{ax+bx^3+cx^5}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.66 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.18

$$\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx =$$
$$-\frac{\sqrt{x}\left(-4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(b^2-2ac+bcx^2)+ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\text{arcs}\right.\right.}{}$$

```
[In] Integrate[x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] -1/4*(Sqrt[x]*(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2 - 2*a*c + b*c*x^2)
 + I*b*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x
], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*S
qrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(a*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b
^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.36

method	result
default	$\frac{\sqrt{x(cx^4+bx^2+a)}\left(-\sqrt{-4ac+b^2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}bcx^3-\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}b^2cx^3+c\sqrt{-\frac{2\left(\sqrt{-4ac+b^2}x^2-bx^2-2a\right)}{a}}\sqrt{\frac{\sqrt{-4ac+b^2}x^2+}{a}}\right.}{}$

```
[In] int(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)*(-(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2)*b*c*x^3-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^2*c*x^3+c*(-2*(
(-4*a*c+b^2)^(1/2)*x^2-b*x^2-2*a)/a)^(1/2)*(1/a*((-4*a*c+b^2)^(1/2)*x^2+b*x
^2+2*a))^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/
2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a*(-4*a*c+b^2)^(1/2
)+b*c*(-2*((-4*a*c+b^2)^(1/2)*x^2-b*x^2-2*a)/a)^(1/2)*(1/a*((-4*a*c+b^2)^(1
/2)*x^2+b*x^2+2*a))^(1/2)*a*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))+2*(-4*
a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*c*x-(-4*a*c+b^2)^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^2*x+2*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*
a*b*c*x-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^3*x)/(c*x^4+b*x^2+a)/a/(4*a*c-b
^2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.23

$$\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \frac{\sqrt{\frac{1}{2}}\left(b^2cx^6+b^3x^4+ab^2x^2-(bc^2x^6+b^2cx^4+abcx^2)\sqrt{\frac{b^2-4\,ac}{c^2}}\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E}{}$$

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*(b^2*c*x^6 + b^3*x^4 + a*b^2*x^2 - (b*c^2*x^6 + b^2*c*x^4 +
a*b*c*x^2)*sqrt((b^2 - 4*a*c)/c^2))*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2)
 - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c
)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((
b^2*c + 2*b*c^2)*x^6 + (b^3 + 2*b^2*c)*x^4 + (a*b^2 + 2*a*b*c)*x^2 - ((b*c^
2 - 2*c^3)*x^6 + (b^2*c - 2*b*c^2)*x^4 + (a*b*c - 2*a*c^2)*x^2)*sqrt((b^2 -
 4*a*c)/c^2))*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(ar
csin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b
^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 2*sqrt(c*x^5 + b*x^3 + a*x)*(2*a*c
^2*x^2 + a*b*c)*sqrt(x))/((a*b^2*c^2 - 4*a^2*c^3)*x^6 + (a*b^3*c - 4*a^2*b*
c^2)*x^4 + (a^2*b^2*c - 4*a^3*c^2)*x^2)
```

Sympy [F]

$$\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{(x\,(a+bx^2+cx^4))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**(3/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
[Out] Integral(x**(3/2)/(x*(a + b*x**2 + c*x**4))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{(cx^5+bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^(3/2)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{(cx^5+bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^(3/2)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \int \frac{x^{3/2}}{(c\,x^5+b\,x^3+a\,x)^{3/2}}\,dx$$

```
[In] int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2),x)
[Out] int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2), x)
```

3.118 $\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}\,dx=\frac{\sqrt{x}(b^2-2ac+bcx^2)}{a(b^2-4ac)\sqrt{ax+bx^3+cx^5}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)*x^(1/2)/a^(1/2)/(c*x^5+b*x^3+a*x)^(1/2))/a^(3/
2)+(b*c*x^2-2*a*c+b^2)*x^(1/2)/a/(-4*a*c+b^2)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1936, 1927, 212}

$$\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}\,dx=\frac{\sqrt{x}(-2ac+b^2+bcx^2)}{a(b^2-4ac)\sqrt{ax+bx^3+cx^5}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

```
[In] Int[Sqrt[x]/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5
]) - ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]
/(2*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 1936

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Dist[(2*a*c - b^2*(p + 2))/(a*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(a*x^q
+ b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2
*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0
] && LtQ[p, -1] && RationalQ[m, p, q] && EqQ[m + p*q + 1, (-(n - q))*(2*p +
 3)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{x}(b^2-2ac+bcx^2)}{a\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx}{a} \\
&= \frac{\sqrt{x}(b^2-2ac+bcx^2)}{a\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}} - \frac{\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{\sqrt{x}(2a+bx^2)}{\sqrt{ax+bx^3+cx^5}}\right)}{a} \\
&= \frac{\sqrt{x}(b^2-2ac+bcx^2)}{a\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}\,dx = -\frac{\sqrt{x}\left(\sqrt{a}(b^2-2ac+bcx^2)+(b^2-4ac)\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)\right)}{a^{3/2}\,(-b^2+4ac)\sqrt{x\,(a+bx^2+cx^4)}}$$

```
[In] Integrate[Sqrt[x]/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] -((Sqrt[x]*(Sqrt[a]*(b^2 - 2*a*c + b*c*x^2) + (b^2 - 4*a*c)*Sqrt[a + b*x^2
+ c*x^4]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]]))/(a^(3/2
)*(-b^2 + 4*a*c)*Sqrt[x*(a + b*x^2 + c*x^4)]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(87) = 174$.

Time = 0.06 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}\,\left(2bc\,x^{2}\sqrt{a}+4\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)ac\sqrt{c\,x^{4}+b\,x^{2}+a}-\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)b^{2}\sqrt{c\,x^{4}+b\,x^{2}+}\right.}{2a^{\frac{3}{2}}\sqrt{x}\,\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)}$

```
[In] int(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(x*(c*x^4+b*x^2+a))^(1/2)/a^(3/2)*(2*b*c*x^2*a^(1/2)+4*ln((2*a+b*x^2+2
*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*a*c*(c*x^4+b*x^2+a)^(1/2)-ln((2*a+b*x^
2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*b^2*(c*x^4+b*x^2+a)^(1/2)-4*a^(3/2)
*c+2*b^2*a^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)/(4*a*c-b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(87) = 174$.

Time = 0.31 (sec) , antiderivative size = 424, normalized size of antiderivative = 4.12

$$\int \frac{\sqrt{x}}{\left(ax+bx^{3}+cx^{5}\right)^{3/2}}\,dx = \left[\frac{\left((b^{2}c-4\,ac^{2})x^{5}+(b^{3}-4\,abc)x^{3}+(ab^{2}-4\,a^{2}c)x\right)\sqrt{a}\log\left(-\frac{(b^{2}+4\,ac)x^{5}+8\,abx^{3}+8}{}\right.}{4\left((a^{2}b^{2}c-4\,a^{3}c^{2})x^{5}+(a^{2}b^{3}-\right.}\right.$$

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*x^5 + (b^3 - 4*a*b*c)*x^3 + (a*b^2 - 4*a^2*c)*x)*s
qrt(a)*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3
 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5) + 4*sqrt(c*x^5 + b*x^3 + a*x)*(
a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(x))/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b
^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x), 1/2*(((b^2*c - 4*a*c^2)*x^5 +
 (b^3 - 4*a*b*c)*x^3 + (a*b^2 - 4*a^2*c)*x)*sqrt(-a)*arctan(1/2*sqrt(c*x^5
+ b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x))
+ 2*sqrt(c*x^5 + b*x^3 + a*x)*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(x))/((a^2*
b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
]
```

Sympy [F]

$$\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \int \frac{\sqrt{x}}{\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**(1/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
[Out] Integral(sqrt(x)/(x*(a + b*x**2 + c*x**4))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \int \frac{\sqrt{x}}{(cx^5+bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(x)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(87) = 174.

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \frac{\frac{abcx^2}{a^2b^2-4\,a^3c}+\frac{ab^2-2\,a^2c}{a^2b^2-4\,a^3c}}{\sqrt{cx^4+bx^2+a}} - \frac{ab^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-4\,a^2c\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}b^2-2\,\sqrt{-a}a^{\frac{3}{2}}c}{\sqrt{-a}a^2b^2-4\,\sqrt{-a}a^3c} + \frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}$$

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] (a*b*c*x^2/(a^2*b^2 - 4*a^3*c) + (a*b^2 - 2*a^2*c)/(a^2*b^2 - 4*a^3*c))/sqr
t(c*x^4 + b*x^2 + a) - (a*b^2*arctan(sqrt(a)/sqrt(-a)) - 4*a^2*c*arctan(sqr
t(a)/sqrt(-a)) + sqrt(-a)*sqrt(a)*b^2 - 2*sqrt(-a)*a^(3/2)*c)/(sqrt(-a)*a^2
*b^2 - 4*sqrt(-a)*a^3*c) + arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/
sqrt(-a))/(sqrt(-a)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}\,dx=\int \frac{\sqrt{x}}{\left(c\,x^5+b\,x^3+a\,x\right)^{3/2}}\,dx$$

```
[In] int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(3/2),x)
[Out] int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(3/2), x)
```

3.119 $\int \frac{1}{\sqrt{x}(ax+bx^3+cx^5)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 468

$$\begin{aligned}
\int \frac{1}{\sqrt{x}\,(ax+bx^3+cx^5)^{3/2}}\,dx &= \frac{b^2-2ac+bcx^2}{a\,(b^2-4ac)\,\sqrt{x}\sqrt{ax+bx^3+cx^5}} \\
&+ \frac{2\sqrt{c}(b^2-3ac)\,x^{3/2}(a+bx^2+cx^4)}{a^2\,(b^2-4ac)\,\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} - \frac{2(b^2-3ac)\sqrt{ax+bx^3+cx^5}}{a^2\,(b^2-4ac)\,x^{3/2}} \\
&- \frac{2\sqrt[4]{c}(b^2-3ac)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{7/4}\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}} \\
&+ \frac{\sqrt[4]{c}(2b^2+\sqrt{a}b\sqrt{c}-6ac)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{7/4}\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}}
\end{aligned}$$

```
[Out] 2*(-3*a*c+b^2)*x^(3/2)*(c*x^4+b*x^2+a)*c^(1/2)/a^2/(-4*a*c+b^2)/(a^(1/2)+x^
2*c^(1/2))/(c*x^5+b*x^3+a*x)^(1/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^(1/
2)/(c*x^5+b*x^3+a*x)^(1/2)-2*(-3*a*c+b^2)*(c*x^5+b*x^3+a*x)^(1/2)/a^2/(-4*a
*c+b^2)/x^(3/2)-2*c^(1/4)*(-3*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^
(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*
x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(-4*a*c+b^2)/(c*x^5+b*x
^3+a*x)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(
2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b^2-6*a*c+b*a^(1/2)*c^
(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(-4*
a*c+b^2)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1938, 1965, 1967, 1211, 1117, 1209}

$$\int \frac{1}{\sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx = \frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\right.\right.}{2a^{7/4}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$
$$-\frac{2\sqrt[4]{c}\sqrt{x}(b^2-3ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{7/4}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$
$$-\frac{2(b^2-3ac)\sqrt{ax+bx^3+cx^5}}{a^2x^{3/2}\left(b^2-4ac\right)}+\frac{2\sqrt{c}x^{3/2}(b^2-3ac)\left(a+bx^2+cx^4\right)}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$
$$+\frac{-2ac+b^2+bcx^2}{a\sqrt{x}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

```
[In] Int[1/(Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5])
 + (2*Sqrt[c]*(b^2 - 3*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(a^2*(b^2 - 4*a*c)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) - (2*(b^2 - 3*a*c)*Sqrt
[a*x + b*x^3 + c*x^5])/(a^2*(b^2 - 4*a*c)*x^(3/2)) - (2*c^(1/4)*(b^2 - 3*a*
c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(a^(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5]) + (c^(1/4)*(2*b^2
 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^
3 + c*x^5])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1938

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n
- q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
+ (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
q] && LtQ[m + p*q + 1, n - q]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 1967

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)
*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\text{integral} = \frac{b^2-2ac+bcx^2}{a(b^2-4ac)\sqrt{x}\sqrt{ax+bx^3+cx^5}} - \frac{\int \frac{-2b^2+6ac-bcx^2}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx}{a(b^2-4ac)}$$

$$= \frac{b^2-2ac+bcx^2}{a(b^2-4ac)\sqrt{x}\sqrt{ax+bx^3+cx^5}} - \frac{2(b^2-3ac)\sqrt{ax+bx^3+cx^5}}{a^2(b^2-4ac)x^{3/2}} + \frac{\int \frac{\sqrt{x}(abc+2c(b^2-3ac)x^2)}{\sqrt{ax+bx^3+cx^5}}\,dx}{a^2(b^2-4ac)}$$

$$= \frac{b^2-2ac+bcx^2}{a(b^2-4ac)\sqrt{x}\sqrt{ax+bx^3+cx^5}} - \frac{2(b^2-3ac)\sqrt{ax+bx^3+cx^5}}{a^2(b^2-4ac)x^{3/2}} + \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{abc+2c(b^2-3ac)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a^2(b^2-4ac)\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{b^2-2ac+bcx^2}{a(b^2-4ac)\sqrt{x}\sqrt{ax+bx^3+cx^5}} - \frac{2(b^2-3ac)\sqrt{ax+bx^3+cx^5}}{a^2(b^2-4ac)x^{3/2}} - \frac{\left(2\sqrt{c}(b^2-3ac)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{a^{3/2}(b^2-4ac)\sqrt{ax+bx^3+cx^5}} + \frac{\left(\left(\sqrt{a}bc^{3/2}+2c(b^2-3ac)\right)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{a^{3/2}\sqrt{c}(b^2-4ac)\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{b^2-2ac+bcx^2}{a(b^2-4ac)\sqrt{x}\sqrt{ax+bx^3+cx^5}} + \frac{2\sqrt{c}(b^2-3ac)x^{3/2}(a+bx^2+cx^4)}{a^2(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} - \frac{2(b^2-3ac)\sqrt{ax+bx^3+cx^5}}{a^2(b^2-4ac)x^{3/2}} - \frac{2\sqrt[4]{c}(b^2-3ac)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{7/4}(b^2-4ac)\sqrt{ax+bx^3+cx^5}} + \frac{\sqrt[4]{c}(2b^2+\sqrt{a}b\sqrt{c}-6ac)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{7/4}(b^2-4ac)\sqrt{ax+bx^3+cx^5}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.41 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(-4a^2c+2b^2x^2(b+cx^2)+a(b^2-7bcx^2-6c^2x^4))-i(b^2-3ac)\left(-b+\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{b^2-\ldots}}{b+\sqrt{b\ldots}}}}{\ldots}$$

```
[In] Integrate[1/(Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] -1/2*(2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-4*a^2*c + 2*b^2*x^2*(b + c*x^2) +
 a*(b^2 - 7*b*c*x^2 - 6*c^2*x^4)) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c]
)*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2
*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*Ar
cSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c
*Sqrt[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c
])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sq
rt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a^2*(b^2 - 4*a*c)*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(454) = 908$.

Time = 4.44 (sec) , antiderivative size = 1136, normalized size of antiderivative = 2.43

method	result	size
default	Expression too large to display	1136
risch	Expression too large to display	1497

```
[In] int(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(x*(c*x^4+b*x^2+a))^(1/2)/x^(3/2)*(12*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*a*c^2*x^4-4*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*b^2*c*x^4+12*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b*c^2*x^4-4*((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^3*c*x^4+a*b*c*(-2*((-4*a*c+b^2)^(1/2)*x^2-
b*x^2-2*a)/a)^(1/2)*(1/a*((-4*a*c+b^2)^(1/2)*x^2+b*x^2+2*a))^(1/2)*Elliptic
F(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b
^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*x*(-4*a*c+b^2)^(1/2)+12*(-2*((-4*a*c+b^2)^
(1/2)*x^2-b*x^2-2*a)/a)^(1/2)*(1/a*((-4*a*c+b^2)^(1/2)*x^2+b*x^2+2*a))^(1/2
)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b
*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a^2*c^2*x-3*a*b^2*c*(-2*((-4*a*c
+b^2)^(1/2)*x^2-b*x^2-2*a)/a)^(1/2)*(1/a*((-4*a*c+b^2)^(1/2)*x^2+b*x^2+2*a)
)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/
2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*x-12*(-2*((-4*a*c+b^2)^(1/
2)*x^2-b*x^2-2*a)/a)^(1/2)*(1/a*((-4*a*c+b^2)^(1/2)*x^2+b*x^2+2*a))^(1/2)*E
llipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-
4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a^2*c^2*x+4*(-2*((-4*a*c+b^2)^(1/2)
*x^2-b*x^2-2*a)/a)^(1/2)*(1/a*((-4*a*c+b^2)^(1/2)*x^2+b*x^2+2*a))^(1/2)*Ell
ipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*
a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a*b^2*c*x+14*(-4*a*c+b^2)^(1/2)*((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b*c*x^2-4*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^
```

```
2)^(1/2))/a)^(1/2)*b^3*x^2+14*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b^2*c*x^2
-4*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^4*x^2+8*(-4*a*c+b^2)^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2)*a^2*c-2*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*a*b^2+8*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a^2*b*c-2*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*a*b^3)/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a^2/((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3+cx^5)^{3/2}}\,dx=\int \frac{1}{(cx^5+bx^3+ax)^{\frac{3}{2}}\sqrt{x}}\,dx$$

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)/(c^2*x^11 + 2*b*c*x^9 + (b^2 + 2
*a*c)*x^7 + 2*a*b*x^5 + a^2*x^3), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3+cx^5)^{3/2}}\,dx=\int \frac{1}{\sqrt{x}\,(x\,(a+bx^2+cx^4))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c*x**5+b*x**3+a*x)**(3/2)/x**(1/2),x)
[Out] Integral(1/(sqrt(x)*(x*(a + b*x**2 + c*x**4))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3+cx^5)^{3/2}}\,dx=\int \frac{1}{(cx^5+bx^3+ax)^{\frac{3}{2}}\sqrt{x}}\,dx$$

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3+cx^5)^{3/2}}\,dx = \int \frac{1}{(cx^5+bx^3+ax)^{\frac{3}{2}}\sqrt{x}}\,dx$$

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="giac")
[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3+cx^5)^{3/2}}\,dx = \int \frac{1}{\sqrt{x}\,(c\,x^5+b\,x^3+a\,x)^{3/2}}\,dx$$

```
[In] int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x)
[Out] int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(3/2)), x)
```

3.120 $\int \frac{1}{x^{3/2}(ax+bx^3+cx^5)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 154

$$\int \frac{1}{x^{3/2}(ax+bx^3+cx^5)^{3/2}}\,dx = \frac{b^2-2ac+bcx^2}{a(b^2-4ac)\,x^{3/2}\sqrt{ax+bx^3+cx^5}} - \frac{(3b^2-8ac)\sqrt{ax+bx^3+cx^5}}{2a^2(b^2-4ac)\,x^{5/2}} + \frac{3b\,\text{arctanh}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{4a^{5/2}}$$

```
[Out] 3/4*b*arctanh(1/2*(b*x^2+2*a)*x^(1/2)/a^(1/2)/(c*x^5+b*x^3+a*x)^(1/2))/a^(5
/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2)-1/2*
(-8*a*c+3*b^2)*(c*x^5+b*x^3+a*x)^(1/2)/a^2/(-4*a*c+b^2)/x^(5/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1938, 1965, 12, 1927, 212}

$$\int \frac{1}{x^{3/2}(ax+bx^3+cx^5)^{3/2}}\,dx = \frac{3b\,\text{arctanh}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{4a^{5/2}} - \frac{(3b^2-8ac)\sqrt{ax+bx^3+cx^5}}{2a^2x^{5/2}(b^2-4ac)} + \frac{-2ac+b^2+bcx^2}{ax^{3/2}(b^2-4ac)\sqrt{ax+bx^3+cx^5}}$$

```
[In] Int[1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5])
 - ((3*b^2 - 8*a*c)*Sqrt[a*x + b*x^3 + c*x^5])/(2*a^2*(b^2 - 4*a*c)*x^(5/2)
```

```
) + (3*b*ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^
5])])/(4*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 1938

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n
- q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
+ (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
q] && LtQ[m + p*q + 1, n - q]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}}-\frac{\int \frac{-3b^2+8ac-2bcx^2}{x^{5/2}\sqrt{ax+bx^3+cx^5}}\,dx}{a\left(b^2-4ac\right)}$$

$$=\frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}}-\frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}}+\frac{\int -\frac{3b\left(b^2-4ac\right)}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx}{2a^2\left(b^2-4ac\right)}$$

$$=\frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}}-\frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}}-\frac{(3b)\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx}{2a^2}$$

$$=\frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}}-\frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}}+\frac{(3b)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{\sqrt{x}\left(2a+bx^2\right)}{\sqrt{ax+bx^3+cx^5}}\right)}{2a^2}$$

$$=\frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}}-\frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{4a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx=\frac{\sqrt{a}\left(-4a^2c+3b^2x^2\left(b+cx^2\right)+a\left(b^2-10bcx^2-8c^2x^4\right)\right)+3b\left(b^2-4ac\right)x^2\sqrt{a}}{2a^{5/2}\left(-b^2+4ac\right)x^{3/2}\sqrt{x\left(a+bx^2+ca\right.}}$$

```
[In] Integrate[1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] (Sqrt[a]*(-4*a^2*c + 3*b^2*x^2*(b + c*x^2) + a*(b^2 - 10*b*c*x^2 - 8*c^2*x^
4)) + 3*b*(b^2 - 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(Sqrt[c]*x^2 -
Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(2*a^(5/2)*(-b^2 + 4*a*c)*x^(3/2)*Sqrt[x
*(a + b*x^2 + c*x^4)])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.43

method	result
default	$\frac{\sqrt{x(c\,x^4+b\,x^2+a)}\left(-16a^{\frac{3}{2}}c^2x^4+6b^2c\,x^4\sqrt{a}+12\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)abc\,x^2\sqrt{c\,x^4+b\,x^2+a}-3\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x}}{x^2}\right.\right.}{4a^{\frac{5}{2}}x^{\frac{5}{2}}(c\,x^4+b\,x^2+a)(4ac-b^2)}$
risch	$-\frac{c\,x^4+b\,x^2+a}{2a^2x^{\frac{3}{2}}\sqrt{x(c\,x^4+b\,x^2+a)}}+\frac{\left(\frac{b^2c\,x^2}{a^2\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}+\frac{b^3}{4a^2\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}-\frac{2c^2x^2}{a\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}-\frac{3b}{4a^2\sqrt{c\,x^4+b\,x^2+a}}\right.}{\sqrt{x(c\,x^4+b\,x^2+a)}}$

```
[In] int(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(x*(c*x^4+b*x^2+a))^(1/2)/a^(5/2)*(-16*a^(3/2)*c^2*x^4+6*b^2*c*x^4*a^(1
/2)+12*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*a*b*c*x^2*(c*x^4
+b*x^2+a)^(1/2)-3*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*b^3*x
^2*(c*x^4+b*x^2+a)^(1/2)-20*a^(3/2)*b*c*x^2+6*a^(1/2)*b^3*x^2-8*a^(5/2)*c+2
*a^(3/2)*b^2)/x^(5/2)/(c*x^4+b*x^2+a)/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 508, normalized size of antiderivative = 3.30

$$\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx=\left[\frac{3\left((b^3c-4\,abc^2)x^7+(b^4-4\,ab^2c)x^5+(ab^3-4\,a^2bc)x^3\right)\sqrt{a}\log\left(-\frac{(b^2+4\,ac}{}\right.}{8\left((a\right.}\right.$$

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*((b^3*c - 4*a*b*c^2)*x^7 + (b^4 - 4*a*b^2*c)*x^5 + (a*b^3 - 4*a^2*b
*c)*x^3)*sqrt(a)*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x + 4*sqrt(c*x
^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5) - 4*sqrt(c*x^5 + b*x^
3 + a*x)*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a
^2*b*c)*x^2)*sqrt(x))/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*
x^5 + (a^4*b^2 - 4*a^5*c)*x^3), -1/4*(3*((b^3*c - 4*a*b*c^2)*x^7 + (b^4 - 4
*a*b^2*c)*x^5 + (a*b^3 - 4*a^2*b*c)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^5 + b
*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x)) + 2
*sqrt(c*x^5 + b*x^3 + a*x)*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c
 + (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(x))/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3
*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx=\int \frac{1}{x^{\frac{3}{2}}\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**(3/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
[Out] Integral(1/(x**(3/2)*(x*(a + b*x**2 + c*x**4))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx=\int \frac{1}{x^{3/2}\,\left(c\,x^5+b\,x^3+a\,x\right)^{3/2}}\,dx$$

```
[In] int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x)
[Out] int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)), x)
```

3.121 $\int \frac{x^{\frac{3}{2}(-1+n)}}{\left(ax^{-1+n}+bx^n+cx^{1+n}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 51

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{\left(ax^{-1+n}+bx^n+cx^{1+n}\right)^{3/2}}\,dx=-\frac{2x^{\frac{1}{2}(-1+n)}(b+2cx)}{(b^2-4ac)\sqrt{ax^{-1+n}+bx^n+cx^{1+n}}}$$

```
[Out] -2*x^(-1/2+1/2*n)*(2*c*x+b)/(-4*a*c+b^2)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1929}

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{\left(ax^{-1+n}+bx^n+cx^{1+n}\right)^{3/2}}\,dx=-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{(b^2-4ac)\sqrt{ax^{n-1}+bx^n+cx^{n+1}}}$$

```
[In] Int[x^((3*(-1 + n))/2)/(a*x^(-1 + n) + b*x^n + c*x^(1 + n))^(3/2),x]
```

```
[Out] (-2*x^((-1 + n)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^(-1 + n) + b*x^n + c*x^(1 + n)])
```

Rule 1929

```
Int[(x_)^(m_.)/((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(3/2), x_Symbol] :> Simp[-2*x^((n - 1)/2)*((b + 2*c*x)/((b^2 - 4*a*c)*Sqrt[a*x^(n - 1) + b*x^n + c*x^(n + 1)])), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, 3*((n - 1)/2)] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral}=-\frac{2x^{\frac{1}{2}(-1+n)}(b+2cx)}{(b^2-4ac)\sqrt{ax^{-1+n}+bx^n+cx^{1+n}}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}}\,dx = -\frac{2x^{\frac{1}{2}(-1+n)}(b+2cx)}{(b^2-4ac)\sqrt{x^{-1+n}(a+x(b+cx))}}$$

```
[In] Integrate[x^((3*(-1 + n))/2)/(a*x^(-1 + n) + b*x^n + c*x^(1 + n))^(3/2),x]
[Out] (-2*x^((-1 + n)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[x^(-1 + n)*(a + x*(b + c*x))])
```

Maple [F]

$$\int \frac{x^{-\frac{3}{2}+\frac{3n}{2}}}{(a\,x^{-1+n}+b\,x^n+c\,x^{1+n})^{\frac{3}{2}}}dx$$

```
[In] int(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x)
[Out] int(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}}\,dx = -\frac{2\,(2\,cx^2+bx)\sqrt{\frac{(cx^2+bx+a)x^{n+1}}{x^2}}}{(ab^2-4\,a^2c+(b^2c-4\,ac^2)x^2+(b^3-4\,abc)x)x^{\frac{1}{2}\,n+\frac{1}{2}}}$$

```
[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x, algorithm="fricas")
[Out] -2*(2*c*x^2 + b*x)*sqrt((c*x^2 + b*x + a)*x^(n + 1)/x^2)/((a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*x^(1/2*n + 1/2))
```

Sympy [F]

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}}\,dx = \int \frac{x^{\frac{3n}{2}-\frac{3}{2}}}{(ax^{n-1}+bx^n+cx^{n+1})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**(-3/2+3/2*n)/(a*x**(-1+n)+b*x**n+c*x**(1+n))**(3/2),x)
[Out] Integral(x**(3*n/2 - 3/2)/(a*x**(n - 1) + b*x**n + c*x**(n + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}n-\frac{3}{2}}}{(cx^{n+1}+ax^{n-1}+bx^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}n-\frac{3}{2}}}{(cx^{n+1}+ax^{n-1}+bx^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}}\,dx = \int \frac{x^{\frac{3n}{2}-\frac{3}{2}}}{(b\,x^n+a\,x^{n-1}+c\,x^{n+1})^{3/2}}\,dx$$

```
[In] int(x^((3*n)/2 - 3/2)/(b*x^n + a*x^(n - 1) + c*x^(n + 1))^(3/2),x)
```

```
[Out] int(x^((3*n)/2 - 3/2)/(b*x^n + a*x^(n - 1) + c*x^(n + 1))^(3/2), x)
```

3.122 $\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 287

$$\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{2dx^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\mathrm{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}} + \frac{2ex^4\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\mathrm{AppellF1}\left(\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7\sqrt{ax+bx^3+cx^5}}$$

```
[Out] 2/3*d*x^2*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2
/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^
2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)+2/7*e*x^4*AppellF1(
7/4,1/2,1/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(
1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^
(1/2)))^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used

$= \{1968, 1349, 1155, 524\}$

$$\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{2dx^2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}} + \frac{2ex^4\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7\sqrt{ax+bx^3+cx^5}}$$

```
[In] Int[(x*(d + e*x^2))/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*d*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2
 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a*x + b*x^3 + c*x^
5]) + (2*e*x^4*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*Sqrt[a*x + b*x^3
 + c*x^5])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1968

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.))^(p_
)*((A_) + (B_.)*(x_)^(q_)), x_Symbol] :> Dist[(a*x^j + b*x^k + c*x^n)^p/(x^
```

```
(j*p)*(a + b*x^(k - j) + c*x^(2*(k - j)))^p), Int[x^(m + j*p)*(A + B*x^(k -
 j))*(a + b*x^(k - j) + c*x^(2*(k - j)))^p, x], x] /; FreeQ[{a, b, c, A, B,
 j, k, m, p}, x] && EqQ[q, k - j] && EqQ[n, 2*k - j] &&  !IntegerQ[p] && Po
sQ[k - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int\frac{\sqrt{x}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int\left(\frac{d\sqrt{x}}{\sqrt{a+bx^2+cx^4}}+\frac{ex^{5/2}}{\sqrt{a+bx^2+cx^4}}\right)dx}{\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\left(d\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int\frac{\sqrt{x}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}+\frac{\left(e\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int\frac{x^{5/2}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\left(d\sqrt{x}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int\frac{\sqrt{x}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{ax+bx^3+cx^5}} \\
&\quad+\frac{\left(e\sqrt{x}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int\frac{x^{5/2}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{ax+bx^3+cx^5}} \\
&= \frac{2dx^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}} \\
&\quad+\frac{2ex^4\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7\sqrt{ax+bx^3+cx^5}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 11.18 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.83

$$\int\frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\left(7dx^2\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+3ex^4\,\text{AppellF1}\left(\frac{7}{4}\right.\right.}{21\sqrt{x\left(a+bx^2+cx^4\right)}}$$

```
[In] Integrate[(x*(d + e*x^2))/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(7*d*x^2*AppellF1[3
```

```
/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])] + 3*e*x^4*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*Sqrt[x*(a + b*x^
2 + c*x^4)])
```

Maple [F]

$$\int \frac{x(e\,x^2+d)}{\sqrt{c\,x^5+b\,x^3+ax}}dx$$

```
[In] int(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x)
[Out] int(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{(ex^2+d)x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

```
[In] integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*(e*x^2 + d)/(c*x^4 + b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{x(d+ex^2)}{\sqrt{x\,(a+bx^2+cx^4)}}\,dx$$

```
[In] integrate(x*(e*x**2+d)/(c*x**5+b*x**3+a*x)**(1/2),x)
[Out] Integral(x*(d + e*x**2)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{(ex^2+d)x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

```
[In] integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)*x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Giac [F]

$$\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{(ex^2+d)x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

```
[In] integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{x\,(e\,x^2+d)}{\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

```
[In] int((x*(d + e*x^2))/(a*x + b*x^3 + c*x^5)^(1/2),x)
[Out] int((x*(d + e*x^2))/(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.123 $\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 45

$$\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(1/6*x*(-3*x^2+6)*3^(1/2)/(x^6-3*x^4+3*x^2)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1918, 212}

$$\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

```
[In] Int[1/Sqrt[3*x^2 - 3*x^4 + x^6],x]
[Out] -1/2*ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
```

```
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{x(6-3x^2)}{\sqrt{3x^2-3x^4+x^6}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx = \frac{x\sqrt{3-3x^2+x^4}\,\text{arctanh}\left(\frac{x^2-\sqrt{3-3x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{x^2(3-3x^2+x^4)}}$$

```
[In] Integrate[1/Sqrt[3*x^2 - 3*x^4 + x^6],x]
[Out] (x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(x^2 - Sqrt[3 - 3*x^2 + x^4])/Sqrt[3]])/(S
qrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\sqrt{3}\,\text{arctanh}\left(\frac{(x^2-2)\sqrt{3}\,x}{2\sqrt{x^2(x^4-3x^2+3)}}\right)}{6}$	34
trager	$-\frac{\text{RootOf}(\_Z^2-3)\ln\left(\frac{-\text{RootOf}(\_Z^2-3)x^3+2\,\text{RootOf}(\_Z^2-3)x+2\sqrt{x^6-3x^4+3x^2}}{x^3}\right)}{6}$	53
default	$\frac{x\sqrt{x^4-3x^2+3}\,\sqrt{3}\,\text{arctanh}\left(\frac{(x^2-2)\sqrt{3}}{2\sqrt{x^4-3x^2+3}}\right)}{6\sqrt{x^6-3x^4+3x^2}}$	58

```
[In] int(1/(x^6-3*x^4+3*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*3^(1/2)*arctanh(1/2*(x^2-2)*3^(1/2)*x/(x^2*(x^4-3*x^2+3))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(-\frac{3\,x^3+2\,\sqrt{3}(x^3-2\,x)+2\,\sqrt{x^6-3\,x^4+3\,x^2}(\sqrt{3}+2)-6\,x}{x^3}\right)$$

```
[In] integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^
2)*(sqrt(3) + 2) - 6*x)/x^3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx = \int \frac{1}{\sqrt{x^6-3x^4+3x^2}}\,dx$$

```
[In] integrate(1/(x**6-3*x**4+3*x**2)**(1/2),x)
[Out] Integral(1/sqrt(x**6 - 3*x**4 + 3*x**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx = \int \frac{1}{\sqrt{x^6-3\,x^4+3\,x^2}}\,dx$$

```
[In] integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(x^6 - 3*x^4 + 3*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx = \frac{\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3\,x^2+3}\right)-\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3\,x^2+3}\right)}{6\,\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2
+ sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3x^2 - 3x^4 + x^6}}\,dx = \int \frac{1}{\sqrt{x^6 - 3\,x^4 + 3\,x^2}}\,dx$$

```
[In] int(1/(3*x^2 - 3*x^4 + x^6)^(1/2),x)
```

```
[Out] int(1/(3*x^2 - 3*x^4 + x^6)^(1/2), x)
```

3.124 $\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 45

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(1/6*x*(-3*x^2+6)*3^(1/2)/(x^6-3*x^4+3*x^2)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2021, 1918, 212}

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

```
[In] Int[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
[Out] -1/2*ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
```

```
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2021

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{x(6-3x^2)}{\sqrt{3x^2-3x^4+x^6}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}}\,dx = \frac{x\sqrt{3-3x^2+x^4}\,\text{arctanh}\left(\frac{x^2-\sqrt{3-3x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{x^2(3-3x^2+x^4)}}$$

```
[In] Integrate[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
[Out] (x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(x^2 - Sqrt[3 - 3*x^2 + x^4])/Sqrt[3]])/(S
qrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\sqrt{3}\, \operatorname{arctanh}\left(\frac{\left(x^{2}-2\right) \sqrt{3}\, x}{2 \sqrt{x^{2} \left(x^{4}-3 x^{2}+3\right)}}\right)}{6}$	34
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right) \ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right) x^{3}-2 \operatorname{RootOf}\left(\_Z^{2}-3\right) x+2 \sqrt{x^{6}-3 x^{4}+3 x^{2}}}{x^{3}}\right)}{6}$	52
default	$\frac{\sqrt{x^{4}-3 x^{2}+3}\, x \sqrt{3}\, \operatorname{arctanh}\left(\frac{\left(x^{2}-2\right) \sqrt{3}}{2 \sqrt{x^{4}-3 x^{2}+3}}\right)}{6 \sqrt{x^{2} \left(x^{4}-3 x^{2}+3\right)}}$	58

```
[In] int(1/(x^2*(x^4-3*x^2+3))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*3^(1/2)*arctanh(1/2*(x^2-2)*3^(1/2)*x/(x^2*(x^4-3*x^2+3))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{x^2 (3-3x^2+x^4)}}\,dx = \frac{1}{6}\sqrt{3}\log\left(-\frac{3\,x^3+2\,\sqrt{3}(x^3-2\,x)+2\,\sqrt{x^6-3\,x^4+3\,x^2}(\sqrt{3}+2)-6\,x}{x^3}\right)$$

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^
2)*(sqrt(3) + 2) - 6*x)/x^3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^2 (3-3x^2+x^4)}}\,dx = \int \frac{1}{\sqrt{x^2 (x^4-3x^2+3)}}\,dx$$

```
[In] integrate(1/(x**2*(x**4-3*x**2+3))**(1/2),x)
[Out] Integral(1/sqrt(x**2*(x**4 - 3*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx=\int \frac{1}{\sqrt{(x^4-3\,x^2+3)x^2}}\,dx$$

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx = \frac{\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3\,x^2+3}\right)-\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3\,x^2+3}\right)}{6\,\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")
[Out] 1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2
+ sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx=\int \frac{1}{\sqrt{x^2\,\left(x^4-3\,x^2+3\right)}}\,dx$$

```
[In] int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2),x)
[Out] int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2), x)
```

3.125 $\int \frac{1}{\sqrt{1-(1-x^2)^3}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}} \, dx = -\frac{\operatorname{arctanh}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(1/6*x*(-3*x^2+6)*3^(1/2)/(x^6-3*x^4+3*x^2)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2021, 1918, 212}

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}} \, dx = -\frac{\operatorname{arctanh}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

```
[In] Int[1/Sqrt[1 - (1 - x^2)^3],x]
```

```
[Out] -1/2*ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2021

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx \\ &= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{x(6-3x^2)}{\sqrt{3x^2-3x^4+x^6}}\right) \\ &= -\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx = \frac{x\sqrt{3-3x^2+x^4}\,\text{arctanh}\left(\frac{x^2-\sqrt{3-3x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{x^2(3-3x^2+x^4)}}$$

```
[In] Integrate[1/Sqrt[1 - (1 - x^2)^3],x]
[Out] (x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(x^2 - Sqrt[3 - 3*x^2 + x^4])/Sqrt[3]])/(S
qrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(x^{2}-2\right)\sqrt{3}\,x}{2\sqrt{x^{2}\left(x^{4}-3x^{2}+3\right)}}\right)}{6}$	34
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)x^{3}-2\operatorname{RootOf}\left(\_Z^{2}-3\right)x+2\sqrt{x^{6}-3x^{4}+3x^{2}}}{x^{3}}\right)}{6}$	52
default	$\frac{x\sqrt{x^{4}-3x^{2}+3}\,\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(x^{2}-2\right)\sqrt{3}}{2\sqrt{x^{4}-3x^{2}+3}}\right)}{6\sqrt{x^{6}-3x^{4}+3x^{2}}}$	58

```
[In] int(1/(1-(-x^2+1)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*3^(1/2)*arctanh(1/2*(x^2-2)*3^(1/2)*x/(x^2*(x^4-3*x^2+3))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx = \frac{1}{6}\sqrt{3}\log\left(-\frac{3\,x^3+2\sqrt{3}(x^3-2\,x)+2\sqrt{x^6-3\,x^4+3\,x^2}\left(\sqrt{3}+2\right)-6\,x}{x^3}\right)$$

```
[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^
2)*(sqrt(3) + 2) - 6*x)/x^3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx = \int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx$$

```
[In] integrate(1/(1-(-x**2+1)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - (1 - x**2)**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx = \int \frac{1}{\sqrt{(x^2-1)^3+1}}\,dx$$

```
[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt((x^2 - 1)^3 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx = \frac{\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3\,x^2+3}\right)-\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3\,x^2+3}\right)}{6\,\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="giac")
[Out] 1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2
+ sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx = \int \frac{1}{\sqrt{(x^2-1)^3+1}}\,dx$$

```
[In] int(1/((x^2 - 1)^3 + 1)^(1/2),x)
[Out] int(1/((x^2 - 1)^3 + 1)^(1/2), x)
```

3.126 $\int \sqrt{3x^2-3x^4+x^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \sqrt{3x^2-3x^4+x^6}\,dx=-\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x}-\frac{3\sqrt{3x^2-3x^4+x^6}\operatorname{arcsinh}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}$$

```
[Out] -1/8*(-2*x^2+3)*(x^6-3*x^4+3*x^2)^(1/2)/x-3/16*arcsinh(1/3*(-2*x^2+3)*3^(1/
2))*(x^6-3*x^4+3*x^2)^(1/2)/x/(x^4-3*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1917, 1121, 626, 633, 221}

$$\int \sqrt{3x^2-3x^4+x^6}\,dx=-\frac{3\sqrt{x^6-3x^4+3x^2}\operatorname{arcsinh}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}-\frac{\sqrt{x^6-3x^4+3x^2}(3-2x^2)}{8x}$$

```
[In] Int[Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

```
[Out] -1/8*((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/x - (3*Sqrt[3*x^2 - 3*x^4 + x^
6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1917

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{3x^2-3x^4+x^6}\int x\sqrt{3-3x^2+x^4}\,dx}{x\sqrt{3-3x^2+x^4}}\\
&= \frac{\sqrt{3x^2-3x^4+x^6}\text{Subst}\left(\int\sqrt{3-3x+x^2}\,dx,x,x^2\right)}{2x\sqrt{3-3x^2+x^4}}\\
&= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x}+\frac{\left(3\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int\frac{1}{\sqrt{3-3x+x^2}}\,dx,x,x^2\right)}{16x\sqrt{3-3x^2+x^4}}\\
&= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x}+\frac{\left(\sqrt{3}\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,-3+2x^2\right)}{16x\sqrt{3-3x^2+x^4}}\\
&= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x}-\frac{3\sqrt{3x^2-3x^4+x^6}\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \sqrt{3x^2-3x^4+x^6}\,dx$$
$$=\frac{x\left(-18+30x^2-18x^4+4x^6-3\sqrt{3-3x^2+x^4}\log\left(3-2x^2+2\sqrt{3-3x^2+x^4}\right)\right)}{16\sqrt{x^2\left(3-3x^2+x^4\right)}}$$

```
[In] Integrate[Sqrt[3*x^2 - 3*x^4 + x^6],x]
[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 - 3*Sqrt[3 - 3*x^2 + x^4]*Log[3 - 2*x^2 +
 2*Sqrt[3 - 3*x^2 + x^4]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

method	result	size
pseudoelliptic	$\frac{4\sqrt{x^2(x^4-3x^2+3)}\,x^2+3\,\operatorname{arcsinh}\left(\frac{\sqrt{3}\,(2x^2-3)}{3}\right)x-6\sqrt{x^2(x^4-3x^2+3)}}{16x}$	62
trager	$\frac{(2x^2-3)\sqrt{x^6-3x^4+3x^2}}{8x}-\frac{3\ln\left(\frac{-2x^3+2\sqrt{x^6-3x^4+3x^2}+3x}{x}\right)}{16}$	64
risch	$\frac{(2x^2-3)\sqrt{x^2(x^4-3x^2+3)}}{8x}+\frac{3\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,(x^2-\frac{3}{2})}{3}\right)\sqrt{x^2(x^4-3x^2+3)}}{16\sqrt{x^4-3x^2+3}\,x}$	74
default	$\frac{\sqrt{x^6-3x^4+3x^2}\left(4\sqrt{x^4-3x^2+3}\,x^2+3\,\operatorname{arcsinh}\left(\frac{\sqrt{3}\,(2x^2-3)}{3}\right)-6\sqrt{x^4-3x^2+3}\right)}{16x\sqrt{x^4-3x^2+3}}$	81

```
[In] int((x^6-3*x^4+3*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/16/x*(4*(x^2*(x^4-3*x^2+3))^(1/2)*x^2+3*arcsinh(1/3*3^(1/2)*(2*x^2-3))*x-
6*(x^2*(x^4-3*x^2+3))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.81

$$\int \sqrt{3x^2-3x^4+x^6}\,dx = -\frac{12\,x\log\left(-\frac{2\,x^3-3\,x-2\,\sqrt{x^6-3\,x^4+3\,x^2}}{x}\right)-8\,\sqrt{x^6-3\,x^4+3\,x^2}(2\,x^2-3)-9\,x}{64\,x}$$

```
[In] integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x^
6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

Sympy [F]

$$\int \sqrt{3x^2-3x^4+x^6}\,dx = \int \sqrt{x^6-3x^4+3x^2}\,dx$$

```
[In] integrate((x**6-3*x**4+3*x**2)**(1/2),x)
[Out] Integral(sqrt(x**6 - 3*x**4 + 3*x**2), x)
```

Maxima [F]

$$\int \sqrt{3x^2-3x^4+x^6}\,dx = \int \sqrt{x^6-3\,x^4+3\,x^2}\,dx$$

```
[In] integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^6 - 3*x^4 + 3*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

$$\int \sqrt{3x^2-3x^4+x^6}\,dx = \frac{1}{16}\left(2\,\sqrt{x^4-3\,x^2+3}(2\,x^2-3)-3\,\log\left(-2\,x^2+2\,\sqrt{x^4-3\,x^2+3}+3\right)\right)\mathrm{sgn}(x) + \frac{3}{16}\left(2\,\sqrt{3}+\log\left(2\,\sqrt{3}+3\right)\right)\mathrm{sgn}(x)$$

```
[In] integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*x
^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3x^2 - 3x^4 + x^6}\,dx = \int \sqrt{x^6 - 3\,x^4 + 3\,x^2}\,dx$$

```
[In] int((3*x^2 - 3*x^4 + x^6)^(1/2),x)
```

```
[Out] int((3*x^2 - 3*x^4 + x^6)^(1/2), x)
```

3.127 $\int \sqrt{x^2 (3-3x^2+x^4)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \sqrt{x^2 (3-3x^2+x^4)}\,dx = -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} - \frac{3\sqrt{3x^2-3x^4+x^6}\operatorname{arcsinh}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}$$

```
[Out] -1/8*(-2*x^2+3)*(x^6-3*x^4+3*x^2)^(1/2)/x-3/16*arcsinh(1/3*(-2*x^2+3)*3^(1/
2))*(x^6-3*x^4+3*x^2)^(1/2)/x/(x^4-3*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2021, 1917, 1121, 626, 633, 221}

$$\int \sqrt{x^2 (3-3x^2+x^4)}\,dx = -\frac{3\sqrt{x^6-3x^4+3x^2}\operatorname{arcsinh}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}} - \frac{\sqrt{x^6-3x^4+3x^2}(3-2x^2)}{8x}$$

```
[In] Int[Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] -1/8*((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/x - (3*Sqrt[3*x^2 - 3*x^4 + x^
6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1917

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rule 2021

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \sqrt{3x^2-3x^4+x^6}\,dx \\
&= \frac{\sqrt{3x^2-3x^4+x^6}\int x\sqrt{3-3x^2+x^4}\,dx}{x\sqrt{3-3x^2+x^4}} \\
&= \frac{\sqrt{3x^2-3x^4+x^6}\,\text{Subst}\left(\int \sqrt{3-3x+x^2}\,dx,x,x^2\right)}{2x\sqrt{3-3x^2+x^4}} \\
&= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x}+\frac{\left(3\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{3-3x+x^2}}\,dx,x,x^2\right)}{16x\sqrt{3-3x^2+x^4}} \\
&= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x}+\frac{\left(\sqrt{3}\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,-3+2x^2\right)}{16x\sqrt{3-3x^2+x^4}}
\end{aligned}
$$

$$= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} - \frac{3\sqrt{3x^2-3x^4+x^6}\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \sqrt{x^2(3-3x^2+x^4)}\,dx = \frac{x\left(-18+30x^2-18x^4+4x^6-3\sqrt{3-3x^2+x^4}\log\left(3-2x^2+2\sqrt{3-3x^2+x^4}\right)\right)}{16\sqrt{x^2(3-3x^2+x^4)}}$$

```
[In] Integrate[Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 - 3*Sqrt[3 - 3*x^2 + x^4]*Log[3 - 2*x^2 +
 2*Sqrt[3 - 3*x^2 + x^4]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

method	result	size
pseudoelliptic	$\frac{4\sqrt{x^2(x^4-3x^2+3)}\,x^2+3\,\operatorname{arcsinh}\left(\frac{\sqrt{3}\,(2x^2-3)}{3}\right)x-6\sqrt{x^2(x^4-3x^2+3)}}{16x}$	62
trager	$\frac{(2x^2-3)\sqrt{x^6-3x^4+3x^2}}{8x}+\frac{3\ln\left(\frac{2x^3+2\sqrt{x^6-3x^4+3x^2}-3x}{x}\right)}{16}$	64
risch	$\frac{(2x^2-3)\sqrt{x^2(x^4-3x^2+3)}}{8x}+\frac{3\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,(x^2-\frac{3}{2})}{3}\right)\sqrt{x^2(x^4-3x^2+3)}}{16\sqrt{x^4-3x^2+3}\,x}$	74
default	$\frac{\sqrt{x^2(x^4-3x^2+3)}\left(4\sqrt{x^4-3x^2+3}\,x^2+3\,\operatorname{arcsinh}\left(\frac{\sqrt{3}\,(2x^2-3)}{3}\right)-6\sqrt{x^4-3x^2+3}\right)}{16x\sqrt{x^4-3x^2+3}}$	81

```
[In] int((x^2*(x^4-3*x^2+3))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/16/x*(4*(x^2*(x^4-3*x^2+3))^(1/2)*x^2+3*arcsinh(1/3*3^(1/2)*(2*x^2-3))*x-
6*(x^2*(x^4-3*x^2+3))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.81

$$\int \sqrt{x^2 (3-3x^2+x^4)}\, dx = -\frac{12\, x \log\left(-\frac{2\, x^3 - 3\, x - 2\, \sqrt{x^6 - 3\, x^4 + 3\, x^2}}{x}\right) - 8\, \sqrt{x^6 - 3\, x^4 + 3\, x^2} (2\, x^2 - 3) - 9\, x}{64\, x}$$

```
[In] integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")
[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x^
6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

Sympy [F]

$$\int \sqrt{x^2 (3-3x^2+x^4)}\, dx = \int \sqrt{x^2 (x^4 - 3x^2 + 3)}\, dx$$

```
[In] integrate((x**2*(x**4-3*x**2+3))**(1/2),x)
[Out] Integral(sqrt(x**2*(x**4 - 3*x**2 + 3)), x)
```

Maxima [F]

$$\int \sqrt{x^2 (3-3x^2+x^4)}\, dx = \int \sqrt{(x^4 - 3\, x^2 + 3)x^2}\, dx$$

```
[In] integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

$$\begin{aligned}\int \sqrt{x^2 (3-3x^2+x^4)}\, dx &= \frac{1}{16} \left(2\, \sqrt{x^4 - 3\, x^2 + 3} (2\, x^2 - 3) - 3\, \log\left(-2\, x^2 + 2\, \sqrt{x^4 - 3\, x^2 + 3} + 3\right)\right) \mathrm{sgn}(x) \\ &\quad + \frac{3}{16} \left(2\, \sqrt{3} + \log\left(2\, \sqrt{3} + 3\right)\right) \mathrm{sgn}(x)\end{aligned}$$

```
[In] integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*x
^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x^2 (3-3x^2+x^4)} \, dx = \int \sqrt{x^2 \ (x^4-3\,x^2+3)} \, dx$$

```
[In] int((x^2*(x^4 - 3*x^2 + 3))^(1/2),x)
```

```
[Out] int((x^2*(x^4 - 3*x^2 + 3))^(1/2), x)
```

3.128 $\int \sqrt{1-(1-x^2)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 86

$$\int \sqrt{1-(1-x^2)^3}\,dx = -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} - \frac{3\sqrt{3x^2-3x^4+x^6}\,\text{arcsinh}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}$$

```
[Out] -1/8*(-2*x^2+3)*(x^6-3*x^4+3*x^2)^(1/2)/x-3/16*arcsinh(1/3*(-2*x^2+3)*3^(1/
2))*(x^6-3*x^4+3*x^2)^(1/2)/x/(x^4-3*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2021, 1917, 1121, 626, 633, 221}

$$\int \sqrt{1-(1-x^2)^3}\,dx = -\frac{3\sqrt{x^6-3x^4+3x^2}\,\text{arcsinh}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}} - \frac{\sqrt{x^6-3x^4+3x^2}(3-2x^2)}{8x}$$

```
[In] Int[Sqrt[1 - (1 - x^2)^3],x]
```

```
[Out] -1/8*((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/x - (3*Sqrt[3*x^2 - 3*x^4 + x^
6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1917

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rule 2021

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sqrt{3x^2-3x^4+x^6}\,dx \\
&= \frac{\sqrt{3x^2-3x^4+x^6}\int x\sqrt{3-3x^2+x^4}\,dx}{x\sqrt{3-3x^2+x^4}} \\
&= \frac{\sqrt{3x^2-3x^4+x^6}\,\text{Subst}\left(\int \sqrt{3-3x+x^2}\,dx,x,x^2\right)}{2x\sqrt{3-3x^2+x^4}} \\
&= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x}+\frac{\left(3\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{3-3x+x^2}}\,dx,x,x^2\right)}{16x\sqrt{3-3x^2+x^4}} \\
&= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x}+\frac{\left(\sqrt{3}\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,-3+2x^2\right)}{16x\sqrt{3-3x^2+x^4}} \\
&= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x}-\frac{3\sqrt{3x^2-3x^4+x^6}\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \sqrt{1-(1-x^2)^3}\,dx = \frac{x\left(-18+30x^2-18x^4+4x^6-3\sqrt{3-3x^2+x^4}\log\left(3-2x^2+2\sqrt{3-3x^2+x^4}\right)\right)}{16\sqrt{x^2(3-3x^2+x^4)}}$$

```
[In] Integrate[Sqrt[1 - (1 - x^2)^3],x]
```

```
[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 - 3*Sqrt[3 - 3*x^2 + x^4]*Log[3 - 2*x^2 +
 2*Sqrt[3 - 3*x^2 + x^4]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

method	result	size
pseudoelliptic	$\frac{4\sqrt{x^2(x^4-3x^2+3)}\,x^2+3\,\operatorname{arcsinh}\left(\frac{\sqrt{3}\,(2x^2-3)}{3}\right)x-6\sqrt{x^2(x^4-3x^2+3)}}{16x}$	62
trager	$\frac{(2x^2-3)\sqrt{x^6-3x^4+3x^2}}{8x}-\frac{3\ln\left(\frac{-2x^3+2\sqrt{x^6-3x^4+3x^2}+3x}{x}\right)}{16}$	64
risch	$\frac{(2x^2-3)\sqrt{x^2(x^4-3x^2+3)}}{8x}+\frac{3\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,(x^2-\frac{3}{2})}{3}\right)\sqrt{x^2(x^4-3x^2+3)}}{16\sqrt{x^4-3x^2+3}\,x}$	74
default	$\frac{\sqrt{x^6-3x^4+3x^2}\left(4\sqrt{x^4-3x^2+3}\,x^2+3\,\operatorname{arcsinh}\left(\frac{\sqrt{3}\,(2x^2-3)}{3}\right)-6\sqrt{x^4-3x^2+3}\right)}{16x\sqrt{x^4-3x^2+3}}$	81

```
[In] int((1-(-x^2+1)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/x*(4*(x^2*(x^4-3*x^2+3))^(1/2)*x^2+3*arcsinh(1/3*3^(1/2)*(2*x^2-3))*x-
6*(x^2*(x^4-3*x^2+3))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.81

$$\int \sqrt{1-(1-x^2)^3}\,dx = -\frac{12\,x\log\left(-\frac{2\,x^3-3\,x-2\,\sqrt{x^6-3\,x^4+3\,x^2}}{x}\right)-8\,\sqrt{x^6-3\,x^4+3\,x^2}(2\,x^2-3)-9\,x}{64\,x}$$

```
[In] integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="fricas")
[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x^
6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

Sympy [F]

$$\int \sqrt{1-(1-x^2)^3}\,dx = \int \sqrt{1-(1-x^2)^3}\,dx$$

```
[In] integrate((1-(-x**2+1)**3)**(1/2),x)
[Out] Integral(sqrt(1 - (1 - x**2)**3), x)
```

Maxima [F]

$$\int \sqrt{1-(1-x^2)^3}\,dx = \int \sqrt{(x^2-1)^3+1}\,dx$$

```
[In] integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((x^2 - 1)^3 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

$$\int \sqrt{1-(1-x^2)^3}\,dx = \frac{1}{16}\left(2\,\sqrt{x^4-3\,x^2+3}(2\,x^2-3)-3\,\log\left(-2\,x^2+2\,\sqrt{x^4-3\,x^2+3}+3\right)\right)\mathrm{sgn}(x) + \frac{3}{16}\left(2\,\sqrt{3}+\log\left(2\,\sqrt{3}+3\right)\right)\mathrm{sgn}(x)$$

```
[In] integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*x
^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-(1-x^2)^3}\,dx = \int \sqrt{(x^2-1)^3+1}\,dx$$

```
[In] int(((x^2 - 1)^3 + 1)^(1/2),x)
```

```
[Out] int(((x^2 - 1)^3 + 1)^(1/2), x)
```

3.129 $\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 38

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {738, 212}

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

```
[In] Int[1/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/Sqrt[a])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int \frac{1}{4a - x^2}\,dx, x, \frac{2a + bx}{\sqrt{a + bx + cx^2}}\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\sqrt{a + bx + cx^2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{c}x - \sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Integrate[1/(x*Sqrt[a + b*x + c*x^2]),x]
[Out] (2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{\sqrt{a}}$	35

```
[In] int(1/x/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.92

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = \left[\frac{\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{a}+8\,a^2}{x^2}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{-a}}{2\,(acx^2+abx+a^2)}\right)}{a}\right]$$

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a
)*sqrt(a) + 8*a^2)/x^2)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*
(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2))/a]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(1/x/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = \frac{2\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = -\frac{\ln\left(\frac{b}{2}+\frac{a}{x}+\frac{\sqrt{a}\sqrt{c\,x^2+b\,x+a}}{x}\right)}{\sqrt{a}}$$

```
[In] int(1/(x*(a + b*x + c*x^2)^(1/2)),x)
[Out] -log(b/2 + a/x + (a^(1/2)*(a + b*x + c*x^2)^(1/2))/x)/a^(1/2)
```

3.130 $\int \frac{1}{\sqrt{x^2(a+bx+cx^2)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 45

$$\int \frac{1}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2021, 1918, 212}

$$\int \frac{1}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

```
[In] Int[1/Sqrt[x^2*(a + b*x + c*x^2)],x]
[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
```

```
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2021

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx\\ &= -\left(2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.51

$$\int \frac{1}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx = \frac{2x\sqrt{a+x(b+cx)}\,\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

```
[In] Integrate[1/Sqrt[x^2*(a + b*x + c*x^2)],x]
[Out] (2*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt
[a]])/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$\frac{\ln(2)-\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)}{\sqrt{a}}$	42
default	$-\frac{x\sqrt{c\,x^2+bx+a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\sqrt{x^2(c\,x^2+bx+a)}\,\sqrt{a}}$	64

```
[In] int(1/(x^2*(c*x^2+b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (ln(2)-ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.89

$$\int \frac{1}{\sqrt{x^2(a+bx+cx^2)}}\,dx = \left[\frac{\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)}{a}\right]$$

```
[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(a))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^2(a+bx+cx^2)}}\,dx = \int \frac{1}{\sqrt{x^2(a+bx+cx^2)}}\,dx$$

```
[In] integrate(1/(x**2*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^2(a+bx+cx^2)}}\,dx = \int \frac{1}{\sqrt{(cx^2+bx+a)x^2}}\,dx$$

```
[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((c*x^2 + b*x + a)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{x^2 (a + bx + cx^2)}}\, dx = -\frac{2 \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) \operatorname{sgn}(x)}{\sqrt{-a}} + \frac{2 \arctan\left(-\frac{\sqrt{c}x - \sqrt{cx^2 + bx + a}}{\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}(x)}$$

```
[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
[Out] -2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(-(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x^2 (a + bx + cx^2)}}\, dx = \int \frac{1}{\sqrt{x^2\,\left(c\,x^2 + b\,x + a\right)}}\, dx$$

```
[In] int(1/(x^2*(a + b*x + c*x^2))^(1/2),x)
[Out] int(1/(x^2*(a + b*x + c*x^2))^(1/2), x)
```

3.131 $\int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh(1/2*(b*x+2*a)*x^(1/2)/a^(1/2)/(c*x^3+b*x^2+a*x)^(1/2))/a^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2022, 1927, 212}

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

```
[In] Int[1/(Sqrt[x]*Sqrt[x*(a + b*x + c*x^2)]),x]
```

```
[Out] -(ArcTanh[(Sqrt[x]*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x + b*x^2 + c*x^3])]/Sqrt
[a])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 2022

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] &&  !General
izedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{\sqrt{x}\sqrt{ax+bx^2+cx^3}}\,dx \\ &= -\left(2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{\sqrt{x}(2a+bx)}{\sqrt{ax+bx^2+cx^3}}\right)\right) \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.49

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx+cx^2\right)}}\,dx = \frac{2\sqrt{x}\sqrt{a+x(b+cx)}\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x(a+x(b+cx))}}$$

```
[In] Integrate[1/(Sqrt[x]*Sqrt[x*(a + b*x + c*x^2)]),x]
[Out] (2*Sqrt[x]*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)]
)/Sqrt[a]])/(Sqrt[a]*Sqrt[x*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

method	result	size
default	$-\frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\sqrt{x(c\,x^2+bx+a)}\,\sqrt{a}}$	64

```
[In] int(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x^(1/2)/(x*(c*x^2+b*x+a))^(1/2)*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*a+b*x+2*
a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.79

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx+cx^2\right)}}\,dx = \left[\frac{\log\left(\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x-4\,\sqrt{cx^3+bx^2+ax}(bx+2\,a)\sqrt{a}\sqrt{x}}{x^3}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^3+bx^2+ax}(bx+2\,a)\sqrt{-a}\sqrt{x}}{2\,(acx^3+abx^2+a^2x)}\right)}{a}\right]$$

```
[In] integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^3 + b*x^2 +
a*x)*(b*x + 2*a)*sqrt(a)*sqrt(x))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*
x^3 + b*x^2 + a*x)*(b*x + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^3 + a*b*x^2 + a^2*x)
)/a]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx+cx^2\right)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**(1/2)/(x*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx+cx^2\right)}}\,dx = \int \frac{1}{\sqrt{(cx^2+bx+a)x}\sqrt{x}}\,dx$$

```
[In] integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x^2 + b*x + a)*x)*sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx+cx^2\right)}}\,dx=\frac{2\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

```
[In] integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a) - 2*arctan
(sqrt(a)/sqrt(-a))/sqrt(-a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx+cx^2\right)}}\,dx=\int \frac{1}{\sqrt{x}\,\sqrt{x\,\left(c\,x^2+b\,x+a\right)}}\,dx$$

```
[In] int(1/(x^(1/2)*(x*(a + b*x + c*x^2))^(1/2)),x)
[Out] int(1/(x^(1/2)*(x*(a + b*x + c*x^2))^(1/2)), x)
```

3.132 $\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh(1/2*x^(3/2)*(b*x+2*a)/a^(1/2)/(c*x^5+b*x^4+a*x^3)^(1/2))/a^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2022, 1927, 212}

$$\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

```
[In] Int[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)],x]
```

```
[Out] -(ArcTanh[(x^(3/2)*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^4 + c*x^5])]/Sq
rt[a])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 2022

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] &&  !General
izedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sqrt{x}}{\sqrt{ax^3+bx^4+cx^5}}\,dx\\ &= -\left(2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x^{3/2}(2a+bx)}{\sqrt{ax^3+bx^4+cx^5}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx+cx^2\right)}}\,dx = \frac{2x^{3/2}\sqrt{a+x(b+cx)}\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^3(a+x(b+cx))}}$$

```
[In] Integrate[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)],x]
[Out] (2*x^(3/2)*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)]
)/Sqrt[a]])/(Sqrt[a]*Sqrt[x^3*(a + x*(b + c*x))])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

method	result	size
default	$-\frac{x^{\frac{3}{2}}\sqrt{c\,x^{2}+bx+a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{\sqrt{x^{3}(c\,x^{2}+bx+a)}\,\sqrt{a}}$	66

```
[In] int(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(x^3*(c*x^2+b*x+a))^(1/2)*x^(3/2)*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*a+b*
x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.84

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx+cx^2\right)}}\,dx = \left[\frac{\log\left(\frac{8\,abx^3+(b^2+4\,ac)x^4+8\,a^2x^2-4\,\sqrt{cx^5+bx^4+ax^3}(bx+2\,a)\sqrt{a}\sqrt{x}}{x^4}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^5+bx^4+ax^3}(bx+2\,a)\sqrt{-a}\sqrt{x}}{2\,(acx^4+abx^3+a^2x^2)}\right)}{a}\right]$$

```
[In] integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a*b*x^3 + (b^2 + 4*a*c)*x^4 + 8*a^2*x^2 - 4*sqrt(c*x^5 + b*x^4
+ a*x^3)*(b*x + 2*a)*sqrt(a)*sqrt(x))/x^4)/sqrt(a), sqrt(-a)*arctan(1/2*sqr
t(c*x^5 + b*x^4 + a*x^3)*(b*x + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^4 + a*b*x^3 +
a^2*x^2))/a]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx+cx^2\right)}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(1/2)/(x**3*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx+cx^2\right)}}\,dx = \int \frac{\sqrt{x}}{\sqrt{(cx^2+bx+a)x^3}}\,dx$$

```
[In] integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt((c*x^2 + b*x + a)*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{x}}{\sqrt{x^3 (a+bx+cx^2)}}\,dx = \frac{2\left(\frac{\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)}{\operatorname{sgn}(x)}$$

```
[In] integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
[Out] 2*(arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a) - arctan(
sqrt(a)/sqrt(-a))/sqrt(-a))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{x^3 (a+bx+cx^2)}}\,dx = \int \frac{\sqrt{x}}{\sqrt{x^3\,\left(c\,x^2+b\,x+a\right)}}\,dx$$

```
[In] int(x^(1/2)/(x^3*(a + b*x + c*x^2))^(1/2),x)
[Out] int(x^(1/2)/(x^3*(a + b*x + c*x^2))^(1/2), x)
```

3.133 $\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1128, 738, 212}

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[In] Int[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$
\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}
$$

```
[In] Integrate[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
[Out] ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]]/Sqrt[a]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{a}}$	39
elliptic	$-\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{a}}$	39
pseudoelliptic	$-\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{a}}$	39

```
[In] int(1/x/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.82

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = \left[\frac{\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)}{2\,a}\right]$$

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] [1/4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
 + 2*a)*sqrt(a) + 8*a^2)/x^4)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2))/a]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(1/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = \frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\ln\left(\frac{1}{x^2}\right)}{2\sqrt{a}} - \frac{\ln\left(2\,a+2\,\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}+b\,x^2\right)}{2\sqrt{a}}$$

```
[In] int(1/(x*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] - log(1/x^2)/(2*a^(1/2)) - log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2) +
b*x^2)/(2*a^(1/2))
```

3.134 $\int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{1}{\sqrt{x^2\left(a+bx^2+cx^4\right)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh(1/2*x*(b*x^2+2*a)/a^(1/2)/(c*x^6+b*x^4+a*x^2)^(1/2))/a^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2021, 1918, 212}

$$\int \frac{1}{\sqrt{x^2\left(a+bx^2+cx^4\right)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

```
[In] Int[1/Sqrt[x^2*(a + b*x^2 + c*x^4)],x]
```

```
[Out] -1/2*ArcTanh[(x*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^4 + c*x^6])]/Sqr
t[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2021

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{ax^2+bx^4+cx^6}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx^2)}{\sqrt{ax^2+bx^4+cx^6}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \frac{1}{\sqrt{x^2\left(a+bx^2+cx^4\right)}}\,dx = \frac{x\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2\left(a+bx^2+cx^4\right)}}$$

```
[In] Integrate[1/Sqrt[x^2*(a + b*x^2 + c*x^4)],x]
[Out] (x*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/
Sqrt[a]])/(Sqrt[a]*Sqrt[x^2*(a + b*x^2 + c*x^4)])
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.47

method	result	size
default	$-\frac{x\sqrt{c\,x^4+b\,x^2+a}\,\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{x^2\left(c\,x^4+b\,x^2+a\right)}\,\sqrt{a}}$	72

```
[In] int(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(x^2*(c*x^4+b*x^2+a))^(1/2)*x*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((2*a+b*
x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.76

$$\int \frac{1}{\sqrt{x^2\left(a+bx^2+cx^4\right)}}\,dx = \left[\frac{\log\left(-\frac{(b^2+4\,ac)x^5+8\,abx^3+8\,a^2x-4\,\sqrt{cx^6+bx^4+ax^2}\,(bx^2+2\,a)\sqrt{a}}{x^5}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^6+bx^4+ax^2}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^5+abx^3+a^2x)}\right)}{2\,a}\right]$$

```
[In] integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^6 + b*x^4 +
 a*x^2)*(b*x^2 + 2*a)*sqrt(a))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c
*x^6 + b*x^4 + a*x^2)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^5 + a*b*x^3 + a^2*x))/a
]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^2\left(a+bx^2+cx^4\right)}}\,dx = \int \frac{1}{\sqrt{x^2\left(a+bx^2+cx^4\right)}}\,dx$$

```
[In] integrate(1/(x**2*(c*x**4+b*x**2+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^2\left(a+bx^2+cx^4\right)}}\,dx = \int \frac{1}{\sqrt{(cx^4+bx^2+a)x^2}}\,dx$$

```
[In] integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((c*x^4 + b*x^2 + a)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + \frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}(x)}$$

```
[In] integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + arctan(-(sqrt(c)*x^2 - sqrt(c*x
^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}}\,dx = \int \frac{1}{\sqrt{x^2\,\left(c\,x^4+b\,x^2+a\right)}}\,dx$$

```
[In] int(1/(x^2*(a + b*x^2 + c*x^4))^(1/2),x)
```

```
[Out] int(1/(x^2*(a + b*x^2 + c*x^4))^(1/2), x)
```

3.135 $\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx^2+cx^4\right)}} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)*x^(1/2)/a^(1/2)/(c*x^5+b*x^3+a*x)^(1/2))/a^(1/
2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2022, 1927, 212}

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx^2+cx^4\right)}} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

```
[In] Int[1/(Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)]),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]
/Sqrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 2022

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] &&  !General
izedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{\sqrt{x}(2a+bx^2)}{\sqrt{ax+bx^3+cx^5}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}}\,dx = \frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x(a+bx^2+cx^4)}}$$

```
[In] Integrate[1/(Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)]),x]
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*
x^4])/Sqrt[a]])/(Sqrt[a]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.41

method	result	size
default	$-\frac{\sqrt{x}\,\sqrt{c\,x^{4}+b\,x^{2}+a}\,\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2\sqrt{x(c\,x^{4}+b\,x^{2}+a)}\,\sqrt{a}}$	72

```
[In] int(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((2*
a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.69

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}}\,dx$$
$$= \left[\frac{\log\left(-\frac{(b^2+4\,ac)x^5+8\,abx^3+8\,a^2x-4\,\sqrt{cx^5+bx^3+ax}\,(bx^2+2\,a)\sqrt{a}\sqrt{x}}{x^5}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\,(bx^2+2\,a)\sqrt{-a}\sqrt{x}}{2\,(acx^5+abx^3+a^2x)}\right)}{2\,a}\right]$$

```
[In] integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 +
 a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*
sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3
 + a^2*x))/a]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**(1/2)/(x*(c*x**4+b*x**2+a))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}}\,dx = \int \frac{1}{\sqrt{(cx^4+bx^2+a)x}\sqrt{x}}\,dx$$

```
[In] integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x^4 + b*x^2 + a)*x)*sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx = \frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

```
[In] integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")
[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) - arctan
(sqrt(a)/sqrt(-a))/sqrt(-a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx = \int \frac{1}{\sqrt{x}\,\sqrt{x\,\left(c\,x^4+b\,x^2+a\right)}}\,dx$$

```
[In] int(1/(x^(1/2)*(x*(a + b*x^2 + c*x^4))^(1/2)),x)
[Out] int(1/(x^(1/2)*(x*(a + b*x^2 + c*x^4))^(1/2)), x)
```

3.136 $\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx^2+cx^4)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 53

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx^2+cx^4\right)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{x^{3/2}(2a+bx^2)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh(1/2*x^(3/2)*(b*x^2+2*a)/a^(1/2)/(c*x^7+b*x^5+a*x^3)^(1/2))/a^(
1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2022, 1927, 212}

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx^2+cx^4\right)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{x^{3/2}(2a+bx^2)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}$$

```
[In] Int[Sqrt[x]/Sqrt[x^3*(a + b*x^2 + c*x^4)],x]
```

```
[Out] -1/2*ArcTanh[(x^(3/2)*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^5 + c*x^7]
)]/Sqrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 2022

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] &&  !General
izedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sqrt{x}}{\sqrt{ax^3+bx^5+cx^7}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x^{3/2}(2a+bx^2)}{\sqrt{ax^3+bx^5+cx^7}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx^2)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{x}}{\sqrt{x^3\,(a+bx^2+cx^4)}}\,dx = \frac{x^{3/2}\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^3\,(a+bx^2+cx^4)}}$$

```
[In] Integrate[Sqrt[x]/Sqrt[x^3*(a + b*x^2 + c*x^4)],x]
```

```
[Out] (x^(3/2)*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*
x^4])/Sqrt[a]])/(Sqrt[a]*Sqrt[x^3*(a + b*x^2 + c*x^4)])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

method	result	size
default	$-\frac{x^{\frac{3}{2}} \sqrt{c\,x^{4}+b\,x^{2}+a}\, \ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2\sqrt{x^{3}\left(c\,x^{4}+b\,x^{2}+a\right)}\,\sqrt{a}}$	74

```
[In] int(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/(x^3*(c*x^4+b*x^2+a))^(1/2)*x^(3/2)*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((
2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.74

$$\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx^2+cx^4)}}\,dx = \left[\frac{\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^4+8\,a^2x^2-4\,\sqrt{cx^7+bx^5+ax^3}\,(bx^2+2\,a)\sqrt{a}\sqrt{x}}{x^6}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^7+bx^5+ax^3}\,(bx^2+2\,a)\sqrt{-a}\sqrt{x}}{2\,(acx^6+abx^4+a^2x^2)}\right)}{2\,a}\right]$$

```
[In] integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")
[Out] [1/4*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^4 + 8*a^2*x^2 - 4*sqrt(c*x^7 + b*x^5
 + a*x^3)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^6)/sqrt(a), 1/2*sqrt(-a)*arctan(
1/2*sqrt(c*x^7 + b*x^5 + a*x^3)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^6 + a
*b*x^4 + a^2*x^2))/a]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx^2+cx^4)}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(1/2)/(x**3*(c*x**4+b*x**2+a))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx^2+cx^4\right)}}\,dx=\int \frac{\sqrt{x}}{\sqrt{(cx^4+bx^2+a)x^3}}\,dx$$

```
[In] integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x)/sqrt((c*x^4 + b*x^2 + a)*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx^2+cx^4\right)}}\,dx=\frac{\frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}}{\operatorname{sgn}(x)}$$

```
[In] integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")
[Out] (arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) - arcta
n(sqrt(a)/sqrt(-a))/sqrt(-a))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx^2+cx^4\right)}}\,dx=\int \frac{\sqrt{x}}{\sqrt{x^3\,\left(c\,x^4+b\,x^2+a\right)}}\,dx$$

```
[In] int(x^(1/2)/(x^3*(a + b*x^2 + c*x^4))^(1/2),x)
[Out] int(x^(1/2)/(x^3*(a + b*x^2 + c*x^4))^(1/2), x)
```

3.137 $\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 40

$$\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{3-3x^2+x^4}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(1/2*(-x^2+2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1128, 738, 212}

$$\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}}$$

```
[In] Int[1/(x*Sqrt[3 - 3*x^2 + x^4]),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[3]*(2 - x^2))/(2*Sqrt[3 - 3*x^2 + x^4])]/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{3-3x+x^2}}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{3(2-x^2)}{\sqrt{3-3x^2+x^4}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{3-3x^2+x^4}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{x^2-\sqrt{3-3x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Integrate[1/(x*Sqrt[3 - 3*x^2 + x^4]),x]
[Out] ArcTanh[(x^2 - Sqrt[3 - 3*x^2 + x^4])/Sqrt[3]]/Sqrt[3]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

method	result	size
pseudoelliptic	$\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(x^{2}-2\right)\sqrt{3}}{2\sqrt{x^{4}-3x^{2}+3}}\right)}{6}$	29
default	$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(-3x^{2}+6\right)\sqrt{3}}{6\sqrt{x^{4}-3x^{2}+3}}\right)}{6}$	31
elliptic	$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(-3x^{2}+6\right)\sqrt{3}}{6\sqrt{x^{4}-3x^{2}+3}}\right)}{6}$	31
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)x^{2}+2\sqrt{x^{4}-3x^{2}+3}-2\operatorname{RootOf}\left(\_Z^{2}-3\right)}{x^{2}}\right)}{6}$	47

```
[In] int(1/x/(x^4-3*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*3^(1/2)*arctanh(1/2*(x^2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(-\frac{3\,x^2+2\,\sqrt{3}(x^2-2)+2\,\sqrt{x^4-3\,x^2+3}(\sqrt{3}+2)-6}{x^2}\right)$$

```
[In] integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log(-(3*x^2 + 2*sqrt(3)*(x^2 - 2) + 2*sqrt(x^4 - 3*x^2 + 3)*(sq
rt(3) + 2) - 6)/x^2)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx = \int \frac{1}{x\sqrt{x^4-3x^2+3}}\,dx$$

```
[In] integrate(1/x/(x**4-3*x**2+3)**(1/2),x)
[Out] Integral(1/(x*sqrt(x**4 - 3*x**2 + 3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.50

$$\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx = -\frac{1}{6}\,\sqrt{3}\,\mathrm{arsinh}\left(-\sqrt{3}+\frac{2\,\sqrt{3}}{x^2}\right)$$

```
[In] integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
[Out] -1/6*sqrt(3)*arcsinh(-sqrt(3) + 2*sqrt(3)/x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3\,x^2+3}\right) - \frac{1}{6}\,\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3\,x^2+3}\right)$$

```
[In] integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="giac")
[Out] 1/6*sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - 1/6*sqrt(3)*log(-x
^2 + sqrt(3) + sqrt(x^4 - 3*x^2 + 3))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx = -\frac{\sqrt{3}\left(\ln\left(x^2-\frac{2\,\sqrt{3}\,\sqrt{x^4-3\,x^2+3}}{3}-2\right)+\ln\left(\frac{1}{x^2}\right)\right)}{6}$$

```
[In] int(1/(x*(x^4 - 3*x^2 + 3)^(1/2)),x)
[Out] -(3^(1/2)*(log(x^2 - (2*3^(1/2)*(x^4 - 3*x^2 + 3)^(1/2))/3 - 2) + log(1/x^2
)))/6
```

3.138 $\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 45

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(1/6*x*(-3*x^2+6)*3^(1/2)/(x^6-3*x^4+3*x^2)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2021, 1918, 212}

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

```
[In] Int[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] -1/2*ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
```

```
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2021

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx \\ &= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{x(6-3x^2)}{\sqrt{3x^2-3x^4+x^6}}\right) \\ &= -\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}}\,dx = \frac{x\sqrt{3-3x^2+x^4}\,\text{arctanh}\left(\frac{x^2-\sqrt{3-3x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{x^2(3-3x^2+x^4)}}$$

```
[In] Integrate[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
[Out] (x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(x^2 - Sqrt[3 - 3*x^2 + x^4])/Sqrt[3]])/(S
qrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(x^{2}-2\right)\sqrt{3}\,x}{2\sqrt{x^{2}\left(x^{4}-3x^{2}+3\right)}}\right)}{6}$	34
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^{2}-3\right)x^{3}+2\operatorname{RootOf}\left(\_Z^{2}-3\right)x+2\sqrt{x^{6}-3x^{4}+3x^{2}}}{x^{3}}\right)}{6}$	53
default	$\frac{\sqrt{x^{4}-3x^{2}+3}\,x\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(x^{2}-2\right)\sqrt{3}}{2\sqrt{x^{4}-3x^{2}+3}}\right)}{6\sqrt{x^{2}(x^{4}-3x^{2}+3)}}$	58

```
[In] int(1/(x^2*(x^4-3*x^2+3))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*3^(1/2)*arctanh(1/2*(x^2-2)*3^(1/2)*x/(x^2*(x^4-3*x^2+3))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx = \frac{1}{6}\sqrt{3}\log\left(-\frac{3\,x^{3}+2\,\sqrt{3}(x^{3}-2\,x)+2\,\sqrt{x^{6}-3\,x^{4}+3\,x^{2}}(\sqrt{3}+2)-6\,x}{x^{3}}\right)$$

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^
2)*(sqrt(3) + 2) - 6*x)/x^3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx = \int \frac{1}{\sqrt{x^2\left(x^4-3x^2+3\right)}}\,dx$$

```
[In] integrate(1/(x**2*(x**4-3*x**2+3))**(1/2),x)
[Out] Integral(1/sqrt(x**2*(x**4 - 3*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx = \int \frac{1}{\sqrt{(x^4-3\,x^2+3)x^2}}\,dx$$

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx = \frac{\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3\,x^2+3}\right)-\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3\,x^2+3}\right)}{6\,\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")
[Out] 1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2
+ sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx = \int \frac{1}{\sqrt{x^2\,\left(x^4-3\,x^2+3\right)}}\,dx$$

```
[In] int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2),x)
[Out] int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2), x)
```

3.139 $\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{3x-3x^2+x^3}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arctanh(1/2*(2-x)*3^(1/2)*x^(1/2)/(x^3-3*x^2+3*x)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2022, 1927, 212}

$$\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{x^3-3x^2+3x}}\right)}{\sqrt{3}}$$

```
[In] Int[1/(Sqrt[x]*Sqrt[x*(3 - 3*x + x^2)]),x]
```

```
[Out] -(ArcTanh[(Sqrt[3]*(2 - x)*Sqrt[x])/(2*Sqrt[3*x - 3*x^2 + x^3])]/Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
```

```
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 2022

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] &&  !General
izedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{x}\sqrt{3x-3x^2+x^3}}\,dx\\ &= -\left(2\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{(6-3x)\sqrt{x}}{\sqrt{3x-3x^2+x^3}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{3x-3x^2+x^3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}}\,dx = \frac{2\sqrt{x}\sqrt{3-3x+x^2}\text{arctanh}\left(\frac{x-\sqrt{3-3x+x^2}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{x(3-3x+x^2)}}$$

```
[In] Integrate[1/(Sqrt[x]*Sqrt[x*(3 - 3*x + x^2)]),x]
[Out] (2*Sqrt[x]*Sqrt[3 - 3*x + x^2]*ArcTanh[(x - Sqrt[3 - 3*x + x^2])/Sqrt[3]])/
(Sqrt[3]*Sqrt[x*(3 - 3*x + x^2)])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

method	result	size
default	$\frac{\sqrt{x}\,\sqrt{x^2-3x+3}\,\sqrt{3}\,\text{arctanh}\left(\frac{(x-2)\sqrt{3}}{2\sqrt{x^2-3x+3}}\right)}{3\sqrt{x(x^2-3x+3)}}$	50

```
[In] int(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x,method=_RETURNVERBOSE)
```

[Out] 1/3*x^(1/2)/(x*(x^2-3*x+3))^(1/2)*(x^2-3*x+3)^(1/2)*3^(1/2)*arctanh(1/2*(x-2)*3^(1/2)/(x^2-3*x+3)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{7x^3+4\sqrt{3}\sqrt{x^3-3x^2+3x}(x-2)\sqrt{x}-24x^2+24x}{x^3}\right)$$

```
[In] integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((7*x^3 + 4*sqrt(3)*sqrt(x^3 - 3*x^2 + 3*x)*(x - 2)*sqrt(x) - 24*x^2 + 24*x)/x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**(1/2)/(x*(x**2-3*x+3))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}}\,dx = \int \frac{1}{\sqrt{(x^2-3x+3)x}\sqrt{x}}\,dx$$

```
[In] integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((x^2 - 3*x + 3)*x)*sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}}\,dx = \frac{1}{3}\sqrt{3}\log\left(x+\sqrt{3}-\sqrt{x^2-3x+3}\right) - \frac{1}{3}\sqrt{3}\log\left(-x+\sqrt{3}+\sqrt{x^2-3x+3}\right)$$

```
[In] integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(3)*log(x + sqrt(3) - sqrt(x^2 - 3*x + 3)) - 1/3*sqrt(3)*log(-x + s
qrt(3) + sqrt(x^2 - 3*x + 3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}}\,dx = \int \frac{1}{\sqrt{x}\,\sqrt{x\,(x^2-3x+3)}}\,dx$$

```
[In] int(1/(x^(1/2)*(x*(x^2 - 3*x + 3))^(1/2)),x)
[Out] int(1/(x^(1/2)*(x*(x^2 - 3*x + 3))^(1/2)), x)
```

3.140 $\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 70

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{bx^n+cx^{2n-q}+ax^q}}\right)}{\sqrt{a}(n-q)}$$

```
[Out] -arctanh(1/2*x^(1/2*q)*(2*a+b*x^(n-q))/a^(1/2)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1
/2))/(n-q)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1927, 212}

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{ax^q+bx^n+cx^{2n-q}}}\right)}{\sqrt{a}(n-q)}$$

```
[In] Int[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q],x]
```

```
[Out] -(ArcTanh[(x^(q/2)*(2*a + b*x^(n - q)))/(2*Sqrt[a]*Sqrt[b*x^n + c*x^(2*n -
q) + a*x^q])]/(Sqrt[a]*(n - q)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x^{q/2}(2a+bx^{n-q})}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\right)}{n-q}\\ &= -\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{bx^n+cx^{2n-q}+ax^q}}\right)}{\sqrt{a}(n-q)}\end{aligned}$$

Mathematica [F]

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx = \int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx$$

```
[In] Integrate[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q],x]
[Out] Integrate[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q], x]
```

Maple [F]

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{b\,x^n+c\,x^{2n-q}+a\,x^q}}dx$$

```
[In] int(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x)
[Out] int(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="fricas
")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n + cx^{2n-q} + ax^q}}\,dx = \int \frac{x^{\frac{q}{2}-1}}{\sqrt{ax^q + bx^n + cx^{2n-q}}}\,dx$$

```
[In] integrate(x**(-1+1/2*q)/(b*x**n+c*x**(2*n-q)+a*x**q)**(1/2),x)
[Out] Integral(x**(q/2 - 1)/sqrt(a*x**q + b*x**n + c*x**(2*n - q)), x)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n + cx^{2n-q} + ax^q}}\,dx = \int \frac{x^{\frac{1}{2}q-1}}{\sqrt{cx^{2n-q} + bx^n + ax^q}}\,dx$$

```
[In] integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="maxima
")
[Out] integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*x^n + a*x^q), x)
```

Giac [F]

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n + cx^{2n-q} + ax^q}}\,dx = \int \frac{x^{\frac{1}{2}q-1}}{\sqrt{cx^{2n-q} + bx^n + ax^q}}\,dx$$

```
[In] integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="giac")
[Out] integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*x^n + a*x^q), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n + cx^{2n-q} + ax^q}}\,dx = \int \frac{x^{\frac{q}{2}-1}}{\sqrt{b\,x^n + a\,x^q + c\,x^{2\,n-q}}}\,dx$$

```
[In] int(x^(q/2 - 1)/(b*x^n + a*x^q + c*x^(2*n - q))^(1/2),x)
[Out] int(x^(q/2 - 1)/(b*x^n + a*x^q + c*x^(2*n - q))^(1/2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                   end if
               else #result contains complex but optimal is not
                   if debug then
                           print("result contains complex but optimal is not");
                   fi;
                   return "C","Result contains complex when optimal does not.";
               fi;
           else # result do not contain complex
                # this assumes optimal do not as well. No check is needed here.
               if debug then
                      print("result do not contain complex, this assumes optimal do not as well")
               fi;
               if leaf_count_result<=2*leaf_count_optimal then
                   if debug then
                       print("leaf_count_result<=2*leaf_count_optimal");
                   fi;
                   return "A"," ";
               else
                   if debug then
                       print("leaf_count_result>2*leaf_count_optimal");
                   fi;
                   return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
               fi;
           fi;
       else #ExpnType(result) > ExpnType(optimal)
           if debug then
               print("ExpnType(result) > ExpnType(optimal)");
           fi;
           return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
       fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```